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Nº 83

ROBINSON'S AMERICAN ARITHMETIC.

Price 50 Cents.

THE

AMERICAN ARITHMETIC,

IN WHICH THE

PRINCIPLES OF NUMBERS

ARE EXPLAINED AND ILLUSTRATED BY A GREAT
VARIETY OF QUESTIONS.

BY JAMES ROBINSON,

PRINCIPAL OF THE MATHEMATICAL DEPARTMENT OF THE BOWDOIN SCHOOL,
BOSTON.

BOSTON:
PUBLISHED BY JOHN P. JEWETT & CO.
1850.

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PREFACE.

THIS edition of the American Arithmetic has been carefully revised, and enlarged by the addition of about forty pages of new and useful matter, comprising more than three hundred practical problems, with rules and explanatory notes for solving a number of them; methods of analyzing problems and cancelling the equal factors of dividends and divisors; rules for computing interest upon notes on which partial payments have been made; rules for ascertaining the tonnage of ships, measuring round timber and spars; descriptions of board rules, used by surveyors of lumber; also, permutation and combination of numbers, exchange, horse power, and a number of useful tables.

It is deemed important that every arithmetical term should be accurately defined; that every operation to be performed should be distinctly stated, and every principle fully illustrated by the solution of one or more problems; that each rule should be deduced from the illustration of the principle, and that the rule should designate the successive steps to be taken in the solution of all similar arithmetical problems.

The peculiar characteristics of this treatise are, its accurate and complete definitions of arithmetical terms, and statement of operations to be performed; its lucid illustrations of the principles and properties of integral and fractional numbers, modes of operation in the solution of problems, and the accuracy and completeness of its rules deduced from those illustrations.

The arrangement of the subjects and articles is such, that the pupil will require no knowledge of any subsequent article, in order to understand each succeeding one.

The arrangement of questions under each head, combining those questions which are to be analyzed and performed *mentally* and *orally*, by the pupil, with those that will require him to use the slate and pencil in performing them, is another important feature of the work; and teachers will find their young pupils interested in studying an arithmetic in which the questions are arranged in a similar manner.

This new edition of the American Arithmetic comprises, *in one volume*, all the principles of numbers, fully explained and illustrated, and is adapted to the use of all classes of pupils in our common and high schools, in every stage of advancement.

Teachers and school committees are respectfully requested to examine the work, and in their examination to notice some of its original features, in Articles 25, 26, 27, 69, 73, 75, 122, 125, 135, 143, 172, and 174.

How far the author has been successful in preparing an arithmetic that will lighten the labor of teachers, and facilitate the progress of pupils, must be left for their decision, until they shall have used it in their schools.

JAMES ROBINSON.

Boston, July 1, 1850.

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ARITHMETIC.

ARITHMETIC is the science of numbers, and the art of computation.

Arithmetic is a science, so far as it explains and illustrates the principles and properties of numbers; it is an art, so far as it furnishes rules for the solution of numerical problems, or for calculation.

Art. 1. Number is either a unit, or a collection of units. Any single whole thing is called a unit, or one. One and one are called two. Two and one are called three. Three and one are called four. Four and one are called five. Five and one are called six. Six and one are called seven. Seven and one are called eight. Eight and one are called nine. Nine and one are called ten. Thus, each successive number may be formed by the continual addition of a unit.

Art. 2. Numbers are the expressions and measures of quantity. Quantity signifies as much of anything as there is, as a quantity of cloth; or as many things as there are, as a quantity of pens.

In measuring quantities of magnitude, some known quantity, as a yard, is taken for the unit of measure. In measuring quantities of multitude, one of the things which compose the quantity is its unit of measure; and the measure of any quantity is the number of times it contains its unit of measure.

Art. 3. When the unit indicates the name of anything, as 1 dollar, it is called a denominate unit; and numbers composed of denominate units are called denominate numbers; as 5 dollars, 10 pounds. When the unit expresses no idea except number, as 1, it is called an abstract unit; and numbers composed of abstract units are called abstract numbers; as 5, 10.

NOTATION AND NUMERATION.

Art. 4. NOTATION is the writing of numbers. Ten characters, or figures, are used in writing numbers. These figures are 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, 0 zero, or cipher. Each of these figures, except the cipher, always expresses the same *simple* number;—but in writing numbers greater than nine, the law or scale of notation assigns also a *local* number to each figure, which depends upon its place.

In the number 111, the 1 in the first place on the right expresses only its *simple* number one, and it is called a unit of the first order;—the 1 in the second place expresses ten times one, or ten, and it is called a unit of the second order;—and the 1 in the third place expresses ten times ten, or one hundred, and it is called a unit of the third order.

If we use the 0, which does not express a number of itself, we can express each of the above numbers singly; thus, 100, 10, 1; all of which are equal to 111. The established law or scale of notation is, that each successive removal of the same figure one place to the left increases or repeats its number ten times.

Art. 5. The units of the second order are written thus: 10, 20, 30, 40, 50, 60, 70, 80, 90. The units of the third order are written thus: 100, 200, 300, 400, 500, 600, 700, 800, 900. The numbers between 10 and 20 are written thus: 11, 12, 13, 14, 15, 16, 17, 18, 19. The numbers between 20 and 30 are written thus: 21, 22, 23, 24, 25, 26, 27, 28, 29. The numbers between all other tens are written in a similar manner.

The units of the fourth order are written thus: 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000. The numbers between the thousands are expressed by their appropriate figures.

The higher orders of units may be written in a similar manner, to an unlimited extent.

Art. 6. NUMERATION is the reading of numbers. The following table exhibits twenty-four places, or orders, in the scale of numbers, divided into eight periods of three figures each, by points. It also exhibits the name of each order and period.

NOTATION AND NUMERATION TABLE.

8th period. Sextillions.			7th period. Quintillions.			6th period. Quadrillions.			5th period. Trillions.			4th period. Billions.			3d period. Millions.			2d period. Thousands.			1st period. Units.		
5	5	5.	5	5	5.	5	5	5.	5	5	5.	5	5	5.	5	5	5.	5	5	5.	5	5	5.
24th order, Hunds. of Sextillions.	23d order, Tens of Sextillions.	22d order, Units of Sextillions.	21st order, Hunds. of Quintillions.	20th order, Tens of Quintillions.	19th order, Units of Quintillions.	18th order, Hunds. of Quadrillions.	17th order, Tens of Quadrillions	16th order, Units of Quadrillions	15th order, Hunds. of Trillions.	14th order, Tens of Trillions.	13th order, Units of Trillions.	12th order, Hunds. of Billions.	11th order, Tens of Billions.	10th order, Units of Billions.	9th order, Hunds. of Millions.	8th order, Tens of Millions.	7th order, Units of Millions.	6th order, Hunds. of Thousands.	5th order, Tens of Thousands.	4th order, Units of Thousands.	3d order, Hundreds.	2d order, Tens.	1st order, Units.

The division of numbers into periods of three figures each, enables us to read large numbers as easily as we can read a number containing only three figures. The figures of each period are to be read in the same manner as we read the figures of the period of units. Beginning at the left, the several periods in the table are read, in succession, as follows: 555 sextillion, 555 quintillion, 555 quadrillion, 555 trillion, 555 billion, 555 million, 555 thousand, 555.

Art. 7. From the arrangement and illustration of the preceding table, we have the following rule for reading numbers:

RULE. *Divide the given numbers into periods of three figures each, by points, counting from the right. Then begin at the left, and read the number of hundreds, tens, and units of each period, in succession, and conclude each period, except the period of units, with its name.*

Learners should be required to read the following numbers:

234178	87654321	987654321	1234567891
456784	45678900	450705430	9080706050
507604	50403020	845064310	8707605400
640530	67400045	614504301	4050634031

Art. 8. *In teaching young children to write numbers, dictate a small number, and require them to name the number of figures necessary to write the number; also, to name each figure, in succession, from left to right. Continue this exercise, until they can readily name the required number of figures, also each figure in succession.*

In teaching children to write large numbers on the slate or blackboard, first require them to name the number of figures necessary to write the proposed number, then the number of periods, and, lastly, the figures of each period, in succession, requiring them to write each figure at the time of naming it.

ILLUSTRATION. Let it be required to write eight hundred and seventy-five trillion, fifty billion, forty-five million, five hundred and forty.

Hundreds of trillions is the highest order named in this number, therefore fifteen figures, or five periods of figures, will be necessary to write it.

Beginning at the left, with the highest order and period, we write 8 in the place of hundreds, 7 in the place of tens, and 5 in the place of units. The orders of hundreds and units of billions being omitted, we write a 0 in the place of hundreds, 5 in the place of tens, and a 0 in the place of units. The order of hundreds of millions being omitted, we write a 0 in the place of hundreds, 4 in the place of tens, and 5 in the place of units. The three orders of thousands being omitted, we write a 0 in the place of each of the orders, thus, 000. In the period of units, we write 5 in the place of hundreds, 4 in the place of tens, and a 0 in the place of units. Arranging all the periods and figures in their proper order, the number is written thus: 875.050.045.000.540.

From the preceding remarks and illustration, we derive the following rule for writing numbers:

RULE. *First, ascertain the number of figures necessary to write the proposed number; also, the number of periods. Then begin at the left, and write a figure expressing the number of hundreds, tens, and units, of each period, in succession, observing to write a cipher in the place of each omitted order.*

Art. 9. *All questions throughout this work which are arranged in double columns, are to be performed both mentally and orally, unless otherwise directed.*

1. How many and what figures are necessary to write seventy-five?
2. How many and what figures are necessary to write ninety-nine?
3. One hundred and twenty-one?
4. Two hundred and eighty-one?
5. Three hundred and fifty-one?
6. Four hundred and sixty-nine?
7. Five hundred and thirty-two?
8. Six hundred and forty-seven?
9. Seven hundred and seventy-three?
10. Eight hundred and eighty-eight?

11. Nine hundred and ninety-nine?

12. One thousand two hundred and one?

13. Three thousand four hundred and ten?

14. Four thousand and fifty-three?

15. Five thousand and seventy-five?

16. Six thousand and five hundred?

17. Seven thousand and seventy-five?

18. Eight thousand and forty-five?

19. Nine thousand and ninety-seven?

20. Ten thousand and nine hundred?

21. Fifteen thousand and twenty-one?

22. Twenty thousand and one hundred?

23. Twenty-five thousand and thirty-five?

24. Thirty thousand and forty-seven?

---

25. Forty-five thousand, seven hundred and fifty-nine.

26. Seventy-nine thousand, nine hundred and ninety-nine.

27. Six hundred and ten thousand, five hundred and forty-seven.

28. Seven hundred and six thousand, four hundred and thirty-two.

29. Nine hundred and ninety-nine thousand, nine hundred and ninety-nine.

30. Two million, one hundred and fifty thousand, three hundred and twenty-one.

31. Thirty-five million, twenty-four thousand, one hundred and fifty-nine.

32. Seventy-six million, five hundred and forty-three thousand, three hundred and twenty-one.

33. Ninety-nine million, eight hundred and seven thousand, six hundred and fifty-four.

34. Three hundred million, two hundred and ten thousand, one hundred and ten.

35. Five hundred and forty million, three hundred and twenty thousand, two hundred and one.

36. Nine hundred and eighty-seven million, six hundred and fifty-four thousand, three hundred and twenty-one.

37. Seventy-five billion, sixty-four million, three hundred thousand, two hundred and seventeen.

38. Five hundred and forty trillion, four hundred and thirty billion, three hundred and twenty million, two hundred and ten thousand, one hundred and one.

39. Eight hundred and nineteen sextillion, seven hundred and sixteen quintillion, six hundred and fifteen quadrillion, five hundred and fourteen trillion, four hundred and thirteen billion, three hundred and twelve million, two hundred and eleven thousand, one hundred and ten.

ADDITION OF SIMPLE NUMBERS.

Art. 10. Addition is the method of finding the amount or sum of two or more numbers.

The amount or sum of two or more numbers is a number that contains the number one as many times as it is contained in all the numbers whose amount or sum is to be found.

Illustration. William has 6 cents, and James has 4 cents. We can find the number of cents which both of them have, by adding 4 cents to 6 cents; 4 cents and 6 cents are 10 cents, the number of cents that they both have. 10 cents is the amount or sum of 4 cents and 6 cents.

1. There are 6 birds on one tree, and 7 on another; how many birds are there on both trees?

2. John gave 7 cents for a pencil, and 8 cents for a book; how many cents did he give for both?

3. How many are 6 and 7?

4. How many are 7 and 8?

5. Frances paid 8 cents for a yard of ribbon, and 9 cents for needles; how many cents did she pay away?

6. There are 9 girls in the first class, and 10 in the second; how many girls are there in the two classes?

7. What is the sum of 8 and 9?

8. What is the sum of 9 and 10?

9. There are 10 houses on one side of a street, and 11 on the other; how many are there on the two sides?

10. William has performed 11 questions in arithmetic, and Henry 12; how many have both of them performed?

11. How many are 10 and 11?

12. How many are 11 and 12?

13. George purchased 12 apples at one time, and 10 at another; what number of apples did he purchase?

14. Julia has recited 11 lessons to her teacher, and Harriet 10; how many lessons have both of them recited?

15. What is the amount of 12 and 10?

16. What is the sum of 11 and 10?

17. A little girl is 7 years old; how old will she be, if living, at the end of 10 years from the present time?

18. William was 12 years old when he came to Boston, and has resided in the city 8 years; how old is William?

19. What is the sum of 7 and 10?

20. What is the amount of 12 and 8?

21. There are 11 boys in one sleigh, and 9 in another; how many boys are there in the two sleighs?

22. John purchased 12 peaches of one man, and 9 of another; how many peaches did he purchase?

23. How many are 11 and 9?

24. How many are 12 and 9?

25. What is the sum of 3, 5, 7, 9, and 11?

26. What is the amount of 2, 4, 6, 8, 10, and 12?

27. What is the sum of 1, 2, 3, 4, 5, and 6?

28. What is the amount of 7, 8, 9, 10, 11, and 12?

ADDITION TABLE.

2 and 0 are 2
2 and 1 are 3
2 and 2 are 4
2 and 3 are 5
2 and 4 are 6
2 and 5 are 7
2 and 6 are 8
2 and 7 are 9
2 and 8 are 10
2 and 9 are 11
2 and 10 are 12
2 and 11 are 13
2 and 12 are 14

3 and 0 are 3
3 and 1 are 4
3 and 2 are 5
3 and 3 are 6
3 and 4 are 7
3 and 5 are 8
3 and 6 are 9
3 and 7 are 10
3 and 8 are 11
3 and 9 are 12
3 and 10 are 13
3 and 11 are 14
3 and 12 are 15

4 and 0 are 4
4 and 1 are 5
4 and 2 are 6
4 and 3 are 7
4 and 4 are 8
4 and 5 are 9
4 and 6 are 10
4 and 7 are 11
4 and 8 are 12
4 and 9 are 13
4 and 10 are 14
4 and 11 are 15
4 and 12 are 16

5 and 0 are 5
5 and 1 are 6
5 and 2 are 7
5 and 3 are 8
5 and 4 are 9
5 and 5 are 10
5 and 6 are 11
5 and 7 are 12
5 and 8 are 13
5 and 9 are 14
5 and 10 are 15
5 and 11 are 16
5 and 12 are 17

6 and 0 are 6
6 and 1 are 7
6 and 2 are 8
6 and 3 are 9
6 and 4 are 10
6 and 5 are 11
6 and 6 are 12
6 and 7 are 13
6 and 8 are 14
6 and 9 are 15
6 and 10 are 16
6 and 11 are 17
6 and 12 are 18

7 and 0 are 7
7 and 1 are 8
7 and 2 are 9
7 and 3 are 10
7 and 4 are 11
7 and 5 are 12
7 and 6 are 13
7 and 7 are 14
7 and 8 are 15
7 and 9 are 16
7 and 10 are 17
7 and 11 are 18
7 and 12 are 19

8 and 0 are 8
8 and 1 are 9
8 and 2 are 10
8 and 3 are 11
8 and 4 are 12
8 and 5 are 13
8 and 6 are 14
8 and 7 are 15
8 and 8 are 16
8 and 9 are 17
8 and 10 are 18
8 and 11 are 19
8 and 12 are 20

9 and 0 are 9
9 and 1 are 10
9 and 2 are 11
9 and 3 are 12
9 and 4 are 13
9 and 5 are 14
9 and 6 are 15
9 and 7 are 16
9 and 8 are 17
9 and 9 are 18
9 and 10 are 19
9 and 11 are 20
9 and 12 are 21

10 and 0 are 10
10 and 1 are 11
10 and 2 are 12
10 and 3 are 13
10 and 4 are 14
10 and 5 are 15
10 and 6 are 16
10 and 7 are 17
10 and 8 are 18
10 and 9 are 19
10 and 10 are 20
10 and 11 are 21
10 and 12 are 22

11 and 0 are 11
11 and 1 are 12
11 and 2 are 13
11 and 3 are 14
11 and 4 are 15
11 and 5 are 16
11 and 6 are 17
11 and 7 are 18
11 and 8 are 19
11 and 9 are 20
11 and 10 are 21
11 and 11 are 22
11 and 12 are 23

12 and 0 are 12
12 and 1 are 13
12 and 2 are 14
12 and 3 are 15
12 and 4 are 16
12 and 5 are 17
12 and 6 are 18
12 and 7 are 19
12 and 8 are 20
12 and 9 are 21
12 and 10 are 22
12 and 11 are 23
12 and 12 are 24

Art. 11. Besides figures, we use several signs in arithmetic indicating equality, or that some operation is to be performed.

Two parallel horizontal lines indicate equality; thus, 1 dollar = 100 cents, which is read 1 dollar is equal to 100 cents.

A cross, one line being horizontal, and the other perpendicular, indicates addition; thus, 5 + 7 = 12, which is read 5 added to 7 is equal to 12, — or 5 plus 7 is equal to 12. There are other signs, which will be explained when we have occasion to use them.

29. How many are 13 + 2? 14 + 2? 15 + 2? 16 + 2? 17 + 2? 18 + 2? 19 + 2?

31. How many are 13 + 4? 14 + 4? 15 + 4? 16 + 4? 17 + 4? 18 + 4? 19 + 4?

33. What is the sum of 13 + 6? 14 + 6? 15 + 6? 16 + 6? 17 + 6? 18 + 6? 19 + 6?

35. What is the amount of 13 + 8? 14 + 8? 15 + 8? 16 + 8? 17 + 8? 18 + 8? 19 + 8?

37. William paid 20 cents for a knife, and 10 cents for a writing-book; how many cents did he pay for both of them?

39. How many are 30 + 10?

41. A man paid 50 dollars for a gold watch, and 20 dollars for a silver one; how many dollars did the two watches cost him?

43. What is the sum of 60 + 30?

45. A merchant borrowed 30 dollars of A., 20 dollars of B., and 10 dollars of C; what was the whole number of dollars that he borrowed?

47. What is the amount of 40 + 30 + 20?

49. A trader paid 70 dollars for a horse, 60 dollars for a yoke of oxen, and 20 dollars for a cow; how many dollars did he pay for all of them?

51. What is the sum of 80 + 60 + 40?

30. How many are 13 + 3? 14 + 3? 15 + 3? 16 + 3? 17 + 3? 18 + 3? 19 + 3?

32. How many are 13 + 5? 14 + 5? 15 + 5? 16 + 5? 17 + 5? 18 + 5? 19 + 5?

34. What is the sum of 13 + 7? 14 + 7? 15 + 7? 16 + 7? 17 + 7? 18 + 7? 19 + 7?

36. What is the amount of 13 + 9? 14 + 9? 15 + 9? 16 + 9? 17 + 9? 18 + 9? 19 + 9?

38. A man lived in the country until he was 30 years old, and has lived in the city 20 years; what is his age?

40. How many are 40 + 20?

42. In a certain school, there are 50 boys, and 30 girls; what is the whole number of pupils in the school?

44. What is the sum of 60 + 40?

46. A trader purchased three pieces of cloth; the first measured 50 yards, the second, 40 yards, and the third, 30 yards; how many yards did he purchase?

48. What is the amount of 60 + 50 + 40?

50. A farmer purchased three pieces of land; the first contained 80 acres, the second, 70 acres, the third, 60 acres; how many acres did he purchase?

52. What is the sum of 90 + 80 + 70?

53. James purchased an arithmetic for 25 cents, and a slate for 15 cents; what number of cents did both of them cost him?

55. What is the amount of 35 + 25?

57. There is a school, in which there are four classes. In the first class there are 65 boys, — in the second, 50, — in the third, 45, — and in the fourth, 40. How many boys are there in *the school?*

59. What is the sum of 65 + 60 + 55 + 50?

61. What is the sum of 240 dollars + 230 dollars + 220 dollars + 210 dollars?

54. A trader borrowed 45 dollars of C., and 35 dollars of D; what number of dollars did he borrow?

56. What is the amount of 55 + 45?

58. A stage driver purchased four horses. He paid 120 dollars for the first, 110 dollars for the second, 90 dollars for the third, and 80 dollars for the fourth. What number of dollars did they cost him?

60. What is the sum of 150 + 140 + 130 + 120?

62. What is the sum of 550 dollars + 460 dollars + 370 dollars + 230 dollars?

Art. 12. A man has three carriages. No. 1 is worth 950 dollars. No. 2 is worth 875 dollars. No. 3 is worth 764 dollars. How many dollars are the three carriages worth?

hund. tens. units.
950
875
764
——
Ans. 2589 dols.

These numbers being large, it is not convenient to add them mentally. We first write them in order, placing units under units, tens under tens, hundreds under hundreds, and then draw a line under them.

We begin at the bottom of the column of units, and say, 4 units and 5 units are 9 units, — the sum of the units being less than 10, we write the 9 units under the column. We then proceed to the column of tens, and say, 6 tens and 7 tens are 13 tens, and 5 tens are 18 tens, or 1 hundred and 8 tens; we write the 8 tens under the column of tens, and add the 1 hundred with the figures in the column of hundreds. We then say, 1 hundred and 7 hundred are 8 hundred, and 8 hundred are 16 hundred, and 9 hundred are 25 hundred, or 2 thousand and 5 hundred; we write the 5 hundred under the column of hundreds, and the 2 thousand at the left of the 5 hundred: and the whole amount is 2589 dollars.

From the preceding illustration, we obtain the following rule for the addition of simple numbers:

Rule. ***Write the numbers, placing units under units, tens under tens, hundreds under hundreds, and draw a line under them.***

Begin at the bottom of the column of units, and add all the figures in that column, in succession; if the amount does not exceed nine, write it under the column; if the amount exceeds nine, write the right hand figure of it under the column, and add the left hand figure, or figures, which are units of the next higher order, with the figures in the next column.

Add the figures in each succeeding column in the same manner, and write down the whole amount of the last column.

Methods of Proof. *Begin at the top of each column, and add the figures downwards in the same manner that they were added upwards; if both operations have been performed correctly, this amount will be equal to the first amount.*

Or separate the numbers into two or more divisions, find the amount of each of these divisions, then find the sum of these amounts; if the several operations have been performed correctly, the sum of these separate amounts will be equal to the first amount.

63. What is the sum of 1, 2, 3, 4, 5, 6, 7, 8, and 9? Ans. 45.

64. What is the amount of 10, 11, 12, 13, 14, 15, 16, 17, 18, and 19? Ans. 145.

65. What is the sum of 20, 25, 30, 35, 40, 45, 50, 55, and 60? Ans. 360.

66. What is the amount of 65, 70, 75, 80, 85, 90, 95, and 100? Ans. 660.

67. Dr. Benjamin Franklin was born in 1706, and was 84 years old when he died; in what year did he die? Ans. 1790.

68. The creation of the world was 4004 years before the Christian era; how many years since the creation to the present time, 1847? Ans. 5851 years.

69. In January there are 31 days, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30, December 31. How many days are there in the twelve months, or one year? Ans. 365 days.

70. A provision dealer purchased eight hogs, weighing as follows: 275 pounds, 286 pounds, 305 pounds, 315 pounds, 334 pounds, 345 pounds, 350 pounds, and 375 pounds. What was their whole weight? Ans. 2585 pounds.

71. A butcher killed five fat oxen. The weight of the first was 1124 pounds; the second, 1235 pounds; the third, 1300

pounds; the fourth, 1440 pounds; the fifth, 1527 pounds. What was the whole weight of all of them?

Ans. 6626 pounds.

72. A man owns four farms. The first is valued at 5750 dollars; the second, at 6500 dollars; the third, at 7225 dollars; the fourth, at 8725 dollars. What are they all worth?

Ans. 28200 dollars.

73. A merchant, engaging in trade, had, in cash, 4550 dollars; in goods, 3125 dollars; in notes, 1500 dollars; he gained 1250 dollars each year during three successive years. What was he worth at the end of the three years?

Ans. 12925 dollars.

74. A mechanic owns six houses. No. 1 is worth 4500 dollars; No. 2, 5400 dollars; No. 3, 5750 dollars; No. 4, 6225 dollars; No. 5, 6540 dollars; No. 6, 7500 dollars. How many dollars are they all worth?

Ans. 35915 dollars.

75. In 1840, the number of inhabitants in Boston was 93383; in Lowell, 20796; in Salem, 15082; in New Bedford, 12087; in Charlestown, 11484; in Springfield, 10985. What was the total number of inhabitants in the above cities and towns in 1840? Ans. 163817 inhabitants.

76. According to the census of 1840, New York contained 312710 inhabitants; Philadelphia, 205580; Baltimore, 102313; New Orleans, 102193; Boston, 93383. What was the total number of inhabitants in these five cities, in 1840?

Ans. 816179 inhabitants.

77. In 1840, the value of articles manufactured from wool in the State of Massachusetts was 7082898 dollars; from cotton, 16553423 dollars; from leather, 10553826 dollars. What was the total value of articles manufactured from wool, cotton, and leather, in 1840?

Ans. 34190147 dollars.

78. In 1840, the State of Massachusetts produced 157923 bushels of wheat, 165319 bushels of barley, 1319680 bushels of oats, 536014 bushels of rye, 87060 bushels of buckwheat, and 1809192 bushels of Indian corn. What was the total number of bushels of grain produced in the State of Massachusetts, in 1840? Ans. 4075188 bushels.

79. The number of regular troops furnished by each of the states in the revolution was as follows: New Hampshire, 12497; Massachusetts, 67907; Rhode Island, 5908; Connecticut, 31939; New York, 17781; New Jersey, 10726; Pennsylvania, 25678; Delaware, 2386; Maryland, 13912; Virginia, 26678; North Carolina, 7263; South Carolina,

6417; Georgia, 2679. What was the number of regular troops engaged during the war? Ans. 231771.

80. The Atlantic Bank in Boston, has a capital of $500000; Atlas, $500000; Boston, $600000; Boylston, $150000; City, $1000000; Columbian, $500000; Eagle, $500000; Freeman's, $200000; Globe, $1000000; Granite, $500000; Hamilton, $500000; Market, 560000; Massachusetts, $800000; Mechanics', $120000; Merchants', $3000000; New England, 1000000; North, 750000; Shawmut, $500000; Shoe and Leather Dealers', $500000; State, $1800000; Suffolk, $1000000; Traders' $400000; Tremont, $500000; Union, $800000; Washington, $500000. What is the whole amount of Bank capital in Boston at the present time, 1847? Ans. $18180000.

81. The following table exhibits the population of the several states and territories, at the taking of each census, to 1840. What was the population of the United States in each of those years?

States.	1790.	1800.	1810.	1820.	1830.	1840.
Maine, . . .	96540	151719	228705	298335	399955	501793
New Hampshire,	141899	183762	214360	244161	269328	284574
Vermont, . . .	85416	154465	217713	235764	280652	291948
Massachusetts, .	378717	423245	472040	523287	610408	737699
Rhode Island, .	69110	69122	77031	83059	97199	108830
Connecticut, .	238141	251002	262042	275202	297665	309978
New York, . .	340120	586756	959949	1372812	1918608	2428921
New Jersey, . .	184139	211949	249555	277575	320823	373303
Pennsylvania, .	434373	602365	810091	1049458	1348233	1724033
Delaware, . .	59098	64273	72674	72749	76748	78085
Maryland, . .	319728	341548	380546	407350	447040	470019
Virginia, . . .	748308	880200	974642	1065379	1211405	1239797
North Carolina, .	393751	478103	555500	638829	737987	753419
South Carolina, .	249073	345591	415115	502741	581185	594398
Georgia, . . .	82548	162101	252433	340987	516823	691392
Alabama, . .	—	—	20845	127901	309527	590756
Mississippi, . .	—	8850	40352	75448	136621	375651
Louisiana, . .	—	—	76556	153407	215739	352411
Arkansas, . .	—	—	—	14273	30388	97574
Tennessee, . .	30791	105602	261727	422813	681904	829210
Kentucky, . .	73077	220955	406511	564317	687917	779828
Ohio, ...	—	45365	230760	581434	937903	1519467
Michigan, . .	—	—	4762	8896	31639	212267
Indiana, . . .	—	4875	24520	147178	343031	685866
Illinois, . . .	—	—	12282	55211	157455	476183
Missouri, . .	—	—	20845	66586	140445	383702
Dist. Columbia,	—	14093	24023	33039	39834	43712
Florida, . . .	—	—	—	—	34730	54477
Wisconsin, . .	—	—	—	—	—	30945
Iowa, ...	—	—	—	—	—	43112

SUBTRACTION OF SIMPLE NUMBERS.

Art. 13. SUBTRACTION is the method of finding how many one number exceeds another, by taking the less number from the greater.

The number found by taking a less number from a greater is called the remainder, or difference, and it shows how many the greater number exceeds the less.

ILLUSTRATION. William has 9 cents, and Henry has 5. *How many more cents has William than Henry?*

If we take 5 cents from 9 cents, there will be 4 cents left; then William has 4 cents more than Henry.

1. Eliza had 9 roses, and she gave 3 of them to her sister; how many had she left?

2. Julia purchased 10 plums, and has eaten 3 of them; how many has she left?

3. Then 9 less 3, are how many?

4. Then 10 less 3, are how many?

5. There were 11 birds in a cage, but a boy has taken 5 of them out; how many birds remain in the cage?

6. A boy bought 13 apples; 4 of them were red, and the others were green; how many of them were green?

7. Then 11 less 5, are how many?

8. Then 13 less 4, are how many?

9. If a man, who owns 14 horses, should sell 5 of them, how many horses would he have left?

10. There are 15 pupils in a class; 6 of them are boys, the others are girls; how many of them are girls?

11. Then 14 less 5, are how many?

12. Then 15 less 6, are how many?

13. A page of a writing-book contains 16 lines, and a boy has written 7 of them; how many lines has he to write to finish the page?

14. A grocer purchased 17 barrels of flour, and has since sold 8 of them; how many barrels has he remaining unsold?

15. Then 16 less 7, are how many?

16. Then 17 less 8, are how many?

17. Emma counted 18 roses on her rosebush this morning; she has since picked off 8 of them; how many roses remain on the bush?

18. There were 19 girls in the second class of a school; 9 of them have since been promoted into the first class; how many girls still remain in the second class?

19. Then 18 less 8, are how many?

20. Then 19 less 9, are how many?

21. Albert is 20 years old, his sister Eliza is only 10 years old; how many years older is Albert than Eliza?

22. A farmer purchased 21 sheep; he afterwards sold 11 of them; how many has he remaining unsold?

23. Then 20 less 10, are how many?

24. Then 21 less 11, are how many?

SUBTRACTION TABLE.

2 from 2 leaves 0
2 from 3 leaves 1
2 from 4 leaves 2
2 from 5 leaves 3
2 from 6 leaves 4
2 from 7 leaves 5
2 from 8 leaves 6
2 from 9 leaves 7
2 from 10 leaves 8
2 from 11 leaves 9
2 from 12 leaves 10
2 from 13 leaves 11
2 from 14 leaves 12

3 from 3 leaves 0
3 from 4 leaves 1
3 from 5 leaves 2
3 from 6 leaves 3
3 from 7 leaves 4
3 from 8 leaves 5
3 from 9 leaves 6
3 from 10 leaves 7
3 from 11 leaves 8
3 from 12 leaves 9
3 from 13 leaves 10
3 from 14 leaves 11
3 from 15 leaves 12

4 from 4 leaves 0
4 from 5 leaves 1
4 from 6 leaves 2
4 from 7 leaves 3
4 from 8 leaves 4
4 from 9 leaves 5
4 from 10 leaves 6
4 from 11 leaves 7
4 from 12 leaves 8
4 from 13 leaves 9
4 from 14 leaves 10
4 from 15 leaves 11
4 from 16 leaves 12

5 from 5 leaves 0
5 from 6 leaves 1
5 from 7 leaves 2
5 from 8 leaves 3
5 from 9 leaves 4
5 from 10 leaves 5
5 from 11 leaves 6
5 from 12 leaves 7
5 from 13 leaves 8
5 from 14 leaves 9
5 from 15 leaves 10
5 from 16 leaves 11
5 from 17 leaves 12

6 from 6 leaves 0
6 from 7 leaves 1
6 from 8 leaves 2
6 from 9 leaves 3
6 from 10 leaves 4
6 from 11 leaves 5
6 from 12 leaves 6
6 from 13 leaves 7
6 from 14 leaves 8
6 from 15 leaves 9
6 from 16 leaves 10
6 from 17 leaves 11
6 from 18 leaves 12

7 from 7 leaves 0
7 from 8 leaves 1
7 from 9 leaves 2
7 from 10 leaves 3
7 from 11 leaves 4
7 from 12 leaves 5
7 from 13 leaves 6
7 from 14 leaves 7
7 from 15 leaves 8
7 from 16 leaves 9
7 from 17 leaves 10
7 from 18 leaves 11
7 from 19 leaves 12

8 from 8 leaves 0
8 from 9 leaves 1
8 from 10 leaves 2
8 from 11 leaves 3
8 from 12 leaves 4
8 from 13 leaves 5
8 from 14 leaves 6
8 from 15 leaves 7
8 from 16 leaves 8
8 from 17 leaves 9
8 from 18 leaves 10
8 from 19 leaves 11
8 from 20 leaves 12

9 from 9 leaves 0
9 from 10 leaves 1
9 from 11 leaves 2
9 from 12 leaves 3
9 from 13 leaves 4
9 from 14 leaves 5
9 from 15 leaves 6
9 from 16 leaves 7
9 from 17 leaves 8
9 from 18 leaves 9
9 from 19 leaves 10
9 from 20 leaves 11
9 from 21 leaves 12

10 from 10 leaves 0
10 from 11 leaves 1
10 from 12 leaves 2
10 from 13 leaves 3
10 from 14 leaves 4
10 from 15 leaves 5
10 from 16 leaves 6
10 from 17 leaves 7
10 from 18 leaves 8
10 from 19 leaves 9
10 from 20 leaves 10
10 from 21 leaves 11
10 from 22 leaves 12

11 from 11 leaves 0
11 from 12 leaves 1
11 from 13 leaves 2
11 from 14 leaves 3
11 from 15 leaves 4
11 from 16 leaves 5
11 from 17 leaves 6
11 from 18 leaves 7
11 from 19 leaves 8
11 from 20 leaves 9
11 from 21 leaves 10
11 from 22 leaves 11
11 from 23 leaves 12

12 from 12 leaves 0
12 from 13 leaves 1
12 from 14 leaves 2
12 from 15 leaves 3
12 from 16 leaves 4
12 from 17 leaves 5
12 from 18 leaves 6
12 from 19 leaves 7
12 from 20 leaves 8
12 from 21 leaves 9
12 from 22 leaves 10
12 from 23 leaves 11
12 from 24 leaves 12

Art. 14. Subtraction is indicated by a short horizontal line, thus 12—8=4, which is read 12 minus 8 equals 4; or 12 less 8, is equal to 4.

25. How many are 13—5? 15—7? 19—9? 21—11? 23—12?

26. How many are 14—6? 16—8? 18—10? 20—12? 22—12?

27. How many are 23—4? 25—6? 27—8? 29—10? 31—12?

28. How many are 22—7? 24—9? 26—11? 28—13? 30—15?

29. How many are 31—5? 33—7? 35—9? 37—11? 39—13?

30. How many are 30—6? 32—8? 34—10? 38—12? 40—14?

31. How many are 20—3? 30—4? 40—5? 50—6? 60—7?

32. How many are 70—8? 80—9? 90—10? 100—11? 110—12?

33. A market woman bought 30 oranges, and she has sold 10 of them; how many remain unsold?

34. A farmer picked 40 barrels of apples, and sold 15 of them; what number of barrels has he left?

35. How many are 35—10?

36. How many are 45—15?

37. A farmer, who owned 50 sheep and lambs, has sold 25 of them; what number has he remaining unsold?

38. A man, who owed another 75 dollars, paid him 25 dollars; what number of dollars remain unpaid?

39. How many are 65—35?

40. How many are 80—25?

41. A farmer purchased a yoke of oxen for 80 dollars, and sold them for 95 dollars; what number of dollars did he gain?

42. Bought a horse for 100 dollars, and sold him for 75 dollars; how many dollars did I lose?

43. How many are 95—20?

44. How many are 120—25?

45. A merchant borrowed 125 dollars, and has paid 85 dollars; how many dollars remain unpaid?

46. A trader purchased 150 yards of sheeting, and has sold 75 yards of it; what number of yards has he remaining unsold?

47. How many are 130—90?

48. How many are 175—150?

Art. 15. Subtraction is performed, when all the figures of the less number are smaller than the corresponding figures of the greater, by taking each figure of the less number, in succession, from the corresponding figure of the greater, and writing the remainder underneath; the several remainders will express the whole difference.

Illustration. A merchant purchased 765 yards of cloth, and has since sold 523 yards of it; what number of yards has he remaining unsold?

765 No. of yds. purchased.
523 No. of yds. sold.
——
242 No. of yds. unsold.

We first write down the greater number, then we write the less number under the greater, placing units under units, tens under tens, and hundreds under hundreds. We first take 3 units from 5 units, and 2 units remain, which we write directly under the units. Then 2 tens from 6 tens, and 4 tens remain. Lastly, 5 hundred from 7 hundred, and 2 hundred remain. The whole remainder, or difference, is 242.

Art. 16. When any figure of the less number is larger than the corresponding figure of the greater number, the following question and its illustration will show the method of performing the operation.

A man borrowed 94 dollars, and has paid 46 dollars; how many dollars remain unpaid?

94 = 8 tens and 14 units.
46 = 4 tens and 6 units.
— — —
48 = 4 tens and 8 units.

We cannot take 6 units from 4 units, but we can take 46 from 94. If 1 of the tens, which is equal to 10 units, be taken from the 9 tens and added to the 4 units, 94 will be decomposed into 8 tens and 14 units.

Decomposing 46 into tens and units, we have 4 tens and 6 units. We now take 6 units from 14 units, and 8 units remain; 4 tens from 8 tens, and 4 tens remain. The whole remainder is 4 tens and 8 units, or 48, the number of dollars unpaid.

Art. 17. If two *unequal* numbers be *equally* increased, their difference will not be altered. As 10 units of any *lower* order are equal to 1 unit of the next *higher* order, whenever a figure of the less number is larger than the corresponding figure of the greater number, if we add 10 to the upper figure, and then add 1 to the next lower figure, the two numbers will be equally increased, yet their difference will remain unaltered.

As 94 is equal to 9 tens and 4 units, if we add 10 units to the 4 units, the amount will be 9 tens and 14 units; — 46 is equal to 4 tens and 6 units, — if we add 1 ten to 4 tens, the amount will be 5 tens and 6 units; — and if we take 5 tens and 6 units from 9 tens and 14 units, the remainder, or difference, will be 4 tens and 8 units, or 48, the number of dollars unpaid.

From the above examples and illustrations, we deduce the *following* rule for subtraction of simple numbers.

Rule. *Write down the greater number, then write the less number under the greater, placing units under units, tens under tens, hundreds under hundreds, and draw a line underneath.*

Begin with the units, and subtract each figure of the less number, in succession, from the figure over it, and write the remainder underneath.

Whenever a figure of the less number is greater than the figure over it, add ten to the upper figure, subtract the lower figure from the amount, then add one to the next lower figure before it is subtracted.

Method of Proof. *Add the remainder to the less number; their sum will be equal to the greater number, if the operation has been correctly performed.*

49. America was discovered by Christopher Columbus in 1492. How many years since to the present time, 1847? Ans. 355 years.

50. The United States were declared independent July 4th, 1776. How many years since to July 4th, 1847? Ans. 71 years.

51. The first settlement at Boston was made in 1630. How many years since to the present time, 1847? Ans. 217 years.

52. The greater of two numbers is seven hundred and fifty million; the less number is five hundred and forty thousand; what is their difference? Ans. 749460000.

53. The mariner's compass was invented in the year 1302; how many years was this before the discovery of America by Columbus, in 1492? Ans. 190 years.

54. A is worth ten thousand dollars; B is worth seven thousand nine hundred and twenty-five dollars. How many dollars is A worth more than B? Ans. 2075 dollars.

55. The planet Saturn is 890000000 miles from the Sun; the Earth is 95000000 miles from the Sun. How many miles further from the sun is Saturn than the Earth? Ans. 795000000 miles.

56. The diameter of Jupiter is eighty-nine thousand miles; the diameter of Venus is seven thousand seven hundred miles. What is the difference of their diameters? Ans. 81300 miles.

57. Massachusetts contains seven thousand five hundred square miles; Connecticut, four thousand seven hundred and sixty-four square miles. How many more square miles does Massachusetts contain than Connecticut? Ans. 2736 square miles.

58. The number of inhabitants in France is 33500000; the number of inhabitants in the United States is 17069453. How many more inhabitants are there in France than in the United States? Ans. 16430547 inhabitants.

59. The library of Harvard University contains 49000 volumes; the library of Yale College contains 35000 volumes. How many more volumes does the former contain than the latter? Ans. 14000 volumes.

60. The first settlement in the United States was made by the English, at Jamestown, in Virginia, in 1607; the United States were declared independent in 1776. How many years between these two events? Ans. 169 years.

61. The population of the United States in 1840 was 17069453, of which 2487355 were slaves. How many free persons were there in the United States in 1840?
Ans. 14582098 free persons.

62. A merchant owns property to the amount of thirty thousand five hundred and seventy-five dollars; but there are demands against him to the amount of seventeen thousand and sixty-five dollars. What number of dollars will he have left after paying his debts? Ans. 13510 dollars.

63. The population of Boston, in 1790, was 18038; in 1800, it was 24927; in 1810, it was 32250; in 1820, it was 43298; in 1830, it was 61392; in 1840, it was 93383. What was the increase in each successive period of 10 years? What was the increase in the whole period of 50 years?

64. The population of New York, in 1790, was 33131; in 1800, it was 60489; in 1810, it was 96373; in 1820, it was 123706; in 1830, it was 202589; in 1840, it was 312710. What was the increase in each successive period of 10 years? What was the increase in the whole period of 50 years?

65. The population of the northern district of New York in 1840, was one million six hundred eighty-three thousand and sixty-eight, and the population of the southern district was seven hundred forty-five thousand eight hundred and fifty-three. How many more inhabitants were there in the northern than in the southern district?

66. The population of the United States, in 1790, was 3929827; in 1800, it was 5305925; in 1810, it was 7239814; in 1820, it was 9638131; in 1830, it was 12866020; in 1840, it was 17069453. What was the increase in each successive period of 10 years? What was the increase in the whole period of 50 years?

PRACTICAL QUESTIONS IN ADDITION AND SUBTRACTION OF SIMPLE NUMBERS.

Art. 18. 1. A merchant purchased 5000 bushels o salt; he has since sold 1500 bushels to one man, and 75 bushels to another. What number of bushels has he remaining unsold?

2. A man's property is worth 10650 dollars. He owe A, 1800 dollars; B, 1260 dollars; C, 750 dollars; and D 500 dollars. What will he have remaining, after paying hi debts?

3. A gentleman purchased 15840 acres of new land. He afterwards sold 2350 acres to one individual, 4500 acres t another, and 3225 to a third. What number of acres has h left?

4. A merchant purchased 4500 yards of cloth; he has since sold 1750 yards to one of his customers, 1275 yards to another, and 950 yards to a third. What number of yards has he remaining unsold?

5. A man died leaving an estate amounting to 12650 dollars, which he bequeathed as follows: 2500 dollars to each of his three daughters, and the remainder to his son. What was the son's share?

6. A flour merchant purchased 1250 barrels of flour for 6250 dollars. He has since sold 500 barrels to A, for 2625 dollars, and 450 barrels to B, for 2475 dollars. How many barrels remain unsold? What was the whole cost of the flour above the amount of sales?

7. A merchant deposited 12000 dollars in the Suffolk Bank. He has since given A a check for 4500 dollars, B a check for 2750 dollars, and C a check for 1675 dollars. What amount has the merchant remaining in the bank?

8. A merchant purchased 12750 yards of cloth of one manufacturer, 10675 yards of another, and 7425 yards of a third. He has since sold 8450 yards to one of his customers, 6745 yards to another, and 5925 yards to a third. How many yards did he purchase? How many yards has he sold? What number of yards has he remaining unsold?

9. A gentleman purchased a house for 12500 dollars, a carriage for 750 dollars, and a span of horses for 500 dollars. He has paid 6500 dollars at one time, 2750 dollars at another time, and 1250 dollars at a third time. What was the whole amount of his purchases? What was the whole amount of payments? And what amount remains unpaid?

MULTIPLICATION OF SIMPLE NUMBERS.

Art. 19. Multiplication is the method of repeating any given number any required number of times.

The terms used in multiplication are the multiplicand, multiplier, and product. The multiplicand is the number to be multiplied or repeated. The multiplier is the number to multiply by, and it expresses the number of times the multiplicand is to be repeated. The product is the number produced by repeating the multiplicand the required number of times. The multiplicand and multiplier are called the factors of the product.

Illustration. If a yard of cloth be worth 10 cents, how many cents are 5 yards worth?

If one yard is worth 10 cents, 5 yards must be worth 5 times 10 cents, or 50 cents. In the above question, 10 cents is the multiplicand, 5 is the multiplier, and 50 cents is the product.

Multiplication is indicated by an inclined cross; thus, $6 \times 5 = 30$; and is read 6 multiplied by 5 is equal to 30?

1. If a quart of milk is worth 5 cents, what are 2 quarts worth?
2. If 1 orange cost 6 cents, how many cents will 3 oranges cost?
3. How many are 2 times 5?
4. How many are 3 times 6?
5. There are 3 feet in one yard; how many feet are there in 5 yards?
6. There are 4 farthings in one penny; how many farthings are there in 4 pence?
7. How many are 5 times 3?
8. How many are 4 times 4?
9. There are 6 shillings in 1 dollar; how many shillings are there in 6 dollars?
10. If one quart of cherries is worth 7 cents, how many cents are 7 quarts worth?
11. How many are 6 times 6?
12. How many are 7 times 7?
13. James has 8 marbles, and Henry has 5 times as many; how many has Henry?
14. If one yard of ribbon cost 9 cents, how many cents will 7 yards cost?
15. How many are 5 times 8?
16. How many are 7 times 9?
17. If one pound of sugar is worth 10 cents, what are 8 pounds worth?
18. There are 10 cents in one dime; how many cents are there in 10 dimes?
19. How many are 8 times 10?
20. How many are 10 times 10?
21. There are 7 days in one week; how many days are there in 11 weeks?
22. How many yards of cloth are there in 10 pieces, each piece measuring 11 yards?
23. How many are 11 times 7?
24. How many are 10 times 11?
25. In one shilling there are 12 pence; how many pence are there in 9 shillings?
26. If a mechanic can earn 12 dollars in one week, how many dollars can he earn in 12 weeks?

MULTIPLICATION TABLE.

2 times 0 are 0
2 times 1 are 2
2 times 2 are 4
2 times 3 are 6
2 times 4 are 8
2 times 5 are 10
2 times 6 are 12
2 times 7 are 14
2 times 8 are 16
2 times 9 are 18
2 times 10 are 20
2 times 11 are 22
2 times 12 are 24

3 times 0 are 0
3 times 1 are 3
3 times 2 are 6
3 times 3 are 9
3 times 4 are 12
3 times 5 are 15
3 times 6 are 18
3 times 7 are 21
3 times 8 are 24
3 times 9 are 27
3 times 10 are 30
3 times 11 are 33
3 times 12 are 36

4 times 0 are 0
4 times 1 are 4
4 times 2 are 8
4 times 3 are 12
4 times 4 are 16
4 times 5 are 20
4 times 6 are 24
4 times 7 are 28
4 times 8 are 32
4 times 9 are 36
4 times 10 are 40
4 times 11 are 44
4 times 12 are 48

5 times 0 are 0
5 times 1 are 5
5 times 2 are 10
5 times 3 are 15
5 times 4 are 20
5 times 5 are 25
5 times 6 are 30
5 times 7 are 35
5 times 8 are 40
5 times 9 are 45
5 times 10 are 50
5 times 11 are 55
5 times 12 are 60

6 times 0 are 0
6 times 1 are 6
6 times 2 are 12
6 times 3 are 18
6 times 4 are 24
6 times 5 are 30
6 times 6 are 36
6 times 7 are 42
6 times 8 are 48
6 times 9 are 54
6 times 10 are 60
6 times 11 are 66
6 times 12 are 72

7 times 0 are 0
7 times 1 are 7
7 times 2 are 14
7 times 3 are 21
7 times 4 are 28
7 times 5 are 35
7 times 6 are 42
7 times 7 are 49
7 times 8 are 56
7 times 9 are 63
7 times 10 are 70
7 times 11 are 77
7 times 12 are 84

8 times 0 are 0
8 times 1 are 8
8 times 2 are 16
8 times 3 are 24
8 times 4 are 32
8 times 5 are 40
8 times 6 are 48
8 times 7 are 56
8 times 8 are 64
8 times 9 are 72
8 times 10 are 80
8 times 11 are 88
8 times 12 are 96

9 times 0 are 0
9 times 1 are 9
9 times 2 are 18
9 times 3 are 27
9 times 4 are 36
9 times 5 are 45
9 times 6 are 54
9 times 7 are 63
9 times 8 are 72
9 times 9 are 81
9 times 10 are 90
9 times 11 are 99
9 times 12 are 108

10 times 0 are 0
10 times 1 are 10
10 times 2 are 20
10 times 3 are 30
10 times 4 are 40
10 times 5 are 50
10 times 6 are 60
10 times 7 are 70
10 times 8 are 80
10 times 9 are 90
10 times 10 are 100
10 times 11 are 110
10 times 12 are 120

11 times 0 are 0
11 times 1 are 11
11 times 2 are 22
11 times 3 are 33
11 times 4 are 44
11 times 5 are 55
11 times 6 are 66
11 times 7 are 77
11 times 8 are 88
11 times 9 are 99
11 times 10 are 110
11 times 11 are 121
11 times 12 are 132

12 times 0 are 0
12 times 1 are 12
12 times 2 are 24
12 times 3 are 36
12 times 4 are 48
12 times 5 are 60
12 times 6 are 72
12 times 7 are 84
12 times 8 are 96
12 times 9 are 108
12 times 10 are 120
12 times 11 are 132
12 times 12 are 144

Art. 20. A man earns 10 dollars in a week, and a boy earns 3 dollars. How many dollars do both of them earn in a week? How many dollars can each of them earn in 4 weeks? How many dollars can both of them earn in 4 weeks?

SOLUTION. 10 dollars plus 3 dollars are 13 dollars. Then both of them earn 13 dollars in a week. If a man earns 10 dollars in one week, in 4 weeks he can earn 4 times 10 dollars, or 40 dollars; — if a boy earns 3 dollars in one week, in 4 weeks he can earn 4 times 3 dollars, or 12 dollars; — and 40 dollars plus 12 dollars are 52 dollars. Then both of them can earn 52 dollars in 4 weeks.

How many are 4 times 13? SOLUTION. 13 is composed of 1 ten and 3 units. 4 times 1 ten are 4 tens, or 40 units; — 4 times 3 units are 12 units; — and 40 units plus 12 units are 52 units. Then 4 times 13 are 52.

27. If a boy pays 10 cents for a writing-book, and 4 cents for a lead pencil, how many cents must he pay for 5 writing-books, and 5 lead pencils?

29. How many are 5 times 14?

31. William gave 10 cents for a ruler, and 6 cents for an inkstand; how many cents must he give for 7 rulers and 7 inkstands?

33. How many are 7 times 16?

35. Jane's brother gave her 10 cents, and her sister gave her 8. If each of them had given her 9 times as many, how many cents would she then have had?

37. How many are 9 times 18?

39. How many are 2 times 13? 3 times 13? 4 times 13? 5 times 13? 6 times 13? 7 times 13? 8 times 13? 9 times 13? 10 times 13?

41. Suppose a ton of hay to be worth 15 dollars; how many dollars are 2 tons worth? 3 tons? 4 tons? 5 tons? 6 tons? 7 tons? 8 tons? 9 tons? 10 tons?

43. How many are 2 times 17? 3 times 17? 4 times 17? 5 times 17? 6 times 17? 7 times 17? 8 times 17? 9 times 17? 10 times 17?

28. Maria paid 10 cents for a yard of ribbon, and 5 cents for a thimble; how many cents will 6 yards of ribbon, and 6 thimbles cost?

30. How many are 6 times 15?

32. When sugar is worth 10 cents a pound, and coffee 7 cents, what will 8 pounds of sugar and 8 pounds of coffee be worth?

34. How many are 8 times 17?

36. Henry can perform 10 questions in arithmetic in an hour, and Charles can perform 9; how many questions can both of them perform in 10 hours?

38. How many are 10 times 19?

40. How many are 2 times 14? 3 times 14? 4 times 14? 5 times 14? 6 times 14? 7 times 14? 8 times 14? 9 times 14? 10 times 14?

42. In one pound there are 16 ounces; how many ounces are there in 2 pounds? 3 pounds? 4 pounds? 5 pounds? 6 pounds? 7 pounds? 8 pounds? 9 pounds? 10 pounds?

44. How many are 2 times 18? 3 times 18? 4 times 18? 5 times 18? 6 times 18? 7 times 18? 8 times 18? 9 times 18? 10 times 18?

45. If a yard of cloth is worth 19 cents, how many cents are 2 yards worth? 3 yards? 4 yards? 5 yards? 6 yards? 7 yards? 8 yards? 9 yards? 10 yards?

47. How many are 2 times 30? 3 times 30? 4 times 30? 5 times 30? 6 times 30? 7 times 30? 8 times 30? 9 times 30? 10 times 30?

49. How many are 2 times 50? 3 times 50? 4 times 50? 5 times 50? 6 times 50? 7 times 50? 8 times 50? 9 times 50? 10 times 50?

46. If an acre of land is worth 20 dollars, how many dollars are 2 acres worth? 3 acres? 4 acres? 5 acres? 6 acres? 7 acres? 8 acres? 9 acres? 10 acres?

48. How many are 2 times 40? 3 times 40? 4 times 40? 5 times 40? 6 times 40? 7 times 40? 8 times 40? 9 times 40? 10 times 40?

50. How many are 2 times 60? 3 times 60? 4 times 60? 5 times 60? 6 times 60? 7 times 60? 8 times 60? 9 times 60? 10 times 60?

Art. 21. If an acre of land is worth 125 dollars, how many dollars are 9 acres worth?

If one acre is worth 125 dollars, 9 acres are worth 9 times 125 dollars; and 9 times 125 dollars can be found by multiplying 125 by 9 in the following manner:

hund. tens. units.
125 dols.
9
——
Ans. 1125 dols.

We write down the multiplicand 125, and write the multiplier 9 under the units of the multiplicand. We then say 9 times 5 units are 45 units, or 4 tens and 5 units; we write down the 5 units, and reserve the 4 tens; then 9 times 2 tens are 18 tens, and 4 tens which we reserved are 22 tens, or 2 hundred and 2 tens; we write down the 2 tens and reserve the 2 hundred; then 9 times 1 hundred are 9 hundred, and 2 hundred which we reserved are 11 hundred, or 1 thousand and 1 hundred; both of which we write down, and the whole product is 1125, the number of dollars that 9 acres are worth.

From the preceding question and its illustration, we derive the following rule for multiplication, when the multiplier does not exceed 12.

RULE. *Write down the multiplicand, then write the multiplier under the multiplicand, placing units under units, tens under tens, and draw a line underneath.*

Multiply, successively, each figure of the multiplicand, beginning with units, by the multiplier; when the product of any figure does not exceed nine, write it under the figure multiplied; when the product exceeds nine, write down the *right hand figure of it, and add the left hand figure, or figures, to the product of the next figure, and write down the whole product of the last figure.*

51. Multiply 4562 by 2. Product 9124
52. Multiply 7083 by 3. Product 21249
53. Multiply 9407 by 4. Product 37628
54. Multiply 10456 by 5. Product 52280
55. Multiply 24104 by 6. Product 144624
56. Multiply 64125 by 7. Product 448875
57. Multiply 75476 by 8. Product 603808
58. Multiply 467109 by 9. Product 4203981
59. Multiply 741376 by 10. Product 7413760
60. Multiply 970614 by 11. Product 10676754
61. Multiply 1417517 by 12. Product 17010204
62. Multiply 5056045 by 12. Product 60672540

63. There are 1760 yards in one mile; how many yards are there in 9 miles?

64. In one year there are 365 days; how many days are there in 11 years?

65. There are 2240 pounds in one ton; how many pounds are there in 12 tons?

Art. 22. What will be the product of 7654 multiplied by 543?

```
   7654
    543
  -----
  22962 = 3 times the multiplicand.
 30616  = 40 times the multiplicand.
38270   = 500 times the multiplicand.
-------
Prod. 4156122 = 543 times the multiplicand.
```

ILLUSTRATION. When the multiplier consists of several figures, we first multiply the multiplicand by the units of the multiplier, as directed in the preceding rule. We next multiply the multiplicand by the tens of the multiplier, and write the first figure of the product in the place of tens, because units multiplied by tens produce tens. Lastly, we multiply the multiplicand by the hundreds of the multiplier, and write the first figure of the product in the place of hundreds, because units multiplied by hundreds produce hundreds. Finally, we find the sum of the several products, and *the total product* is 4156122.

From the preceding illustration, we deduce the following rule for multiplication, when the multiplier consists of several figures.

RULE. *Write down the multiplicand, then write the multiplier under the multiplicand, placing units under units, tens under tens, and hundreds under hundreds.*

Multiply the multiplicand by each significant figure of the multiplier, in succession, beginning with units, and write the first figure of each product directly under the figure by which you are multiplying.

Find the sum of the several products; their sum will be the total product required.

METHODS OF PROOF. *Multiplication is proved by division. If the product be divided by either of the factors, the quotient will be equal to the other factor.*

Multiplication may be proved by the following method. Find the excess of 9s in the multiplicand, then find the excess of 9s in the multiplier; multiply one excess by the other; if the excess of 9s in their product is equal to the excess of 9s in the total product, the work is supposed to be right. This method of proof depends upon a property of the number 9, viz., that any number divided by 9, will leave the same remainder, as the sum of its figures divided by 9.

66. Multiply 48125 by 35. Product 1684375
67. Multiply 75452 by 47. Product 3546244
68. Multiply 94543 by 58. Product 5483494
69. Multiply 75015 by 63. Product 4725945
70. Multiply 87417 by 76. Product 6643692
71. Multiply 54302 by 89. Product 4832878
72. Multiply 80704 by 432. Product 34864128
73. Multiply 70408 by 7605. Product 535452840
74. Multiply 479856 by 59807. Product 28698747792
75. Multiply 8706544 by 67089. Product 584113330416
76. Multiply 8496427 by 874359. Product 7428927415293
77. Multiply 98765432 by 98765432. Product 9754610558146624

78. If an acre of land is worth 225 dollars, what is the value of 475 acres?

79. If an acre of land produce 32 bushels of wheat, how many bushels will 64 acres produce?

80. If a man travels uniformly 35 miles each day, how many miles will he travel in 35 days?

81. What number of dollars would be required to pay 925 men 96 dollars each?

82. If 45 men perform a piece of work in 25 days, how many men will be required to perform the same piece of work in 1 day?

Art. 23. When any of the right hand figures of the multiplicand, or multiplier, are ciphers.

According to the principle of notation, annexing one cipher to any number, multiplies it by 10; annexing two ciphers, multiplies it by 100; annexing three ciphers, multiplies it by 1000; and so on.

ILLUSTRATION. Multiply 475 by 10, by 100, and by 1000.

475	475	475
10	100	1000
4750 product.	47500	475000

From the above principle and illustration we derive the following

RULE. *Place the significant figures of the multiplier under the significant figures of the multiplicand, then, omitting the ciphers, multiply the significant figures of the multiplicand by the significant figures of the multiplier, and annex as many ciphers to the product as were omitted in both factors.*

83. Multiply	425	by	30.	Product	12750
84. Multiply	75600	by	40.	Product	3024000
85. Multiply	804000	by	50.	Product	40200000
86. Multiply	905040	by	1200.	Product	1086048000
87. Multiply	9045200	by	70650.	Product	639043380000
88. Multiply	50750000	by	75000.	Product	3806250000000

89. A farmer purchased 20 cows at 25 dollars each, what did the 20 cows cost?

90. What is the value of a farm containing 200 acres, at 30 dollars an acre?

91. What will 750 shares of railroad stock amount to, at 500 dollars a share?

92. If a passenger train of railroad cars travel 20 miles an hour, how many miles will it travel in 50 hours?

93. There are 320 rods in one mile; what number of rods are there in 200 miles?

94. What would be the cost of a railroad 250 miles in length, at 30,000 dollars a mile?

95. What is the capital stock of Boston and Lowell Railroad, there being 3600 shares, at 500 dollars a share?

96. What is the capital stock of Merchants' Bank, in Boston, consisting of 30000 shares of 100 dollars each?

Art. 24. When the multiplier is equal to the product of any two numbers, each of which does not exceed 12, it is often more convenient to multiply by the factors of the multiplier.

ILLUSTRATION. If a yard of cloth be worth 17 cents, what are 15 yards worth?

In this question, the multiplier 15, is equal to the product of the factors 3 and 5. Multiplying 17 cents, the value of one yard, by 3, the product is 51 cents, the value of 3 yards. Then multiplying 51 cents by 5, the product is 255 cents, or 2 dollars and 55 cents, the value of 15 yards.

From the preceding illustration we obtain the following

RULE. *Find any two numbers whose product is equal to the multiplier. Multiply the multiplicand by one of these numbers, then multiply the product by the other; the last product is the total product required.*

97.	Multiply	1615	by	21.	Product	33915
98.	Multiply	2765	by	24.	Product	66360
99.	Multiply	3864	by	28.	Product	108192
100.	Multiply	4927	by	36.	Product	177372
101.	Multiply	5325	by	45.	Product	239625
102.	Multiply	32435	by	64.	Product	2075840
103.	Multiply	41732	by	72.	Product	3004704
104.	Multiply	56217	by	84.	Product	4722228
105.	Multiply	64541	by	108.	Product	6970428
106.	Multiply	75644	by	144.	Product	10892736

107. What will 120 yards of cloth come to, at 6 dollars a yard?

108. What is the value of 132 acres of land, at 34 dollars an acre?

109. If 1 acre of land produce 36 bushels of corn, what number of bushels will 18 acres produce?

110. If an acre of land produce 225 bushels of potatoes, what number of bushels will 25 acres produce?

111. If a ship sails uniformly 175 miles each day, what number of miles will she sail in 27 days?

112. A man purchased a wood lot containing 33 acres, at 35 dollars an acre; what did it cost him?

113. If a bale of sheeting contains 32 pieces, and each piece measures 36 yards, what number of yards does the bale contain?

114. A merchant purchased 42 pieces of broadcloth, each piece containing 38 yards; what number of yards did he purchase?

DIVISION OF SIMPLE NUMBERS.

Art. 25. Division is the method of finding the number of times, or part of a time, one number is contained in another. By division, we can also find any required part of any given number.

The terms used in division, are the dividend, divisor, quotient, and remainder.

The dividend is the number to be divided. The divisor is the number to divide by, and it indicates what part of the dividend is required by the question. The quotient is the number found by the operation, and it expresses the number of times, or part of a time, the divisor is contained in the dividend; — it also expresses the number of units, or part of a unit, in that part of the dividend required by the question. The remainder, if any, is the undivided part of the dividend, and is always less than the divisor; consequently, it will contain the divisor only a part of a time, which may be expressed by writing the remainder over a short line, and the divisor under it, at the right of the quotient figures already obtained. This part is called a fraction.

Illustration. How many times is 5 contained in 20? We know that 4 times 5 are 20; hence, it is plain that 5 is contained in 20, 4 times. In this question, 20 is the dividend, 5 is the divisor, and it indicates the part of the dividend required, viz., 1 fifth. The quotient, 4, is the number of times the divisor is contained in the dividend; and it is also the number of units in 1 fifth of the dividend.

What is 1 fifth of 32? 1 fifth of 30 is 6. 1 fifth of the remainder 2, is 2 fifths of 1. Then 1 fifth of 32 is 6$\frac{2}{5}$.

A short horizontal line between two points, indicates that the number before it is to be divided by the number after it; thus, $30 \div 6 = 5$.

1. How many peaches, at 2 cents each, can you buy with 4 cents?

2. How many oranges, at 3 cents each, can you buy with 6 cents?

3. 4 are how many times 2?

4. 6 are how many times 3?

5. 1 half of 4 is how many?

6. 1 third of 6 is how many?

7. How many yards of cloth, at 4 dollars a yard, can you buy with 12 dollars?

8. In 1 gallon there are 4 quarts; how many gallons are there in 20 quarts?

9. 12 are how many times 4?

10. 20 are how many times 4?

11. 1 fourth of 12 is how many?

12. 1 fifth of 20 is how many?

DIVISION TABLE.

2 in 2 1 time.
2 in 4 2 times.
2 in 6 3 times.
2 in 8 4 times.
2 in 10 5 times.
2 in 12 6 times.
2 in 14 7 times.
2 in 16 8 times.
2 in 18 9 times.
2 in 20 10 times.
2 in 22 11 times.
2 in 24 12 times.

3 in 3 1 time.
3 in 6 2 times.
3 in 9 3 times.
3 in 12 4 times.
3 in 15 5 times.
3 in 18 6 times.
3 in 21 7 times.
3 in 24 8 times.
3 in 27 9 times.
3 in 30 10 times.
3 in 33 11 times.
3 in 36 12 times.

4 in 4 1 time.
4 in 8 2 times.
4 in 12 3 times.
4 in 16 4 times.
4 in 20 5 times.
4 in 24 6 times.
4 in 28 7 times.
4 in 32 8 times.
4 in 36 9 times.
4 in 40 10 times.
4 in 44 11 times.
4 in 48 12 times.

5 in 5 1 time.
5 in 10 2 times.
5 in 15 3 times.
5 in 20 4 times.
5 in 25 5 times.
5 in 30 6 times.
5 in 35 7 times.
5 in 40 8 times.
5 in 45 9 times.
5 in 50 10 times.
5 in 55 11 times.
5 in 60 12 times.

6 in 6 1 time.
6 in 12 2 times.
6 in 18 3 times.
6 in 24 4 times.
6 in 30 5 times.
6 in 36 6 times.
6 in 42 7 times.
6 in 48 8 times.
6 in 54 9 times.
6 in 60 10 times.
6 in 66 11 times.
6 in 72 12 times.

7 in 7 1 time.
7 in 14 2 times.
7 in 21 3 times.
7 in 28 4 times.
7 in 35 5 times.
7 in 42 6 times.
7 in 49 7 times.
7 in 56 8 times.
7 in 63 9 times.
7 in 70 10 times.
7 in 77 11 times.
7 in 84 12 times.

8 in 8 1 time.
8 in 16 2 times.
8 in 24 3 times.
8 in 32 4 times.
8 in 40 5 times.
8 in 48 6 times.
8 in 56 7 times.
8 in 64 8 times.
8 in 72 9 times.
8 in 80 10 times.
8 in 88 11 times
8 in 96 12 times.

9 in 9 1 time.
9 in 18 2 times.
9 in 27 3 times.
9 in 36 4 times.
9 in 45 5 times.
9 in 54 6 times.
9 in 63 7 times.
9 in 72 8 times.
9 in 81 9 times.
9 in 90 10 times.
9 in 99 11 times.
9 in 108 12 times.

10 in 10 1 time.
10 in 20 2 times.
10 in 30 3 times.
10 in 40 4 times.
10 in 50 5 times.
10 in 60 6 times.
10 in 70 7 times.
10 in 80 8 times.
10 in 90 9 times.
10 in 100 10 times.
10 in 110 11 times.
10 in 120 12 times.

11 in 11 1 time.
11 in 22 2 times.
11 in 33 3 times.
11 in 44 4 times.
11 in 55 5 times.
11 in 66 6 times.
11 in 77 7 times.
11 in 88 8 times.
11 in 99 9 times.
11 in 110 10 times
11 in 121 11 times.
11 in 132 12 times.

12 in 12 1 time.
12 in 24 2 times.
12 in 36 3 times.
12 in 48 4 times.
12 in 60 5 times.
12 in 72 6 times.
12 in 84 7 times.
12 in 96 8 times.
12 in 108 9 times.
12 in 120 10 times.
12 in 132 11 times.
12 in 144 12 times.

DIVISION TABLE.

1 half of 2 is 1
1 half of 4 is 2
1 half of 6 is 3
1 half of 8 is 4
1 half of 10 is 5
1 half of 12 is 6
1 half of 14 is 7
1 half of 16 is 8
1 half of 18 is 9
1 half of 20 is 10
1 half of 22 is 11
1 half of 24 is 12

1 third of 3 is 1
1 third of 6 is 2
1 third of 9 is 3
1 third of 12 is 4
1 third of 15 is 5
1 third of 18 is 6
1 third of 21 is 7
1 third of 24 is 8
1 third of 27 is 9
1 third of 30 is 10
1 third of 33 is 11
1 third of 36 is 12

1 fourth of 4 is 1
1 fourth of 8 is 2
1 fourth of 12 is 3
1 fourth of 16 is 4
1 fourth of 20 is 5
1 fourth of 24 is 6
1 fourth of 28 is 7
1 fourth of 32 is 8
1 fourth of 36 is 9
1 fourth of 40 is 10
1 fourth of 44 is 11
1 fourth of 48 is 12

1 fifth of 5 is 1
1 fifth of 10 is 2
1 fifth of 15 is 3
1 fifth of 20 is 4
1 fifth of 25 is 5
1 fifth of 30 is 6
1 fifth of 35 is 7
1 fifth of 40 is 8
1 fifth of 45 is 9
1 fifth of 50 is 10
1 fifth of 55 is 11
1 fifth of 60 is 12

1 sixth of 6 is 1
1 sixth of 12 is 2
1 sixth of 18 is 3
1 sixth of 24 is 4
1 sixth of 30 is 5
1 sixth of 36 is 6
1 sixth of 42 is 7
1 sixth of 48 is 8
1 sixth of 54 is 9
1 sixth of 60 is 10
1 sixth of 66 is 11
1 sixth of 72 is 12

1 seventh of 7 is 1
1 seventh of 14 is 2
1 seventh of 21 is 3
1 seventh of 28 is 4
1 seventh of 35 is 5
1 seventh of 42 is 6
1 seventh of 49 is 7
1 seventh of 56 is 8
1 seventh of 63 is 9
1 seventh of 70 is 10
1 seventh of 77 is 11
1 seventh of 84 is 12

1 eighth of 8 is 1
1 eighth of 16 is 2
1 eighth of 24 is 3
1 eighth of 32 is 4
1 eighth of 40 is 5
1 eighth of 48 is 6
1 eighth of 56 is 7
1 eighth of 64 is 8
1 eighth of 72 is 9
1 eighth of 80 is 10
1 eighth of 88 is 11
1 eighth of 96 is 12

1 ninth of 9 is 1
1 ninth of 18 is 2
1 ninth of 27 is 3
1 ninth of 36 is 4
1 ninth of 45 is 5
1 ninth of 54 is 6
1 ninth of 63 is 7
1 ninth of 72 is 8
1 ninth of 81 is 9
1 ninth of 90 is 10
1 ninth of 99 is 11
1 ninth of 108 is 12

1 tenth of 10 is 1
1 tenth of 20 is 2
1 tenth of 30 is 3
1 tenth of 40 is 4
1 tenth of 50 is 5
1 tenth of 60 is 6
1 tenth of 70 is 7
1 tenth of 80 is 8
1 tenth of 90 is 9
1 tenth of 100 is 10
1 tenth of 110 is 11
1 tenth of 120 is 12

1 eleventh of 11 is 1
1 eleventh of 22 is 2
1 eleventh of 33 is 3
1 eleventh of 44 is 4
1 eleventh of 55 is 5
1 eleventh of 66 is 6
1 eleventh of 77 is 7
1 eleventh of 88 is 8
1 eleventh of 99 is 9
1 eleventh of 110 is 10
1 eleventh of 121 is 11
1 eleventh of 132 is 12

1 twelfth of 12 is 1
1 twelfth of 24 is 2
1 twelfth of 36 is 3
1 twelfth of 48 is 4
1 twelfth of 60 is 5
1 twelfth of 72 is 6
1 twelfth of 84 is 7
1 twelfth of 96 is 8
1 twelfth of 108 is 9
1 twelfth of 120 is 10
1 twelfth of 132 is 11
1 twelfth of 144 is 12

Art. 28. 13. If 4 oranges cost 24 cents, what will 1 orange cost? What will 2 oranges cost?

If 4 oranges cost 24 cents, 1 orange will cost 1 fourth of 24 cents, or 6 cents. 2 oranges will cost 2 times 6 cents, or 12 cents.

15. If 6 pieces of tape are worth 48 cents, what is 1 piece worth? What are 9 pieces worth? What are 10 pieces worth?

17. If 2 pounds of cheese cost 18 cents, what will 5 pounds cost?

19. If 18 is 2 fifths of some number, what is 5 fifths, or the whole of the number?

21. If 6 pounds of beef cost 72 cents, what will 10 pounds cost?

23. If 72 is 6 tenths of some number, what is 10 tenths, or the whole of the number?

25. If 8 yards of flannel cost 96 cents, what will 10 yards cost? What will 12 yards cost?

27. If 96 is 8 tenths of some number, what is 10 tenths, or the whole of the number?

29. If 2 pounds of butter are worth 40 cents, what are 6 pounds worth?

31. If 6 yards of linen are worth 120 cents, what are 2 yards worth?

33. If 4 horses are worth 320 dollars, what are 8 horses worth? What are 10 horses worth?

35. If 8 yoke of oxen are worth 640 dollars, what are 5 yoke worth?

37. What is 1 third of 60? of 90? of 120? of 150?

39. What is 1 fifth of 90? of 100? of 110? of 120?

41. What is 1 seventh of 140? of 210? of 280? of 350?

43. What is 1 ninth of 180? of 270? of 360? of 540?

45. What is 1 eleventh of 330? of 440? of 550? of 660?

14. If 5 yards of ribbon cost 40 cents, how many cents will 1 yard cost? How many cents will 6 yards cost?

If 5 yards cost 40 cents, 1 yard will cost 1 fifth of 40 cents, or 8 cents. 6 yards will cost 6 times 8 cents, or 48 cents.

16. If 7 pounds of sugar are worth 56 cents, what is 1 pound of it worth? What is the value of 10 pounds?

18. If 5 yards of cloth are worth 50 cents, what are 8 yards worth?

20. If 50 is 5 eighths of some number, what is 8 eighths, or the whole of the number?

22. If 7 yards of cloth cost 77 cents, what will 11 yards cost?

24. If 77 is 7 elevenths of some number, what is 11 elevenths, or the whole of the number?

26. If 9 barrels of flour are worth 63 dollars, what are 12 barrels worth?

28. If 63 is 9 twelfths of some number, what is 12 twelfths, or the whole of the number?

30. If 3 acres of land are worth 90 dollars, what are 8 acres worth?

32. If 8 cows are worth 240 dollars, what are 3 of them worth?

34. If a railroad train travel 240 miles in 12 hours, how many miles will it travel in 5 hours?

36. How many miles will it travel in 20 hours? How many in 30 hours?

38. What is 1 fourth of 60? of 80? of 100? of 120?

40. What is 1 sixth of 90? of 120? of 180? of 240?

42. What is 1 eighth of 160? of 240? of 320? of 400?

44. What is 1 tenth of 200? of 250? of 300? of 350?

46. What is 1 twelfth of 240? of 360? of 480? of 600?

Art. 27. *In the preceding article, we have proposed only such questions as required us to find the number of times a greater number contained a less, or to find one or more of the equal parts of any given number. In this article, we shall propose questions which will require us to find what part of one time a less number contains a greater;*

47. If I can buy an orange with 2 cents, what part of an orange can I buy with 1 cent?

As I can buy a whole orange with 2 cents, and 1 cent being 1 half of 2 cents, I can buy 1 half of an orange with 1 cent.

49. What part of one time is 2 contained in 1?

51. If a yard of cloth is worth 4 dollars, what part of a yard can be purchased with 1 dollar? what part, with 2 dollars? what part, with 3 dollars?

53. What part of one time is 4 contained in 1? what part, in 2? what part, in 3?

55. When wheat is worth 6 shillings a bushel, what part of a bushel can you purchase with 1 shilling? what part, with 2 shillings? what part, with 3 shillings? what part, with 4 shillings? what part, with 5 shillings?

57. What part of one time is 6 contained in 1? what part, in 2? what part, in 3? what part, in 4? what part, in 5?

59. When coal is selling at 8 dollars a ton, what part of a ton can you buy with 1 dollar? what part, with 2 dollars? what part, with 3 dollars? what part, with 4 dollars? what part, with 5 dollars? what part, with 6 dollars? what part, with 7 dollars?

61. What is 1 eighth of 1? what is 1 eighth of 2? what is 1 eighth of 3? what is 1 eighth of 4? what is 1 eighth of 5? what is 1 eighth of 6? what is 1 eighth of 7?

48. If a yard of tape is worth 3 cents, what part of a yard can you buy with 1 cent? what part, with 2 cents?

50. What part of one time is 3 contained in 1? what part, in 2?

52. When flour is worth 5 dollars a barrel, what part of a barrel can you buy with 1 dollar? what part, with 2 dollars? what part, with 3 dollars? what part, with 4 dollars?

54. What part of one time is 5 contained in 1? what part, in 2? what part, in 3? what part, in 4?

56. When wood is worth 7 dollars a cord, what part of a cord can be purchased with 1 dollar? what part, with 2 dollars? what part, with 3 dollars? what part, with 4 dollars? what part, with 5 dollars? what part, with 6 dollars?

58. What is 1 seventh of 1? what is 1 seventh of 2? what is 1 seventh of 3? what is 1 seventh of 4? what is 1 seventh of 5? what is 1 seventh of 6?

60. When sugar is worth 9 dollars a hundred, what part of a hundred can you buy with 1 dollar? what part, with 2 dollars? what part, with 3 dollars? what part, with 4 dollars? what part, with 5 dollars? what part, with 6 dollars? what part, with 7 dollars? what part, with 8 dollars?

62. What is 1 ninth of 1? what is 1 ninth of 2? what is 1 ninth of 3? what is 1 ninth of 4? what is 1 ninth of 5? what is 1 ninth of 6? what is 1 ninth of 7? what is 1 ninth of 8?

Art. 28. *This article contains questions, which require us to find the number of times and parts of a time that a greater number contains a less.*

63. If an orange is worth 2 cents, how many oranges can you buy with 5 cents?

As many times as 2 cents is contained in 5 cents, so many oranges you can buy. 5 cents are 2 and 1 half times 2 cents; therefore you can buy 2 and 1 half oranges with 5 cents.

65. What is 1 half of 5? of 7? of 9? of 11?

67. In 1 penny there are 4 farthings. How many pence are there in 13 farthings? how many, in 19 farthings? how many, in 27 farthings? how many, in 31 farthings?

69. What is 1 fourth of 29? of 34? of 43?

71. In 1 dollar there are 6 shillings. What number of dollars are there in 27 shillings? what number, in 35 shillings? what number, in 45 shillings? what number, in 50 shillings?

73. What is 1 sixth of 59? of 63? of 70?

75. What number of pounds of raisins, at 8 cents a pound, can you purchase with 65 cents? what number, with 75 cents? what number, with 95 cents?

77. What is 1 eighth of 74? of 84? of 94?

79. When beef is worth 10 cents a pound, what number of pounds can you buy with 75 cents? what number, with 88 cents? what number, with 99 cents?

81. What is 1 tenth of 104? of 115? of 125?

83. What is 1 third of 20? of 40? of 50? of 70? of 80?

85. What is 1 seventh of 30? of 50? of 70? of 90? of 110?

64. In 1 yard there are 3 feet. How many yards are there in 11 feet?

As 3 feet are equal to 1 yard, there must be 1 third as many yards as there are feet. 1 third of 11 is 3 and 2 thirds; therefore there are 3 and 2 thirds yards in 11 feet.

66. What is 1 third of 13? of 17? of 19?

68. How many skeins of silk can you purchase with 37 cents, at 5 cents a skein? how many, with 48 cents? how many, with 54 cents? how many, with 62 cents?

70. What is 1 fifth of 53? of 57? of 63?

72. In 1 week there are 7 days. What number of weeks are there in 37 days? what number, in 45 days? what number, in 48 days? what number, in 54 days?

74. What is 1 seventh of 47? of 54? of 65?

76. What number of yards of ribbon, at 9 cents a yard, can you buy with 65 cents? what number, with 75 cents? what number, with 85 cents?

78. What is 1 ninth of 84? of 94? of 100?

80. When poultry is selling at 12 cents a pound, what number of pounds can you purchase with 100 cents? what number, with 125 cents? what number, with 136 cents?

82. What is 1 twelfth of 117? of 127? of 137?

84. What is 1 fourth of 30? of 50? of 70? of 90? of 110?

86. What is 1 sixth of 20? of 40? of 50? of 70? of 80?

Art. 29. How many times is 9 contained in 675?

As we cannot easily find an answer to this question mentally, we write down the dividend and draw a curve line on each side of it, and then write the divisor at the left of the dividend.

```
   hund.
   tens.
   units.   tens.
            units.
9)675(75
  63
  --
   45
   45
   --
```

We first resolve the dividend, 675, into 67 tens and 5 units. We next find the number of times that the divisor, 9, is contained in 67 tens, which is 7 (tens) times; we write the 7 (tens) at the right of the dividend and multiply the divisor by the 7, (tens,) the product is 63 tens, which we write under the 67 tens; we then subtract 63 tens from 67 tens, the remainder is 4 tens. We then place the 5 units of the dividend at the right of the remainder, and we have the number 45. The divisor, 9, is contained 5 times in 45; we write the 5 units at the right of the 7 tens in the quotient, and multiply the divisor by the 5 units, the product is 45, which we write under the partial dividend, 45. There being no remainder, the operation is completed, and we have found that 9 is contained 75 times in 675.

88. If 10624 dollars be equally divided among 15 men, what number of dollars will each man receive?

We first write down the dividend, and then write the divisor at its left as before. The two highest orders of figures in the dividend, 10, (thousand,) being less than the divisor, 15, we resolve the dividend into 106 hundred, 2 tens, and 4 units.

```
Divisor. Dividend. Quotient.                 Method of proof.
 15)10624(708 4/15 Ans.                     708        4
   *105                                      15       15
    ----                                    ----     -----
     124                                    3540    15)60(4
    *120                                    708        60
     ---                                   -----      ---
      *4 remainder.                        10620
   -------                                     4
   10624 proof by addition.                -----
                                           10624
```

We now perceive that the divisor, 15, is contained in 106 (hundred) 7 (hundred) times; we write 7 (hundred) in the quotient, and multiply the divisor by it, the product is 105 hundred, which we write under 106 hundred; we then subtract and the remainder is 1 hundred; we then place the 2 tens of the dividend at the right of the remainder and the number is 12 tens, which being less than the divisor, we write a cipher in the place of tens in the quotient, and then place the 4 units of the dividend at the right of the 12 tens, and the number is 124; we find that 15 is contained in 124, 8 times; we write 8 in the quotient, and multiply the divisor by it, the product is 120, which we subtract from 124, and the remainder is 4. This remainder, 4, will contain 15, 4 fifteenths of 1 time, which we write in a fractional form at the right of the quotient figures before found, and we have the complete quotient, $708\frac{4}{15}$, which is the number of dollars that each man will receive.

Method of Proof. We first multiply 4 fifteenths by the divisor, 15, the product is 60 fifteenths; we then divide 60 fifteenths by 15, the number of fifteenths which are equal to a unit; the quotient is 4 units, which is the product of 4 fifteenths multiplied by 15. We then multiply 708 by 15, the product is 10620. We then add the 4 units to 10620, and the sum is 10624, which is the total product of the quotient multiplied by the divisor. This product being equal to the dividend, proves that the operation has been performed correctly.

As the operation of division may be proved by multiplying the divisor by the quotient, and this is always done during the operation, we have only to add the several products in the order they are placed, the sum of these products will be equal to the dividend if the operation has been rightly performed. When there is a remainder, it must be added with the several products. The stars indicate the numbers to be added.

From the preceding questions and illustrations we derive the following general rule for division.

Rule. *Write down the dividend, draw a curve line on each side of it, and write the divisor at its left.*

Take so many of the highest orders of figures of the dividend as will contain the divisor, find the number of times the divisor is contained in them, write a figure expressing the number at the right of the dividend, then multiply the divisor by this quotient figure, and write the product under those figures of the dividend taken.

Subtract this product from those figures, and place the next undivided figure of the dividend at the right of the *remainder; then divide these orders of figures as before, and thus proceed until all the figures of the dividend are divided.*

If there be a final remainder, write it over a short line at the right of the quotient figures already found, and place the divisor under it; this fraction will express what part of a time the remainder contains the divisor, and completes the quotient.

Whenever a figure of the dividend has been annexed to the remainder, if this partial dividend is less than the divisor, write a cipher in the quotient, and annex another figure.

Whenever the product of the divisor by any quotient figure is greater than the partial dividend, the quotient figure is too great, and must be made less.

Whenever a remainder is equal to or greater than the divisor, the quotient figure is too small, and must be made greater.

89. Divide 4056 by 13. Quotient, 312.
90. Divide 5475 by 15. Quotient, 365.
91. Divide 6548 by 17. Quotient, $385\frac{3}{17}$.
92. Divide 7649 by 19. Quotient, $402\frac{11}{19}$.
93. Divide 8757 by 21. Quotient, 417.
94. What is 1 twenty-fourth of 17472? Ans. 728.
95. What is 1 twenty-seventh of 62100? Ans. 2300.
96. What is 1 thirty-fifth of 75435? Ans. $2155\frac{10}{35}$.
97. Divide 84625 by 47. Quotient, $1800\frac{25}{47}$.
98. Divide 96108 by 59. Quotient, $1628\frac{56}{59}$.
99. Divide 112648 by 64. Quotient, $1760\frac{8}{64}$.
100. What is 1 seventy-fifth of 215419? Ans. $2872\frac{19}{75}$.
101. What is 1 eighty-ninth of 417645? Ans. $4692\frac{57}{89}$.
102. What is 1 ninety-sixth of 444372? Ans. $4628\frac{84}{96}$.
103. Divide 557642 by 142. Quotient, $3927\frac{8}{142}$.
104. Divide 641144 by 276. Quotient, $2322\frac{272}{276}$.
105. Divide 864325 by 394. Quotient, $2193\frac{283}{394}$.
106. How many times is 912 contained in 1893312? Ans. 2076.
107. How many times is 2076 contained in 3786624? Ans. 1824.
108. How many times is 6474 contained in 47254149? Ans. $7299\frac{423}{6474}$.
109. How many times is 8465 contained in 761858465? Ans. 90001.
110. What number of times is 912314 contained in 280208122081? Ans. $307140\frac{121}{912314}$.

111. A man purchased a farm containing 275 acres for 6875 dollars; how many dollars did he pay for each acre?

112. If a man's income be 5110 dollars a year, what is that a day, allowing the year to contain 365 days?

113. There are 144 square inches in 1 square foot; how many square feet are there in 10800 square inches?

114. In 1 acre there are 160 square rods; how many acres are there in 28000 square rods?

115. There are 320 rods in 1 mile; how many miles are there in 320640 rods?

116. In 1 cubic foot there are 1728 cubic inches; how many cubic feet are there in 2985984 cubic inches?

117. The distance from Boston to Liverpool is about 3000 miles; in how many days will a steamship sail from Boston to Liverpool, if she sail 225 miles each day?

118. The circumference of the earth is about 25000 miles; if a railroad car travel at the rate of 480 miles a day, in how many days would it travel round the earth?

Art. 30. *When the divisor does not exceed 12, the operation may be shortened by performing the operation mentally, and writing only the quotient.*

Illustration. If 109025 acres of land be equally divided among 12 men, what number of acres will each man have?

12)109025

Ans. $9085\frac{5}{12}$ acres.

The two highest orders of figures being less than the divisor, we take the three highest, 109, (thousand,) and find that 12 is contained in 109 (thousand) 9 (thousand) times; we write the 9 under the order of thousands in the dividend, and, mentally, multiply the divisor by it, the product is 108, (thousand,) which we subtract from 109, (thousand,) the remainder is 1, (thousand;) this remainder and next figure of the dividend annexed, 10, (hundred,) being less than the divisor, we write a cipher in the place of hundreds in the quotient; we then find that the divisor is contained in the remainder and two next figures of the dividend annexed, 102, (tens,) 8 times; we write 8 in the quotient in the place of tens, and multiply the divisor by it; the product is 96, (tens,) which we subtract from 102, (tens,) the remainder is 6, (tens;) we then find that the divisor is contained in this remainder and last figure of the dividend annexed, 65, 5 times; we write 5 in the quotient in the place of units, we then multiply the divisor by it, and the product is 60, which we subtract from 65, the remainder is 5; this remainder will contain the divisor 5 twelfths of 1 time, which we place at the right of the quotient figures before found, and the complete quotient is $9085\frac{5}{12}$, the number of acres which each man will have.

From the preceding illustration, we derive the following rule for division, when the divisor does not exceed 12:

RULE. *Write down the dividend, draw a curve line at its left, and a horizontal line under it, then write the divisor at the left of the dividend.*

Find the number of times the divisor is contained in so many of the highest orders of figures of the dividend as will contain it, write the quotient figure under the number divided, then, mentally, multiply the divisor by this quotient figure and subtract the product from the number divided; then find the number of times the divisor is contained in the remainder and next figure of the dividend, or in the next figure of the dividend only, when there is no remainder, write the quotient figure underneath, multiply and subtract as before directed; proceed in the same manner until all the figures of the dividend are divided.

If the remainder and next figure of the dividend, or next figure of the dividend only, when there is no remainder, be less than the divisor, write a cipher in the quotient, and then find the number of times the divisor is contained in the remainder and two next figures of the dividend.

119. Divide 235464 by 2.	Quotient, 117732.
120. Divide 347675 by 3.	Quotient, $115891\frac{2}{3}$.
121. Divide 235464 by 4.	Quotient, 58866.
122. Divide 347675 by 5.	Quotient, 69535.
123. Divide 456148 by 6.	Quotient, $76024\frac{4}{6}$.
124. Divide 514763 by 7.	Quotient, $73537\frac{4}{7}$.
125. Divide 456148 by 8.	Quotient, $57018\frac{4}{8}$.
126. Divide 749999 by 9.	Quotient, $83333\frac{2}{9}$.
127. Divide 645448 by 10.	Quotient, $64544\frac{8}{10}$.
128. Divide 514763 by 11.	Quotient, $46796\frac{7}{11}$.
129. Divide 749995 by 12.	Quotient, $62499\frac{7}{12}$.

130. If 11 acres of land be worth 1210 dollars, what is 1 acre worth?

131. The salary of the president of the United States is 25000 dollars a year; what is his salary for 1 month?

Art. 31. ***When the divisor is equal to the product of any two numbers, or factors, each of which does not exceed 12.***

132. If 15 acres of land are worth 300 dollars, how many dollars is 1 acre worth?

$$3\times5=15\left\{\begin{array}{l}3)300 \text{ dollars, value of 15 acres.}\\ \hline 5)100 \text{ dollars, value of 5 acres.}\end{array}\right.$$

$$\overline{20} \text{ dollars, value of 1 acre.}$$

The factors of 15 are 3 and 5. Dividing 300 dollars by 3, the quotient is 100 dollars, the value of 1 third of 15 acres, or 5 acres; dividing 100 dollars by 5, the quotient is 20 dollars, the value of 1 acre.

In dividing by the factors of the divisor, remainders frequently occur, and the quotient or answer being incomplete without them, we shall explain the method of finding the true remainder.

What will be the quotient of 395 divided by 16?

$$4\times4=16.\left\{\begin{array}{l}4)395\\ \hline 4)98-3\end{array}\right.$$

$$\overline{24-2}\times4=8+3=11, \text{ total remainder.}$$

Ans. $24\frac{11}{16}$.

We may observe that every unit in the first quotient may be considered as containing 4 of the units of the given dividend, consequently, every unit which remains after dividing this quotient will contain the same number of units; therefore, the last remainder must be multiplied by the first divisor, 4, to find the number of units it contains of the given dividend. The units in the first remainder being of the same kind as the units of the dividend, must be added to the product, the sum will be the total remainder.

From the preceding illustration we derive the following rule:

RULE. *Find two numbers whose product is equal to the divisor. Divide the given dividend by one of these numbers, then divide the quotient by the other, the last quotient will be the answer or quotient required.*

To find the total remainder, multiply the last remainder by the first divisor, add the first remainder to the product, their sum will be the total remainder.

The units in the first remainder being of the same kind as the units of the dividend, it is plain that the first remainder must be the total remainder, when there is no other remainder.

It is evident that the last remainder must be multiplied by the first divisor, to find the number of units it contains of the same kind as those of the dividend; therefore, the product must be the total remainder, when there is no first remainder.

133. Divide 2304 by 16. Quotient, 144.
134. Divide 4728 by 28. Quotient, $168\frac{24}{28}$.
135. Divide 5435 by 35. Quotient, $155\frac{10}{35}$.
136. Divide 6444 by 48. Quotient, $134\frac{12}{48}$.
137. Divide 7128 by 54. Quotient, 132.
138. Divide 8548 by 64. Quotient, $133\frac{36}{64}$.
139. Divide 9672 by 72. Quotient, $134\frac{24}{72}$.
140. Divide 9884 by 84. Quotient, $117\frac{56}{84}$.

141. If 108 yards of broadcloth be worth 540 dollars, how many dollars is 1 yard worth?

142. If a farm containing 132 acres be worth 5280 dollars, what is the value of 1 acre?

Art. 32. *To divide by* 10, 100, 1000, *or a unit with any number of ciphers annexed.*

By the rule or law of notation, the same figure expresses a number 10, 100, &c., times greater or less, according to its distance from the place of units. By cutting off figures from the right, the place of units is removed towards the left; consequently, cutting off the right hand figure of a number divides it by 10; cutting off the two right hand figures, divides it by 100, &c. Hence the following

RULE. *Cut off so many of the right hand figures of the dividend as there are ciphers in the divisor, the other figures of the dividend will be the quotient, and the figures cut off will be the remainder.*

143. Divide 4347 by 10. Quotient, $434\frac{7}{10}$.
144. Divide 74563 by 100. Quotient, $745\frac{63}{100}$.
145. Divide 195617 by 1000. Quotient, $195\frac{617}{1000}$.

146. A prize of 15500 dollars is owned equally by 100 men; what is each man's share?

147. Suppose 125000 acres of land are to be divided equally among 1000 soldiers; what number of acres will each soldier receive?

Art. 33. *To divide by a number whose right hand figures are ciphers.*

Let it be required to divide 94674 by 6500.

$$
\begin{array}{l}
65|00)946|74(14\frac{3674}{6500} \\
\quad\quad\;\; 65 \\
\quad\quad\;\; \overline{296} \\
\quad\quad\;\; 260 \\
\quad\quad\;\; \overline{\quad\quad\quad} \\
\quad\quad\;\; 3674, \text{ total rem.}
\end{array}
$$

The factors of 6500 are 100 and 65. We first divide the dividend by 100, by cutting off the two right hand figures, the quotient is 946, and the remainder 74. We then divide the quotient, 946, by 65, the quotient is 14, and the remainder 36. To find the total remainder, we multiply the last remainder, 36, by the first divisor, 100, and then add the first remainder, 74, to the product, their sum is 3674, the total remainder. If we annex the first remainder, or figures cut off, to the last remainder, we shall obtain the same total remainder.

From the above illustration we derive the following rule:

Rule. *Separate the divisor into factors by cutting off the ciphers. Then cut off an equal number of figures from the right of the dividend. Divide the remaining figures of the dividend by the remaining figures of the divisor; lastly, annex the figures cut off from the dividend to the remainder, this number will be the total remainder. If there be no last remainder, the first remainder, or figures cut off from the dividend, will be the total remainder.*

148. Divide 15420 by 20, 30, 40, 50, 60, 70, 80, and 90.

Quotients, 771, 514, $385\frac{20}{40}$, $308\frac{20}{50}$, 257, $220\frac{20}{70}$, $192\frac{60}{80}$, $171\frac{30}{90}$.

149. Divide 125432 by 110, 120, 130, 140, 150, 160, and 190.

Quotients, $1140\frac{32}{110}$, $1045\frac{32}{120}$, $964\frac{112}{130}$, $895\frac{132}{140}$, $836\frac{32}{150}$, $783\frac{152}{160}$, $660\frac{32}{190}$.

150. What will be the quotient of 750000 divided by 7500?

151. How many times is 125000 contained in 75000000?

152. If 4750 pounds of beef be equally divided among 950 soldiers, what number of pounds would each soldier have?

153. Suppose that prize-money to the amount of 16250 dollars is to be equally distributed among 650 sailors; what number of dollars will each sailor receive?

PRACTICAL QUESTIONS IN MULTIPLICATION AND DIVISION.

Art. 34. 1. A gentleman gave 120 dollars for a horse, which was 4 ninths of what he gave for a chaise; what did he give for the chaise?

2. What is 9 times 1 fourth of 120?

3. What is 1 fourth of 9 times 120?

4. If a train of railroad cars run 200 miles in 8 hours, what number of miles will it run in 15 hours?

5. What is 15 times 1 eighth of 200?

6. What is 1 eighth of 15 times 200?

7. If 40 tons of hay will keep 30 cows throughout the winter, what number of tons will be sufficient to keep 45 cows the same time?

8. What is 45 times 1 thirtieth of 40?

9. What is 1 forty-fifth of 30 times 40?

10. When flour is worth 6 dollars a barrel, what number of barrels will pay for 120 acres of land which is worth 30 dollars an acre?

11. What is 1 sixth of 120 times 30?

12. What is 120 times 1 sixth of 30?

13. A bookseller exchanged 75 arithmetics, which were worth 42 cents each, for 90 grammars; what was the value of each grammar?

14. What is 1 ninetieth of 75 times 42?

15. What is 1 seventy-fifth of 90 times 35?

16. If 30 men can perform a piece of work in 25 days, what number of men will be required to perform the same work in 15 days?

17. What is 1 fifteenth of 25 times 30?

18. What is 1 twenty-fifth of 15 times 50?

19. A merchant sold 12 yards of silk, at 90 cents a yard, and received payment in potatoes, at 40 cents a bushel. What number of bushels did he receive?

20. What is 1 fortieth of 12 times 90?

21. What is 1 twelfth of 40 times 27?

22. If 25 men can build a bridge in 90 days, what number of men would be required to build the same in 30 days?

23. What is 1 thirtieth of 90 times 25?

24. What is 1 ninetieth of 30 times 75?

25. If 24 men can build 720 rods of wall in 10 days, how many rods can 40 men build in the same number of days?

26. What is 40 times 1 twenty-fourth of 720?

27. What is 1 fortieth of 24 times 1200?

FEDERAL MONEY.

Art. 25. MONEY is a measure of the value of things. Federal Money is the national currency of the United States. Every system of national currency has its integer, or unit of measure. In the United States, the integer, or unit of measure, is the dollar.

The denominations of Federal Money are the eagle, the dollar, the dime, the cent, and the mill.

The gold coins are the eagle, the half-eagle, and the quarter-eagle. The silver coins are the dollar, the half-dollar, the quarter-dollar, the dime, and the half-dime. The copper coins are the cent, and the half-cent. The mill is imaginary, there being no coin of a less value than the half-cent.

The denominations of Federal Money increase in a tenfold proportion, like simple numbers, as is shown in the following table:

1 eagle = 10 dollars.	$.	1 dollar = $\frac{1}{10}$ of 1 eagle.
1 dollar = 10 dimes.	d.	1 dime = $\frac{1}{10}$ of 1 dollar.
1 dime = 10 cents.	c.	1 cent = $\frac{1}{10}$ of 1 dime.
1 cent = 10 mills.	m.	1 mill = $\frac{1}{10}$ of 1 cent.

NOTATION AND NUMERATION OF FEDERAL MONEY.

Art. 26. Dollars being integers or whole numbers, we separate them from cents by a point;—all the figures at the left of the point express dollars; the first two figures at the right of the point express cents; the third, mills; and the fourth, tenths of a mill. Thus: $25.7525 are read, twenty-five dollars, seventy-five cents, two mills, and five tenths.

To express any sum of Federal Money by figures, we first write the number of dollars, and place a point at the right; we then write the number of cents and mills at the right of the point. When the number of cents is less than ten, we write a cipher in the place of tens of cents.

Fifty-five dollars, twenty-five cents, and five mills, are written thus: $55.255. Twenty-five dollars and nine cents are written thus: $25.09. Fifteen dollars and five mills are written thus: $15.005.

Halves and fourths of a cent are sometimes used instead of mills and tenths of a mill.

REDUCTION OF FEDERAL MONEY.

Art. 37. Reduction of Federal Money is changing it from one denomination to another of equal value.

Since 1 dollar is equal to 100 cents, any number of dollars must be equal to 100 times as many cents as there are dollars.

As 1 dollar is equal to 1000 mills, any number of dollars must be equal to 1000 times as many mills as there are dollars.

Since 100 cents are equal to 1 dollar, any number of cents must be equal to the hundredth part as many dollars as there are cents.

As 1000 mills are equal to 1 dollar, any number of mills must be equal to the thousandth part as many dollars as there are mills.

Hence we have the following rules:

TO REDUCE DOLLARS TO CENTS.

RULE. *Multiply the number of dollars by* 100, *by annexing two ciphers to the given number. The product will be cents.*

1. Reduce 25 dollars to cents.

2. What number of cents are there in 75 dollars? What number, in 145 dollars?

TO REDUCE DOLLARS TO MILLS.

RULE. *Multiply the number of dollars by* 1000, *by annexing three ciphers to the given number. The product will be mills.*

3. Reduce 45 dollars to mills.

4. What number of mills are there in 125 dollars? What number, in 450 dollars?

TO REDUCE ANY SUM OF FEDERAL MONEY, CONSISTING OF SEVERAL DENOMINATIONS, TO THE LOWEST DENOMINATION NAMED IN THE GIVEN SUM.

RULE. *Read the whole sum in the lowest denomination named.*

5. Reduce $12.24 to cents.

6. What number of mills are there in $15.375?

7. What number of mills are there in $75.875?

TO CHANGE CENTS TO DOLLARS.

RULE. *Divide the number of cents by* 100, *by pointing off the two right hand figures of the given number. The quotient, or figures at the left of the point, will be dollars; the remainder, or figures at the right of the point, will be cents.*

8. Change 2500 cents to dollars.

9. What number of dollars are there in 7500 cents?

10. What number of dollars are there in 1224 cents?

TO CHANGE MILLS TO DOLLARS.

RULE. *Divide the number of mills by* 1000, *by pointing off the three right hand figures of the given number. The quotient, or figures at the left of the point, will be dollars; the remainder, or figures at the right of the point, will be mills, or the first two will be cents, and the third, mills.*

11. Change 45000 mills to dollars.

12. What number of dollars are there in 125000 mills?

13. What number of dollars are there in 15375 mills?

ADDITION OF FEDERAL MONEY.

Art. 38. 1. What is the amount of 75 cents, 25 cents, 15 cents, and 10 cents?

2. Henry purchased an arithmetic for 50 cents, a slate for 25 cents, and a knife for 20 cents. How many cents did they all cost him?

3. A man owes A 75 dollars, B 50 dollars, C 25 dollars, and D 15 dollars. What is the whole amount of his debts?

4. A merchant borrowed 125 dollars of A, 75 dollars of B, 50 dollars of C, and 25 dollars of D. What was the whole amount that he borrowed?

5. A trader received 25 dollars 50 cents of one man, 15 dollars 25 cents of another, and 10 dollars 25 cents of a third. What amount did he receive?

6. What is the sum of 575 dollars 75 cents, and 25 dollars 25 cents?

RULE. *Write dollars under dollars, cents under cents, and mills under mills; — then add as in addition of simple numbers, and place a point in the amount directly under the separating points above.*

When parts of a cent are used instead of mills, divide the sum of them by so many of such parts as are equal to one cent; the quotient will be cents.

7. What is the amount of \$175.125, \$347.375, \$548.875, \$601.625, and \$1000.000?

\$ c. m.
175.125
347.375
548.875
601.625
1000.000

\$2673.000 Ans.

8. What is the amount of \$215.12½, \$375.37½, \$410.62½, \$540.87½, and \$600.15¼?

\$ c.
215.12½
375.37½
410.62½
540.87½
600.15¼

\$2142.15¼ Ans.

9. What is the amount of \$45.25, \$75.45, \$125.62, \$264.44, \$450.66, and \$500.00? Ans. \$1461.42.

10. What is the amount of \$4025.25, \$5640.50, \$6345.75, \$7075.87, and \$10000.00? Ans. \$33087.37.

11. A trader commenced business with 7500 dollars, and he gained 1275 dollars 50 cents each year, during three successive years. What was the amount of his property at the end of three years? Ans. \$11326.50.

12. A merchant purchased a quantity of goods for \$4750, paid for freight \$175.50, the duties amounted to \$750.75. What was the whole cost of the goods? Ans. \$5676.25.

13. A gentleman owns four farms. The first is worth 7500 dollars; the second, 6275 dollars 75 cents; the third, 5000 dollars; and the fourth, 4560 dollars 50 cents. How much are they all worth? Ans. \$23336.25.

SUBTRACTION OF FEDERAL MONEY.

Art. 39. 1. Thomas had 35 cents, and gave 10 of them to his brother. How many has he left?

2. A boy, having 20 cents, gave 8 of them for two oranges; he afterwards lost 4 cents. What number of cents has he left?

3. A man borrowed 50 dollars; he paid at one time 25 dollars; at another, 15 dollars. How many dollars remain unpaid?

4. A man, who owed 75 dollars, paid all except 15 dollars. How many dollars did he pay?

5. Charles bought a book of Henry, for which he agreed to give him one dollar. Charles, at one time, paid 50 cents; at another, 25 cents; at a third, 12½ cents. How many cents does he still owe Henry?

6. A man purchased a chaise for 150 dollars; and paid 25 dollars for repairing it; he then sold it for 200 dollars. What number of dollars did he gain by trading?

7. What is the difference between 75 dollars, and 75 cents?

RULE. *Write dollars under dollars, cents under cents, and mills under mills. Subtract as in subtraction of simple numbers, and place a point in the remainder directly under the separating points above.*

8. From $475.25 subtract $190.12½.

$ c. m.
475.250
190.125

$285.125 Ans.

9. Subtract $125.625 from $1000.

$ cts.
1000.000
125.625

$874.375 Ans.

10. Subtract ten dollars and five cents from fifty dollars.
Ans. $39.95.

11. From one hundred dollars take seventy-five cents and five mills. Ans. $99.245.

12. A man sold a horse for a hundred and fifty dollars, which was thirty-seven dollars and fifty cents more than the horse cost him. What did he give for the horse?
Ans. $112.50.

13. A merchant bought a piece of cloth for $120.50, and sold it for $135. How much did he gain by trading?
Ans. $14.50.

14. If a man's income be 2500 dollars a year, and his expenses 1850 dollars 75 cents, how much does his income exceed his expenses? Ans. $649.25.

15. A man purchased a farm for 5000 dollars. He paid at one time $1500; at another, 1200 dollars; at a third, 750 dollars 75 cents. How much did he still owe for his farm?
Ans. $1549.25.

MULTIPLICATION OF FEDERAL MONEY.

Art. 40. 1. When sugar is worth 8 cents a pound, what is the value of 7 pounds?

2. What will 5½ yards of cloth come to, at 12 cents a yard?

3. What is the value of 12½ pounds of coffee, at 12 cents a pound?

4. What is the value of 11 pounds of beef, at 10 cents a pound?

5. What will 3 fourths of a pound of tea come to, at 60 cents a pound?

6. When butter is worth 20 cents a pound, what is the value of 8 pounds?

7. What is the value of 7 cords of wood, at 6½ dollars a cord?

8. If coal be worth 8 dollars a ton, what is the value of 10¾ tons?

9. If a yard of silk be worth 64 cents, what is 7 eighths of a yard worth?

10. When flour is worth 5½ dollars a barrel, what are 15 barrels worth?

RULE. *Multiply the given sum or price as in multiplication of simple numbers. The product will be the answer in the lowest denomination named in the multiplicand or given sum.*

When there are parts of a cent in the given sum or price, it is more convenient to use the number of mills in such parts.

11. If a yard of broadcloth be worth $5.62½, what is the value of 25 yards?

```
 $ c. m.
 5.625
    25
 -----
 28125
11250
--------
$140.625 Ans.
```

12. If a barrel of flour be worth $6.37½, what is the value of 75 barrels?

```
 $ c. m.
 6.375
    75
 -----
 31875
44625
--------
$478.125 Ans.
```

13. What will 15 pounds of butter cost, at 17 cents a pound? Ans. $2.55.

14. If a ream of paper be worth $3.75, what is the value of 35 reams? Ans. $131.25.

15. What is the value of 25 pair of walking-shoes, at $1.62½ a pair? Ans. $40.625.

16. What is the value of 750 bushels of corn, at 62½ cents a bushel? Ans. $468.75.

17. What is the value of a piece of linen measuring 24¾ yards, at 36 cents a yard? Ans. $8.91.

18. Bought a piece of silk measuring 35⅞ yards, at 64 cents a yard; what was the whole cost? Ans. $22.96.

19. Sold 45 cords of wood, at $5.75 a cord; what was the amount of sales? Ans. $258.75.

20. A gentleman purchased a farm containing 325 acres, at $25.75 an acre; what did the farm cost him? Ans. $8368.75.

DIVISION OF FEDERAL MONEY.

Art. 41. 1. If 9 yards of cloth be worth 72 cents, what is 1 yard worth?

2. If 5½ yards of cloth be worth 66 cents, what is 1 yard worth?

3. If I pay $1.10 for 11 pounds of beef, what is 1 pound of it worth?

4. If 12½ pounds of coffee be worth $1.50, what is the value of 1 pound?

5. If 3 fourths of a pound of tea be worth 45 cents, what is the value of 1 pound?

6. Purchased 8 pounds of butter, for which I paid $1.60; what is the value of 1 pound?

7. Purchased 7 cords of wood for $45.50; what did it cost per cord?

8. Bought 10¾ tons of coal for $86; what did it cost per ton?

9. If 7 eighths of a yard of silk be worth 56 cents, what is the value of 1 yard?

10. Paid $82.50 for 15 barrels of flour; what is 1 barrel of it worth?

Rule. *Divide the given sum or price as in division of simple numbers. The quotient will be the answer in the lowest denomination named in the dividend or given sum.*

11. If $84.37½ be equally divided among 15 boys, what will each boy receive?

```
      $  c. m.  $  c. m.
15 ) 84.375 ( 5.625 Ans.
     75
     --
      93
      90
      --
       37
       30
       --
        75
        75
        --
```

12. If 64 yards of broadcloth be worth $392, what is the value of 1 yard?

```
      $    $  c. m.
64 ) 392 ( 6.125 Ans.
     384
     ---
       800
        64
       ---
       160
       128
       ---
        320
        320
        ---
```

13. If 15 pounds of butter be worth $2.55, what is the value of 1 pound? Ans. 17 cents.

14. Purchased 35 reams of paper for $131.25; what did it cost per ream? Ans. $3.75.

15. Bought 25 pair of shoes, for which I paid $40.62½; what did they cost per pair? Ans. $1.625.

16. Sold a cargo of corn, containing 750 bushels, for $468.75; what was it sold for per bushel? Ans. $.625.

17. If a piece of silk measuring 35⅞ yards be worth $22.96, what is the value of 1 yard? Ans. 64 cents.

18. Bought 45 cords of wood for $258.75; what did it cost per cord? Ans. $5.75.

19. A gentleman purchased a farm containing 325 acres, for which he paid $8368.75; what did his farm cost him per *acre?* Ans. $25.75.

PRACTICAL QUESTIONS IN FEDERAL MONEY.

Art. 42. 1. A borrowed $175 of B; he paid $75.50 at one time, and $65.75 at another. How much remains unpaid?

2. A gentleman purchased a house, for which he gave his note for $5000; he has since paid $2325 at one time, and $1500 at another. What is the balance due on the note?

3. A lady purchased 15 yards of silk, at 87½ cents a yard, and gave the shopkeeper a $20 bill in payment. How much is due the purchaser?

4. A farmer purchased 5 cows, at $25 each; 15 sheep, at $2.25 each; and 2 garden hoes, at 37½ cents each. What was the amount of his purchase?

5. A gentleman gave $7325 to his son and daughter; the daughter was to have 2 thirds as much as the son. What was the share of each?

6. Can you divide $46.50 among A, B, and C, and give B, $5.25 more than C, and A, $5.25 more than B, and inform me what number of dollars each of them will receive?

7. A grocer purchased 25 firkins of butter, weighing 56 pounds each, at 17 cents a pound. What was the whole cost of the butter?

8. What is the value of 4 pieces of broadcloth, No. 1 measuring 36 yards; No. 2, 38 yards; No. 3, 42 yards; and No. 4, 44 yards; at $3.87½ a yard?

9. A trader purchased 36 yards of broadcloth for $108, and sold the same at $4 a yard. Did he gain, or lose, by the transaction; and how much?

10. A gentleman purchased an estate for $10344, and agreed to pay 1 half in 3 months, 1 third in 6 months, and the remainder in 9 months. What was the amount of each payment?

11. A farmer sold a grocer 1250 pounds of cheese, at 10 cents a pound; and 75 pounds of butter, at 18 cents a pound. He received in payment two barrels of flour, at $7.50 a barrel; 175 pounds of sugar, at 8 cents a pound; and the balance in cash. How much money did he receive?

12. A merchant purchased 750 barrels of flour, for which he paid $4875; he sold it, after keeping it a few days, at $7.25 a barrel. What was the profit on each barrel, and what was the whole amount of profit?

13. A merchant purchased 5 bales of sheeting, each bale containing 30 pieces, and each piece measuring 36 yards, at 17 cents a yard. He sold 2 bales, at 17½ cents a yard; and the other 3 bales, at 17¼ cents a yard. What was the sum gained?

BILLS OF PARCELS.

Art. 43. A Bill of Parcels is a written paper, given by the seller of merchandise to the buyer, specifying each article, its quantity and price, with the amount and date of the transaction.

What is the cost of each of the several articles, and what is the amount of each of the following bills?

Boston, May 15th, 1847.

Mr. Henry Richards,

Bought of Silas Pierce,

25 lbs. Brown Sugar,	@	$.09	$
40 lbs. Coffee,	@	.10	
12 lbs. Green Tea,	@	.75	
14 lbs. Raisins,	@	.12	
10 lbs. Butter,	@	.17	
		Amount,	$

Received payment,

Silas Pierce.

Boston, May 25th, 1847.

Samuel Greene, Esq.

Bought of Phillips & Sampson,

12 Sherwin's Algebra,	@	$ .75	$
18 Emerson's National Spelling-Book,	@	.20	
6 Pierpont's National Reader,	@	.30	
6 Goodrich's History of the U. States,	@	.35	
			$

Received Payment,

Phillips & Sampson.

Boston, June 10th, 1847.

Thomas C. Wales, Esq.

Bought of George Warren & Co.,

15 yds. Irish Linen,	@	$ .87	$
32 yds. Sheeting,	@	.18	
12 yds. Black Silk,	@	.92	
10 yds. Cambric,	@	.12	
			$

Received payment,

George Warren & Co.

REDUCTION.

Art. 44. Reduction is the changing of any denomination of money, weight, or measure, from its number in one denomination to its number in any other denomination of equal value. Thus: 5 pounds, changed to shillings, are equal to 100 shillings. 500 cents, changed to dollars, are equal to 5 dollars.

ENGLISH MONEY.

The denominations of English money are the pound, £; the shilling, s.; the penny, d.; and the farthing, qr.

TABLE.

1 pound = 20 shillings.
1 shilling = 12 pence.
1 penny = 4 farthings.

1 shilling = $\frac{1}{20}$ of 1 £.
1 penny = $\frac{1}{12}$ of 1 s.
1 farthing = $\frac{1}{4}$ of 1 d.

1. What number of shillings are there in £2? In £3? In £4? In £5? In £6? In £7? In £8? In £9? In £10? In £11? In £12?

2. What number of pounds are there in 40s.? In 60s.? In 80s.? In 100s.? In 120s.? In 140s.? In 160s.? In 180s.? In 200s.? In 220s.? In 240s.?

3. What number of pence are there in 2s.? In 3s.? In 4s.? In 5s.? In 6s.? In 7s.? In 8s.? In 9s.? In 10s.? In 11s.? In 12s.?

4. What number of shillings are there in 24d.? In 36d.? In 48d.? In 60d.? In 72d.? In 84d.? In 96d.? In 108d.? In 120d.? In 132d.? In 144d.?

5. What number of farthings are there in 2d.? In 3d.? In 4d.? In 5d.? In 6d.? In 7d.? In 8d.? In 9d.? In 10.? In 11d.? In 12d.?

6. What number of pence are there in 8qrs.? In 12qrs.? In 16qrs.? In 20qrs.? In 24qrs.? In 28qrs.? In 32qrs.? In 36qrs.? In 40qrs.? In 44qrs.? In 48qrs.?

7. Reduce £62. 15s. 6d. 3qrs. to farthings.

```
 £.  s.  d. qr.
 62  15  6  3.
 20
 ----
 1255 shillings.
 12
 ----
 15066 pence.
 4
 ----
Ans. 60267 farthings.
```

8. Change 60267 farthings to pounds.

```
  qr.
4)60267
  ------
12)15066 - 3qrs.
  ------
2|0)125|5 - 6d.
  ------
Ans. £62. 15s. 6d. 3qrs.
```

Since 1 pound is equal to 20 shillings, there must be 20 times as many shillings as pounds; therefore, we multiply £62 by 20, the product is 1240s. and 15s. are 1255 shillings. As 1 shilling is equal to 12 pence, there must be 12 times as many pence as shillings; we multiply 1255 shillings by 12, the product is 15060d. and 6d. are 15066 pence. As 1 penny is equal to 4 farthings, there must be 4 times as many farthings as pence; we multiply 15066d. by 4, the product is 60264qrs. and 3qrs. are 60267 farthings.

Since 4 farthings are equal to 1 penny, there must be 1 fourth as many pence as farthings; therefore, we divide 60267 farthings by 4, the quotient is 15066 pence, and 3qrs. remain. As 12 pence are equal to 1 shilling, there must be 1 twelfth as many shillings as pence; we divide 15066d. by 12, the quotient is 1255 shillings, and 6d. remain. As 20 shillings are equal to 1 pound, there must be 1 twentieth as many pounds as shillings; we divide 1255s. by 20, the quotient is £62, and 15s. remain.

From the preceding illustrations, we have the following rules.

Rule. I. *When a higher denomination is to be reduced to a lower;—multiply the number of the higher denomination by that number of the next lower denomination which is equal to a unit of the higher, adding to the product so many of the lower denomination as are expressed in the given quantity; the product, or sum, will be the number of the next lower denomination. Continue to multiply in a similar manner until the required denomination is obtained.*

Rule. II. *When a lower denomination is to be changed to a higher;—divide the number of the lower denomination by that number of the lower denomination which is equal to a unit of the next higher, the quotient will be the number of the next higher denomination; and the remainder, if any, is of the same denomination as the dividend. Continue to divide in a similar manner until the required denomination is obtained.*

9. Reduce £434 to farthings. Ans. 416640qrs.

11. Reduce £529. 18s. 9d. 2qrs. to farthings.
Ans. 508742qrs.

13. Reduce 675 dollars, at 6 shillings each, to pence.
Ans. 48600d.

15. Reduce £765. 15s. to shillings.

17. Reduce £1250. 13s. 4d. to pence.

10. Change 416640qrs. to pounds. Ans. £434.

12. Change 508742qrs. to pounds.
Ans. £529. 18s. 9d. 2qrs.

14. Change 48600d. to dollars, of 6 shillings each.
Ans. 675 dollars.

16. Change 15315 shillings to pounds.

18. Change 300160 pence to pounds.

TROY WEIGHT.

Art. 45. Troy weight is used in weighing gold, silver, platina, diamonds, and other precious stones. The denominations of troy weight are the pound, lb.; the ounce, oz.; the pennyweight, pwt.; and the grain, gr. The standard troy pound of the United States is the weight of 22.794377 cubic inches of distilled water weighed in air.

TABLE.

1 pound = 12 ounces.	1 ounce = $\frac{1}{12}$ of 1 lb.
1 ounce = 20 pwts.	1 pwt. = $\frac{1}{20}$ of 1 oz.
1 pwt. = 24 grains.	1 grain = $\frac{1}{24}$ of 1 pwt.

1. How many ounces are there in 4 pounds? In 5 lbs.? In 6 lbs.? In 7 lbs.? In 8 lbs.? In 9 lbs.? In 10 lbs.? In 11 lbs.? In 12 lbs.?

2. How many pounds are there in 48 ounces? In 60 oz.? In 72 oz.? In 84 oz.? In 96 oz.? In 108 oz.? In 120 oz.? In 132 oz.?

3. How many pennyweights are there in 2 ounces? In 3 oz.? In 4 oz.? In 5 oz.? In 6 oz.? In 7 oz.? In 8 oz.? In 9 oz.?

4. How many ounces are there in 40 pennyweights? In 60 pwts.? In 80 pwts.? In 100 pwts.? In 120 pwts.? In 140 pwts.? In 160 pwts.? In 180 pwts.?

5. How many grains are there in 2 pennyweights? In 3 pwts.? In 4 pwts.? In 5 pwts.? In 6 pwts.?

6. How many pennyweights are there in 48 grains? In 72 grs.? In 96 grs.? In 120 grs.?

7. Reduce 18 lbs. 5 oz. 9 pwts. 21 grs. to grains.
Ans. 106317 grains.

8. Change 106317 grains to pounds.
Ans. 18 lbs. 5 oz. 9 pwts. 21 grs.

9. Reduce 210 lbs. 8 oz. 12 pwts. to pennyweights.
Ans. 50572 pwts.

10. Change 50572 pennyweights to pounds.
Ans. 210 lbs. 8 oz. 12 pwts.

APOTHECARIES' WEIGHT.

Art. 46. This weight is used only by apothecaries and physicians in compounding medicines. Its denominations are the pound, lb.; the ounce, ℥; the dram, ʒ; the scruple, ℈; and the grain, gr.

TABLE.

1 pound = 12 ounces.	1 ounce = $\frac{1}{12}$ of 1 lb.
1 ounce = 8 drams.	1 dram = $\frac{1}{8}$ of 1 ℥.
1 dram = 3 scruples.	1 scruple = $\frac{1}{3}$ of 1 ʒ.
1 scruple = 20 grains.	1 grain = $\frac{1}{20}$ of 1 ℈.

1. How many ounces are there in 4 pounds? In 5 lbs.? In 6 lbs.? In 7 lbs.? In 8 lbs.? In 9 lbs.? In 10 lbs.? In 11 lbs.? In 12 lbs.?

2. How many pounds are there in 48 ounces? In 60 ℥? In 72 ℥? In 84 ℥? In 96 ℥? In 108 ℥? In 120 ℥? In 132 ℥? In 144 ℥?

3. How many drams are there in 4 ounces? In 5 ℥? In 6 ℥? In 7 ℥? In 8 ℥? In 9 ℥? In 10 ℥? In 11 ℥? In 12 ℥?

4. How many ounces are there in 32 drams? In 40 ʒ? In 48 ʒ? In 56 ʒ? In 64 ʒ? In 72 ʒ? In 80 ʒ? In 88 ʒ? In 96 ʒ?

5. How many grains are there in 37 lbs. 6 ℥? Ans. 216000 grs.

6. In 216000 grains, how many pounds? Ans. 37 lbs. 6 ℥.

7. Reduce 118 pounds to grains. Ans. 679680 grs.

8. Change 679680 grains to pounds. Ans. 118 lbs.

9. In 9 lbs. 8 ℥, 1 ʒ, 2 ℈, 19 grs., how many grains? Ans. 55799 grs.

10. In 55799 grains, how many pounds? Ans. 9 lbs. 8 ℥, 1 ʒ, 2 ℈, 19 grs.

AVOIRDUPOIS WEIGHT.

Art. 47. Avoirdupois weight is used in weighing most kinds of merchandise, and all metals, except silver and gold.

Its denominations are the ton, t.; the hundred-weight, cwt.; the quarter, qr.; the pound, lb.; the ounce, oz.; and the dram, dr.

TABLE.

1 ton	= 20 hund. wt.	1 hund. wt.	= $\frac{1}{20}$ of 1 ton.
1 hund. wt.	= 4 quarters.	1 quarter	= $\frac{1}{4}$ of 1 cwt.
1 quarter	= 28 pounds.	1 pound	= $\frac{1}{28}$ of 1 qr.
1 pound	= 16 ounces.	1 ounce	= $\frac{1}{16}$ of 1 lb.
1 ounce	= 16 drams.	1 dram	= $\frac{1}{16}$ of 1 oz.

The ton of 2240 pounds, the cwt. of 112 pounds, and the quarter of 28 pounds, are not much used. The ton of 2000 pounds, the hundred of 100 pounds, and the quarter of 25 pounds, are now generally used.

The standard avoirdupois pound of the United States is the weight of 27.7015 cubic inches of distilled water weighed in air. 144 avoirdupois pounds are equal to 175 troy pounds. 192 avoirdupois ounces are equal to 175 troy ounces. 1 avoirdupois pound is equal to 7000 troy grains.

1. How many cwt. are there in 4 tons? In 5 t.? In 6 t.? In 7 t.? In 8 t.?

2. How many tons are there in 80 cwt.? In 100 cwt.? In 120 cwt.? In 140 cwt.? In 160 cwt.?

3. How many quarters are there in 8 cwt.? In 9 cwt.? In 10 cwt.? In 11 cwt.? In 12 cwt.?

4. How many cwt. are there in 32 quarters? In 36 qrs.? In 40 qrs.? In 44 qrs.? In 48 qrs.?

5. How many pounds are there in 2 quarters? In 3 qrs.? In 4 qrs.? In 5 qrs.? In 6 qrs.?

6. How many quarters are there in 56 pounds? In 84 lbs.? In 112 lbs.? In 140 lbs.?

7. How many ounces are there in 4 pounds? In 5 lbs.? In 6 lbs.? In 7 lbs.? In 8 lbs.?

8. How many pounds are there in 64 ounces? In 80 oz.? In 96 oz.? In 112 oz.? In 128 oz.?

9. Reduce 5 t. 12 cwt. 3 qrs. 21 lbs. 14 oz. to ounces.

Ans. 202398 ounces.

10. Change 202398 ounces to tons.

Ans. 5 t. 12 cwt. 3 qrs. 21 lbs. 14 oz.

11. In 39 cwt. 2 qrs. 14 lbs. 8 oz. 4 drs. how many drams?

Ans. 1136260 drams.

12. In 1136260 drams, how many cwt.?

Ans. 39 cwt. 2 qrs. 14 lbs 8 oz. 4 drs.

CLOTH MEASURE.

Art. 48. This measure is used for measuring cloth and other goods which are sold by the yard or ell.

Its denominations are the English ell, E. ell; the French ell, Fr. ell; the Flemish ell, Fl. ell; the yard, yd.; the quarter, qr.; and the nail, na.

TABLE.

1 English ell = 5 quarters.
1 French ell = 6 quarters.
1 Flemish ell = 3 quarters.
1 yard = 4 quarters.
1 quarter = 4 nails.

1 quarter = $\frac{1}{5}$ of 1 E. ell.
1 quarter = $\frac{1}{6}$ of 1 Fr. ell.
1 quarter = $\frac{1}{3}$ of 1 Fl. ell.
1 quarter = $\frac{1}{4}$ of 1 yard.
1 nail = $\frac{1}{4}$ of 1 quarter.

1. How many quarters are there in 8 E. ells? In 9 ells? In 10 ells? In 11 ells? In 12 ells?

2. How many E. ells are there in 40 quarters? In 45 qrs.? In 50 qrs.? In 55 qrs.? In 60 qrs.?

3. How many quarters are there in 8 yards? In 9 yds.? In 10 yds.? In 11 yds.? In 12 yds.?

4. How many yards are there in 32 quarters? In 36 qrs.? In 40 qrs.? In 44 qrs.? In 48 qrs.?

5. How many quarters are there in 8 Fl. ells? In 9 Fl. ells? In 10 Fl. ells? In 11 Fl. ells? In 12 Fl. ells?

6. How many Flemish ells are there in 24 quarters? In 27 qrs.? In 30 qrs.? In 33 qrs.? In 36 qrs.?

7. Reduce 34 English ells to nails.
Ans. 680 nails.

8. In 680 nails, how many English ells?
Ans. 34 E. ells.

9. Reduce 83 yards, 3 quarters to quarters.
Ans. 335 quarters.

10. Change 335 quarters to yards. Ans. 83 yds. 3 qrs.

11. Reduce 75 Flemish ells to nails.
Ans. 900 nails.

12. Change 900 nails to Flemish ells.
Ans. 75 Fl. ells.

LINEAR MEASURE.

Art. 49. This measure is used in measuring distances, lengths, breadths, heights, and depths. Its denominations are the degree, deg.; the league, lea.; the mile, m.; the furlong, fur.; the rod, rd.; the yard, yd.; the foot, ft.; and the inch, in.

TABLE.

1 degree = 60 G. miles.
1 degree = 69½ S. miles.
1 league = 3 miles.
1 mile = 8 furlongs.
1 furlong = 40 rods.
1 rod = 5½ yards.
1 rod = 16½ feet.
1 yard = 3 feet.
1 foot = 12 inches.

1 G. mile = $\frac{1}{60}$ of 1 degree.
1 S. mile = $\frac{2}{139}$ of 1 degree.
1 mile = $\frac{1}{3}$ of 1 league.
1 furlong = $\frac{1}{8}$ of 1 mile.
1 rod = $\frac{1}{40}$ of 1 furlong.
1 yard = $\frac{2}{11}$ of 1 rod.
1 foot = $\frac{2}{33}$ of 1 rod.
1 foot = $\frac{1}{3}$ of 1 yard.
1 inch = $\frac{1}{12}$ of 1 foot.

A hand is equal to 4 inches, used in measuring the height of horses. A fathom is equal to 6 feet, used in measuring the depth of water, and the length of cordage.

"With a view to an invariable standard, and one which cannot be lost or destroyed, the linear inch has been referred to a pendulum vibrating seconds in a non-resisting medium at the level of the sea at Greenwich or London. The pendulum being divided into 391392 equal parts, the linear inch is defined to be 10000 of these parts; therefore, the length of the pendulum vibrating seconds is 39.1392 inches. The length of the standard yard is 360000 of the equal parts of the pendulum, or 36 inches."

1. How many miles are there in 4 degrees? In 5 deg.? In 6 deg.? In 8 deg.?

3. How many furlongs are there in 4 miles? In 5 m.? In 6 m.? In 8 m.?

5. How many feet are there in 2 rods? In 3 rds.? In 4 rds.? In 6 rds.?

2. How many degrees are there in 240 miles? In 300 m.? In 360 m.? In 480 m.?

4. How many miles are there in 32 furlongs? In 40 fur.? In 48 fur.? In 64 fur.?

6. How many rods are there in 33 feet? In 49½ ft.? In 66 ft.? In 99 ft.?

7. In 43 miles, how many inches?

Ans. 2724480 inches.

9. How many inches are there in the circumference of the earth?

Ans. 1585267200 inches.

11. The railroad from Boston to Albany is 200 miles in length; how many feet of iron rails are there in a single track?

Ans. 2112000 feet.

8. In 2724480 inches, how many miles?

Ans. 43 miles.

10. In 1585267200 inches, how many degrees?

Ans. 360 degrees.

12. In 2112000 feet, how many miles?

Ans. 400 miles.

13. In 15713280 inches, how many miles?

Ans. 248 miles.

SUPERFICIAL OR SQUARE MEASURE.

Art. 50. Superficial or square measure is used in measuring all kinds of surfaces, such as land, paving, flooring, plastering, and everything which has length and breadth. Its denominations are the mile, m.; the acre, a.; the rood, r.; the rod, rd.; the yard, yd.; the foot, ft.; and the inch, in.

Gunter's chain, used by surveyors in measuring land, also in measuring distances, is 4 rods, or 66 feet, in length, and is composed of 100 links.

TABLE.

1 sq. mile = 640 sq. acres.
1 sq. acre = 4 sq. roods.
1 sq. acre = 160 sq. rods.
1 sq. rood = 40 sq. rods.
1 sq. rod = 30¼ sq. yards.
1 sq. rod = 272¼ sq. feet.
1 sq. yard = 9 sq. feet.
1 sq. foot = 144 sq. inches.

1 sq. acre = $\frac{1}{640}$ of one sq. m.
1 sq. rood = $\frac{1}{4}$ of 1 sq. acre.
1 sq. rod = $\frac{1}{160}$ of 1 sq. acre.
1 sq. rod = $\frac{1}{40}$ of 1 sq. rood.
1 sq. yard = $\frac{4}{121}$ of 1 sq. rod.
1 sq. foot = $\frac{4}{1089}$ of 1 sq. rod.
1 sq. foot = $\frac{1}{9}$ of 1 sq. yard.
1 sq. inch = $\frac{1}{144}$ of 1 sq. foot.

1. In 9 square acres, how many sq. roods? In 10 sq. a.? In 11 sq. a.? In 12 sq. a.?

3. In 2 sq. rods, how many sq. yards? In 3 sq. rds.? In 4 sq. rds.? In 5 sq. rds.?

5. In 5 sq. yards, how many sq. feet? In 6 sq. yds.? In 7 sq. yds.? In 8 sq. yds.?

2. In 36 sq. roods, how many sq. acres? In 40 sq. r.? In 44 sq. r.? In 48 sq. r.?

4. In 60½ sq. yards, how many sq. rods? In 90¾ sq. yds.? In 121 sq. yds.? In 151¼ sq. yds.?

6. In 45 square feet, how many sq. yards? In 54 sq. ft.? In 63 sq. ft.? In 72 sq. feet?

7. In 25 a. 2 r. 20 rds., how many rods?

Ans. 4100 rods.

9. In 24 sq. rods, how many sq. inches?

Ans. 940896 sq. inches.

11. Reduce 64 sq. miles to sq. rods?

Ans. 6553600 sq. rods.

8. In 4100 rods, how many acres?

Ans. 25 a. 2 r. 20 rods.

10. In 940896 sq. inches, how many square rods?

Ans. 24 sq. rods.

12. Change 6553600 sq. rods to sq. miles.

Ans. 64 sq. miles.

A square inch, foot, &c., is the superficial magnitude of a square, of which each of the sides is a linear inch, foot, &c.; — and the number of square inches, feet, &c., contained in any square, or rectangular surface, is the product obtained by multiplying its length by its breadth.

CUBIC MEASURE.

Art. 51. Cubic measure is used in measuring solids and capacities, or anything that has three dimensions, length, breadth, and thickness. Its denominations are the cord, c.; the ton, t.; the yard, yd.; the foot, ft.; and the inch, in.

TABLE.

1 cord of wood = 128 cub. ft.

1 foot of wood = 16 cub. ft.

1 ton of timber = 40 cub. ft.

1 cubic yard = 27 cub. ft.

1 cubic foot = 1728 cubic inches.

1 cubic foot = $\frac{1}{128}$ of 1 c. of wood.

1 cubic foot = $\frac{1}{16}$ of 1 ft. of wood.

1 cubic foot = $\frac{1}{40}$ of 1 ton of timber.

1 cubic foot = $\frac{1}{27}$ of 1 cub. yd.

1 cubic inch = $\frac{1}{1728}$ of 1 cubic foot.

A perch of stone is equal to 24¾ cubic feet, used by masons in measuring stone walls. A square of earth is equal to 216 cubic feet. A cord of wood = 8 feet of wood.

1. In 4 tons of timber, how many cubic feet? In 5 tons? In 6 tons? In 7 tons? In 8 tons?

2. In 160 cubic feet, how many tons? In 200 c. ft.? In 240 c. ft.? In 280 c. ft.? In 320 c. ft.?

3. In 2 cubic yards, how many cubic feet? In 3 c. yds.? In 4 c. yds.? In 5 c. yds.? In 6 c. yds.?

4. In 54 cubic feet, how many cubic yards? In 81 c. ft.? In 108 c. ft.? In 135 c. ft.? In 162 c. ft.?

5. In 25 tons of timber, how many cubic inches?
Ans. 1728000 c. in.

6. In 1728000 cubic inches, how many tons of timber?
Ans. 25 tons.

7. In 12 cords of wood, how many cubic inches?
Ans. 2654208 c. in.

8. In 2654208 cubic inches, how many cords of wood?
Ans. 12 cords.

9. In 48 perch of stone, how many cubic feet?
Ans. 1188 c. feet.

10. In 1188 cubic feet, how many perch of stone?
Ans. 48 perch.

A cubic inch, foot, &c., is the solid contents of a cube, or figure contained by six equal squares, of which each of the edges is a linear inch, foot, &c.; and the number of cubic inches, feet, &c., contained in any solid body in the form of a cube or parallelopiped, is the product obtained by multiplying its length by its breadth, and this product by its thickness.

WINE MEASURE.

Art. 52. Wine measure is used in measuring wine and all spirituous liquors, except porter, ale, and beer. Its denominations are the tun, t.; the pipe, p.; the hogshead, hhd.; the barrel, bbl.; the gallon, gal.; the quart, qt.; the pint, pt.; and the gill, gl.

The standard gallon of the United States is the wine gallon, which measures 231 cubic inches, and contains 8.3388822 pounds avoirdupois of distilled water.

The British standard imperial gallon measures 277.274 cubic inches, and contains 10 pounds avoirdupois of distilled water.

TABLE.

1 tun = 2 pipes.
1 pipe = 2 hogsheads.
1 hogshead = 63 gallons.
1 barrel = 31½ gallons.
1 gallon = 4 quarts.
1 quart = 2 pints.
1 pint = 4 gills.

1 pipe = $\frac{1}{2}$ of 1 tun.
1 hogshead = $\frac{1}{2}$ of 1 pipe.
1 gallon = $\frac{1}{63}$ of 1 hhd.
1 gallon = $\frac{2}{63}$ of 1 bbl.
1 quart = $\frac{1}{4}$ of 1 gal.
1 pint = $\frac{1}{2}$ of 1 qt.
1 gill = $\frac{1}{4}$ of 1 pint.

1. In 2 barrels, how many gallons? In 3 bbls.? In 4 bbls.? In 5 bbls.?

3. In 15 gallons, how many quarts? In 16 gals.? In 17 gals.? In 18 gals.?

5. In 9 pints, how many gills? In 10 pts.? In 11 pts.? In 12 pts.?

2. In 63 gallons, how many barrels? In 94½ gals.? In 126 gals.? In 157½ gals.?

4. In 60 quarts, how many gallons? In 64 qts.? In 68 qts.? In 72 qts.?

6. In 36 gills, how many pints? In 40 gills? In 44 gills? In 48 gills?

7. In 15 tuns of wine, how many gallons?
Ans. 3780 gallons.

9. In 5 pipes of wine, how many gills?
Ans. 20160 gills.

11. Reduce 8 barrels of cider to pints. Ans. 2016 pts.

8. In 3780 gallons of wine, how many tuns?
Ans. 15 tuns.

10. In 20160 gills of wine, how many pipes?
Ans. 5 pipes.

12. Change 2016 pints of cider to barrels. Ans. 8 bbls.

ALE AND BEER MEASURE.

Art. 53. This measure is used in measuring porter, ale, beer, milk, and water. Its denominations are the hogshead, hhd.; the barrel, bbl.; the gallon, gal.; the quart, qt.; and the pint, pt.

The beer gallon measures 282 cubic inches.

TABLE.

1 hogshead = 54 gallons.
1 barrel = 36 gallons.
1 gallon = 4 quarts.
1 quart = 2 pints.

1 gallon = $\frac{1}{54}$ of 1 hhd.
1 gallon = $\frac{1}{36}$ of 1 bbl.
1 quart = $\frac{1}{4}$ of 1 gal.
1 pint = $\frac{1}{2}$ of 1 qt.

1. In 2 barrels, how many gallons? In 3 bbls.? In 4 bbls.? In 5 bbls.?

2. In 72 gallons, how many barrels? In 108 gals.? In 144 gals.? In 180 gals.?

3. In 5 hogsheads of beer, how many pints?
Ans. 2160 pints.

5. In 47 barrels 18 gallons of beer, how many quarts?
Ans. 6840 quarts.

7. Change 20 hogsheads of beer to pints.
Ans. 8640 pints.

4. In 2160 pints of beer, how many hogsheads?
Ans. 5 hhds.

6. In 6840 quarts of beer, how many barrels?
Ans. 47 bbls. 18 gals.

8. Change 8640 pints of beer to hogsheads.
Ans. 20 hogsheads.

DRY MEASURE.

Art. 54. This measure is used in measuring grain, fruit, seeds, roots, salt, sand, oysters, coal, &c. Its denominations are the chaldron, ch.; the bushel, bu.; the peck, pk.; the quart, qt.; and the pint, pt.

The bushel is $18\frac{1}{2}$ inches in diameter, 8 inches in depth, and measures $2150\frac{2}{5}$ cubic inches.

TABLE.

1 chaldron = 36 bushels.
1 bushel = 4 pecks.
1 peck = 8 quarts.
1 quart = 2 pints.

1 bushel = $\frac{1}{36}$ of 1 ch.
1 peck = $\frac{1}{4}$ of 1 bu.
1 quart = $\frac{1}{8}$ of 1 pk.
1 pint = $\frac{1}{2}$ of 1 qt.

1. In 17 bushels, how many pecks? In 18 bu.? In 19 bu.? In 20 bu.?

2. In 68 pecks, how many bushels? In 72 pks.? In 76 pks.? In 80 pks.?

3. In 9 pecks, how many quarts? In 10 pks.? In 11 pks.? In 12 pks.?

4. In 72 quarts, how many pecks? In 80 qts.? In 88 qts.? In 96 qts.?

5. In 136 bushels, how many pints?
Ans. 8704 pints.

6. In 8704 pints, how many bushels?
Ans. 136 bushels.

7. In 75 bu. 3 pks. 5 qts., how many quarts?
Ans. 2429 quarts.

8. In 2429 quarts, how many bushels?
Ans. 75 bu. 3 pks. 5 qts.

9. In 24 chaldrons, how many bushels?
Ans. 864 bushels.

10. In 864 bushels, how many chaldrons?
Ans. 24 chaldrons.

MEASURE OF TIME.

Art. 55. Time is the measure of duration, or existence. Its denominations are the century, c.; the year, yr.; the month, mo.; the week, wk.; the day, d.; the hour, h.; the minute, m.; and the second, s.

TABLE.

1 century = 100 years.
1 year = 12 cal. months.
1 year = $13\frac{5}{112}$ w. months.
1 year = $365\frac{1}{4}$ days.
1 month = 4 weeks.
1 week = 7 days.
1 day = 24 hours.
1 hour = 60 minutes.
1 *minute* = *60 seconds.*

1 year = $\frac{1}{100}$ of 1 c.
1 c. month = $\frac{1}{12}$ of 1 yr.
1 w. month = $\frac{112}{1461}$ of 1 yr.
1 day = $\frac{4}{1461}$ of 1 yr.
1 week = $\frac{1}{4}$ of 1 mo.
1 day = $\frac{1}{7}$ of 1 wk.
1 hour = $\frac{1}{24}$ of 1 d.
1 minute = $\frac{1}{60}$ of 1 h.
1 second = $\frac{1}{60}$ of 1 m.

The year is divided into 12 calendar months, as follows:

January,	1st month, has	31 days.	July,	7th month, has	31 days.
February,	2nd " "	28 "	August,	8th " "	31 "
March,	3d " "	31 "	September,	9th " "	30 "
April,	4th " "	30 "	October,	10th " "	31 "
May,	5th " "	31 "	November,	11th " "	30 "
June,	6th " "	30 "	December,	12th " "	31 "

When any year can be divided by 4 without a remainder, it is called leap year, in which February has 29 days.

1. In 5 days, how many hours? In 6 days? In 7 days? In 8 days? In 10 days?

2. In 120 hours, how many days? In 144 hours? In 168 hours? In 192 hours?

3. In 5 hours, how many minutes? In 6 hours? In 7 hours? In 8 hours?

4. In 300 minutes, how many hours? In 360 m.? In 420 m.? In 480 m.?

5. How many seconds are there in 1824 years?
Ans. 57561062400 seconds?

6. How many years are there in 57561062400 seconds?
Ans. 1824 years.

7. In 15 yrs. 19 d. 11 h. 37 m. 45 s., how many seconds?
Ans. 475047465 seconds.

8. In 475047465 seconds, how many years?
Ans. 15 yrs. 19 d. 11 h. 37 m. 45 s.

9. What number of seconds are there in an astronomical or solar year, which is 365 d. 5 h. 48 m. 51 s.?
Ans. 31556931 seconds.

10. What number of astronomical years are there in 631138620 seconds?
Ans. 20 yrs.

CIRCULAR MEASURE.

Art. 56. Circular measure is used in measuring circles, latitude and longitude, and in computing the revolution of the earth and other planets round the sun. Its denominations are the circle, c.; the sign, s.; the degree, °; the minute, ′; and the second, ″.

TABLE.

1 circle = 12 signs.
1 circle = 360 degrees.
1 sign = 30 degrees.
1 degree = 60 minutes.
1 minute = 60 seconds.

1 sign = $\frac{1}{12}$ of 1 circle.
1 degree = $\frac{1}{360}$ of 1 circle.
1 degree = $\frac{1}{30}$ of 1 sign.
1 minute = $\frac{1}{60}$ of 1 degree.
1 second = $\frac{1}{60}$ of 1 minute.

1. In 9 signs, how many degrees? In 10 signs? In 11 signs? In 12 signs?

2. In 270 degrees, how many signs? In 300°? In 330°? In 360°?

3. In 9 minutes, how many seconds? In 10 minutes? In 11 minutes? In 12 minutes?

4. In 540 seconds, how many minutes? In 600″? In 660″? In 720″?

5. In 9s. 13° 25′, how many minutes?

Ans. 17005 minutes.

6. In 17005 minutes, how many signs?

Ans. 9 s. 13° 25′.

7. How many seconds in 360 degrees?

Ans. 1296000 seconds.

8. How many degrees in 1296000 seconds?

Ans. 360 degrees.

Art. 57. MISCELLANEOUS MEASURES.

TABLE.

12 single things = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gr.
144 dozen = 1 great gr.

24 sheets of paper = 1 quire.
20 quires = 1 ream.
2 reams = 1 bundle.
20 single things = 1 score.

1. In 9 dozen, how many single things? In 10 dozen? In 11 dozen? In 12 dozen?

2. In 108 single things, how many dozen? In 120 s. th.? In 132 s. th.? In 144 s. th.?

3. In 5 quires of paper, how many sheets? In 6 quires? In 7 quires? In 8 quires?

4. In 120 sheets of paper, how many quires? In 144 sheets? In 168 sheets? In 192 sheets?

5. In 9 score, how many pounds? In 10 score? In 11 score? In 12 score?

6. In 180 pounds, how many score? In 200 lbs.? In 220 lbs.? In 240 lbs.?

BOOKS.

A sheet folded in two leaves is called a folio.

A sheet folded in four leaves is called a quarto, 4to.

A sheet folded in eight leaves is called an octavo, or 8vo.

A sheet folded in twelve leaves is called a duodecimo, or 12mo.

A sheet folded in eighteen leaves is called an 18mo.

A sheet folded in twenty-four leaves is called a 24mo.

PRACTICAL QUESTIONS IN REDUCTION.

Art. 58. 1. In £75. 12s., how many dollars, at 6 shillings each?

2. How many spoons, weighing 2 ounces 15 pennyweights each, can be made of a silver tankard weighing 68 oz. 15 pwts.?

3. In 3 boxes of Havana sugar, each containing 4 cwt. 3 qrs. 14 lbs., how many parcels of 7 pounds each?

4. If it take 4 yards 2 quarters to make a suit of clothes, what number of suits can be made from 324 yards?

5. What number of times will a wheel, which is 16 feet 6 inches in circumference, turn round in travelling 42 miles?

6. What number of square yards of carpeting will be required to cover a floor 18 feet in length, and 15 feet in breadth? (Art. 50.)

7. How many bricks will be required to pave a sidewalk 24 feet in length, and 6 feet in breadth; each brick being 8 inches in length, and 4 inches in breadth?

8. If a piece of land, in the form of a parallelogram, is 120 rods in length, and 90 rods in breadth, what number of acres does it contain?

9. What number of shingles will cover the roof of a house, which is 44 feet long, and 36 feet wide, with rafters 24 feet in length; allowing each shingle to cover 24 square inches?

10. What number of cubic feet are there in a log of mahogany, which is twelve feet in length, 18 inches in width, and 15 inches in depth? (Art. 51.)

11. There are 4 piles of wood; each pile is 48 feet in length, 4 feet in width, and 4 feet in height. What number of cords are there in the 4 piles?

12. What number of bricks will be required to build the walls of a house 44 feet long, 40 feet wide, and 20 feet high; each brick being 8 inches long, 4 inches wide, and 2½ inches thick; the thickness of the walls to be 12 inches, and no deduction to be made for doors and windows?

13. A merchant wishes to ship 570 bushels of flax-seed, in casks containing 7 bushels 2 pecks each; what number of casks will be required?

14. Suppose the number of inhabitants in the United States to be 20 million; how long would it take you to count them, counting 12 hours each day, and at the rate of 50 in a minute?

15. The sun is 96 millions of miles from the earth, and a cannon ball flies a mile in 7½ seconds; how long would a *cannon ball be*, at that rate, in flying from the earth to the sun?

ADDITION OF COMPOUND NUMBERS.

Art. 59. Addition of Compound Numbers is the method of finding the amount of two or more sums or quantities, each of which is composed of several denominations.

The several denominations of compound numbers may be regarded like the different orders of units in simple numbers, and the numbers of each denomination may be made the subject of a distinct operation, the result of which may be united to the next, and so on through all the denominations.

1. What is the amount of £4. 12s. and £9. 15s.?

3. James paid 4s. 6d. for a book, 2s. 6d. for a slate, and 1s. 9d. for a knife; how much did he pay for all of them?

5. What is the sum of 9 oz. 15 pwts. and 10 oz. 17 pwts.?

7. The American eagle weighs 10 pwts. 18 grs.; the dollar, 17 pwts. 4½ grs.; and the cent, 7 pwts. What is the whole weight of the 3 coins?

9. Bought 3 boxes of sugar; No. 1 weighed 4 cwt. 2 qrs. 14 lbs.; No. 2, 4 cwt. 1 qr. 7 lbs.; No. 3, 4 cwt. 3 qrs. What was the whole weight of the 3 boxes?

11. If it takes 25 yds. 3 qrs. of carpeting to cover the floor of one room, and 24 yds. to cover the floor of another, how many yards will be required for both?

13. If a schoolroom is 36 ft. 6 in. long, and 24 ft. 6 in. wide, what is the whole length of the two sides and two ends of the room?

15. What is the sum of 15 ft. 72 in., 12 ft. 36 in., and 10 ft. 36 inches?

17. Purchased 2 logs of mahogany; No. 1 measured 30 ft. 432 in.; No. 2, 25 ft. 1296 in. How many feet in the two logs?

2. What is the amount of 15s. 9d. and 12s. 6d.?

4. Purchased 2 loads of hay; the first cost £5. 10s.; the second, £4. 15s. What was the whole cost of the 2 loads?

6. What is the amount of 10 oz. 12 pwts. and 5 oz. 18 pwts.?

8. Purchased a silver pitcher weighing 15 oz. 17 pwts., and 6 tumblers weighing 18 oz. 15 pwts.; what was the weight of the whole purchase?

10. Purchased 2 loads of hay; the first weighed 18 cwt. 3 qrs. 14 lbs.; the second, 17 cwt. 1 qr. 21 lbs. What was the whole weight of the 2 loads?

12. Purchased 2 pieces of sheeting; the first measured 32 yds. 3 qrs.; the second, 30 yds. 1 qr. What was the whole number of yards purchased?

14. A piece of land is 60 rods in length, and 40 rods in breadth; how many rods of fence will be required to enclose this piece of land?

16. What is the amount of 123 a. 2 r. 15 rods, and 115 a. 3 r. 25 rods?

18. Bought 2 loads of wood; the first measured 1 c. 3½ ft.; the second, 1 c. 2¼ ft. How many cords are there in the 2 loads?

19. Purchased 2 casks of wine; the first contained 35 gals. 2 qts.; the second, 32 gals. 2 qts. What number of gallons did I purchase?

20. Purchased 2 barrels of vinegar; the first contained 31 gals. 2 qts.; the second, 30 gals. 2 qts. How many gallons did I purchase?

21. Bought 2 casks of beer; the first contained 18 gals. 2 qts.; the second, 17 gals. 2 qts. How many gallons did I buy?

22. Purchased 2 cans of milk; the first contained 3 gals. 2 qts.; the second, 2 gals. 3 qts. How much milk did I purchase?

23. Purchased 25 bu. 3 pks. of corn, 50 bu. 1 pk. of barley, and 24 bu. of oats; how many bushels of grain did I purchase?

24. Bought 2 barrels of apples; the first contained 2 bu. 3 pks.; the second, 2 bu. 1 pk. What number of bushels did I buy?

25. William was 9 yrs. and 6 mo. old when he came to Boston; he has lived in Boston 7 yrs. and 6 mo. How old is William?

26. John's age is 15 yrs. and 9 mo.; Henry's age is 12 yrs. and 7 mo. What is the sum of their ages?

27. What is the difference of latitude between Boston and St. Helena? The latitude of Boston is 42° 23′ north, the latitude of St. Helena is 15° 55′ south.

28. What is the difference of longitude between Boston and Calcutta? The longitude of Boston is 71° 4′ west, the longitude of Calcutta is 88° 28′ east.

Art. 60. 29. What is the amount of £47. 17s. 11d.; £24. 13s. 9d.; £36. 10s. 6d.; £21. 15s. 7d.; £17. 14s. 6d.; £15. 11s. 5d.; and £10. 10s. 2d.?

£.	s.	d.
47	17	11
24	13	9
36	10	6
21	15	7
17	14	6
15	11	5
10	10	2
£174	13	10 Ans.

We first write down the several sums, placing numbers of the same denomination under one another in separate columns. We then add the numbers of the lowest denomination, the amount is 46 pence, which are equal to 3s. 10d.; we write the 10d. under the column of pence, and then add the 3s. with the numbers in the column of shillings. The amount of the numbers in the column of shillings is 93s., which are equal to £4. 13s.; we write the 13s. under the column of shillings, and then add the £4. with the numbers in the column of pounds. The amount of the numbers in the column of pounds is £174, which we write under the column of pounds. The whole amount of the several sums is £174. 13s. 10d.

From the method of finding the amount of the several sums of money, in the preceding question, we deduce the following rule.

RULE. *Write down the given sums or quantities, placing numbers of the same denomination under one another, in separate columns.*

Add the numbers of the lowest denomination, and change the amount to the next higher denomination, by dividing it by so many of the same denomination as are equal to a unit of the next higher; write the remainder under the column, and add the quotient with the numbers of the next higher denomination.

Add the numbers of each succeeding higher denomination in the same manner, and write down the whole amount of the highest denomination.

30. What is the amount of £47. 14s. 7½d.; £52. 5s. 4½d.; £60. 0s. 11d.; £39. 19s. 1d.; and £40. 4s. 6¼d.?
Ans. £240. 4s. 6¼d.

31. A man's real estate is worth £4750. 14s. 6d.; his personal property amounts to £2600. 13s. 4d.; and he has debts due him amounting to £500. 15s. 10d. What is the whole amount of his property? Ans. £7852. 3s. 8d.

32. A gentleman's farm is valued at £2958. 16s. 10d.; he has a ship which is worth £2309. 6s. 8d.; he has notes amounting to £812. 8s. 9½d.; and he has cash to the amount of £408. 12s. 1¾d. What is the whole amount of his property? Ans. £6489. 4s. 5¼d.

33. Purchased a silver tea-pot, weighing 23 oz. 17 pwts. 11 grs.; a sugar-bowl, weighing 8 oz. 13 pwts. 19 grs.; a cream-pitcher, weighing 5 oz. 11 grs.; six cups, weighing 18 oz. 18 grs.; twelve table-spoons, weighing 27 oz. 9 pwts.; and twelve tea-spoons, weighing 9 oz. 15 pwts. 14 grs. What was the weight of the whole?
Ans. 92 oz. 17 pwts. 1 gr.

34. Purchased 4 hogsheads of sugar, weighing as follows: No. 1, 12 cwt. 3 qrs. 17 lbs.; No. 2, 12 cwt. 1 qr. 14 lbs.; No. 3, 11 cwt. 2. qrs. 21 lbs.; No. 4, 10 cwt. 3 qrs. 24 lbs. What was the whole weight of the 4 hogsheads?
Ans. 47 cwt. 3 qrs. 20 lbs.

35. Bought 4 pieces of cloth; No. 1 measured 38 yds. 2 qrs. 3 na.; No. 2, 37 yds. 3 qrs. 2 na.; No. 3, 36 yds. 1 qr. 2 na.; No. 4, 35 yds. 3 qrs. 3 na. What was the whole number of yards purchased? Ans. 148 yds. 3 qrs. 2 na.

36. A lady purchased 32 yds. 3 qrs. of sheeting; 31 yds. 1 qr. of shirting; 14 yds. 2 qrs. of linen; and 6 yds. 2 qrs. of cambric. What was the total number of yards purchased? Ans. 85 yards.

37. An ironmonger sold 94 tons 15 cwt. 3 qrs. of iron to one man; 75 tons 12 cwt. 2 qrs. to another; 64 tons 10 cwt. 2 qrs. 7 lbs. to another; and 45 tons 1 qr. 21 lbs. to another. What number of tons did he sell?

Ans. 279 t. 19 cwt. 1 qr.

38. What is the sum of 145 m. 3 fur. 25 rds.; 96 m. 4 fur. 15 rds.; 88 m. 5 fur. 30 rds.; 54 m. 2 fur. 10 rds.; and 106 miles? Ans. 491 miles.

39. A man has 4 farms; the first contains 420 a. 2 r. 25 rds.; the second, 375 a. 1 r. 15 rds.; the third, 280 a. 3 r. 35 rds.; and the fourth, 123 a. 0 r. 5 rds. What is the whole number of acres in the 4 farms? Ans. 1200 acres.

40. Purchased 4 sticks of timber; No. 1 contained 37 ft. 432 in.; No. 2, 35 ft. 864 in.; No. 3, 33 ft. 1296 in.; No. 4, 32 ft. 864 in. What was the whole number of feet purchased? Ans. 139 cubic feet.

41. Sold 4 hogsheads of molasses; the first contained 108 gals. 2 qts.; the second, 110 gals. 3 qts.; the third, 112 gals. 1 qt.; and the fourth, 118 gals. 2 qts. What was the whole quantity sold? Ans. 450 gallons.

42. Purchased 74 hhds. 37 gals. 2 qts. of beer at one time; 45 hhds. 41 gals. 3 qts. at another; 34 hhds. 15 gals. 1 qt. at a third; and 17 hhds. 15 gals. at a fourth. What was the whole quantity purchased?

Ans. 172 hhds. 1 gal. 2 qts.

43. A farmer raised 200 bu. 2 pks. of wheat; 175 bu. 3 pks. of rye; 320 bu. 1 pk. of corn; and 225 bu. 2 pks. of oats. What was the whole quantity of grain raised?

Ans. 922 bushels.

44. From the first discovery of America by Columbus to the settlement of Plymouth, in Massachusetts, was 128 yrs. 1 mo. 30 da.; from the settlement of Plymouth to the declaration of independence, 155 yrs. 6 m. 13 da.; from the declaration of independence to July 4th, 1847, 71 yrs. How long was it from the discovery of America to July 4th, 1847?

Ans. 354 yrs. 8 mo. 12 da.

45. What is the breadth of the torrid zone, which extends from the tropic of Cancer, in 23° 28′ north latitude, to the tropic of Capricorn, in 23° 28′ south latitude?

Ans. 46° 56′.

SUBTRACTION OF COMPOUND NUMBERS.

Art. 61. Subtraction of Compound Numbers is the method of finding the difference between any two sums or quantities, each of which is composed of several denominations.

The subtraction of compound numbers is performed in the same manner as the subtraction of simple numbers, with the exception of the number which it is necessary to add to the upper number of any denomination, whenever the lower number exceeds the upper.

1. James had 7s. 6d., and has paid 3s. 9d. of it for a knife; how much money has he left?
2. Mary has 15s. 6d.; Jane has 12s. 8d. How much more money has Mary than Jane?
3. From 9 oz. 10 pwts. take 5 oz. 15 pwts.
4. Subtract 4 lbs. 5 oz. 10 pwts. from 10 lbs.
5. From 5 lbs. 4 ℥, take 3 lbs. 8 ℥.
6. Subtract 2 ℈, 15 grs. from 5 ʒ, 10 grs.
7. From 2 cwt. take 3 qrs. 14 lbs.
8. Subtract 17 cwt. 3 qrs. from 5 tons.
9. From 25 yds. take 15 yds. 3 qrs.
10. Subtract 18 yds. 3 qrs. from 36 yds. 1 qr.
11. From 25 m. take 5 fur. 25 rods.
12. Subtract 9 ft. 6 in. from 5 rds.
13. From 36 ft. take 17 ft. 72 in.
14. Subtract 74 a. 1 r. 20 rds. from 150 a.
15. From 31 gals. 2 qts. take 15 gals. 3 qts.
16. Subtract 29 gals. 2 qts. from 2 hhds.
17. From 5 bar. of beer take 25 gals. 3 qts.
18. Subtract 45 gals. 2 qts. from 5 hhds.
19. From 10 bu. take 5 bu. 2 pks.
20. Subtract 3 pks. 5 qts. from 5 bu.
21. From 5 days take 15 h. 40 m.
22. Subtract 3 w. 5 d. from 6 mo.
23. From 45° take 14° 30′.
24. Subtract 44° 25′ from 90°.

Art. 62. If £41. 15s. 4d. 1qr. be taken from £75. 12s. 6d. 2 qrs., what will be the remainder or difference?

	£.	s.	d.	qrs.
	75	12	6	2
	41	15	4	1
Ans.	33	17	2	1

We first write down the greater sum, £75. 12s. 6d. 2qrs.; we then write the less sum, £41. 15s. 4d. 1qr., under the greater, placing those numbers which are of the same denomination under each other. We then begin with the lowest denomination, and take 1 farthing from 2 farthings, the remainder is 1 farthing, which we write under the farthings. We then take 4 pence from 6 pence, the remainder is 2 pence, which we write under the pence. As we cannot take 15 shillings from 12 shillings, we add 20 shillings, which are equal to 1 pound, to the 12 shillings, the amount is 32 shillings; we then take the 15 shillings from 32 shillings, the remainder is 17 shillings, which we write under the shillings. We next add 1 pound to 41 pounds, the amount is 42 pounds, which we subtract from 75 pounds, the remainder is 33 pounds, which we write under the pounds. The whole remainder or difference is £33. 17s. 2d. 1qr.

By adding 20s. to the 12s. of the greater sum, and £1 to the £41 of the less sum, we have added the same amount to each; consequently, their difference is not altered. From the above illustration we deduce the following rule for subtraction of compound numbers.

RULE. *Write down the greater sum or quantity; then write the less sum or quantity under the greater, placing those numbers which are of the same denomination directly under each other.*

Begin with the lowest denomination, and take the lower number of each denomination, in succession, from the upper, and write each remainder under its respective denomination.

Whenever the lower number of any denomination is greater than the upper, add so many units of the same denomination to the upper number as are equal to a unit of the next higher, subtract the lower number from the amount, write the remainder underneath, then add a unit to the lower number of the next higher denomination before it is subtracted.

25. A man borrowed £50. 10s. 6d., and has paid £25. 18s. 9d.; how much remains unpaid? Ans. £24. 11s. 9d.

26. A merchant sold goods to the amount of £136. 15s. 6d. 2qrs., and has received in payment £50. 18s. 9d. 3qrs.; how much remains due? Ans. £85. 16s. 8d. 3qrs.

27. A man purchased a farm for £1250; he has paid at one time £750. 10s., at another, £250. 15s. How much remains unpaid? Ans. £248. 15s.

28. From 46 lbs. 2 oz. 3 pwts. 8 grs. take 19 lbs. 8 oz 15 pwts. 12 grs. Ans. 26 lbs. 5 oz. 7 pwts. 20 grs.

29. From 15 lbs. 6 ℥, 3 ʒ, 1 ℈, 13 grs. take 10 lbs. 11 ℥, 6 ʒ, 2 ℈, 18 grs. Ans. 4 lbs. 6 ℥, 4 ʒ, 1 ℈, 15 grs.

30. Bought a hogshead of sugar weighing 12 cwt. 1 qr. 14 lbs., and sold 7 cwt. 2 qrs. of it; what quantity remains unsold? Ans. 4 cwt. 3 qrs. 14 lbs.

31. A merchant purchased 151 yds. 2 qrs. of broadcloth, and has sold 53 yds. 3 qrs. of it; how many yards has he remaining unsold? Ans. 97 yds. 3 qrs.

32. The distance from Boston to Albany by railroad is 200 miles, from Boston to Worcester it is 44 miles; what is the distance from Worcester to Albany? Ans. 156 miles.

33. A man, who owned a farm containing 325 a. 2 roods, gave his son 150 a. 1 r. 25 rds.; how many acres had he remaining? Ans. 175 a. 0 r. 15 rds.

34. A man purchased 275 cords of wood, and has sold 115 cords and 64 cubic feet of it; what quantity remains unsold? Ans. 159 cords, 64 feet.

35. From a pipe of wine containing 126 gallons, 31 gal. 2 qts. 1 pt. have leaked out; how much remains in the pipe? Ans. 94 gals. 1 qt. 1 pt.

36. A merchant purchased 1500 bushels of Indian corn, and has sold 750 bu. 3 pks. of it; what number of bushels has he left? Ans. 749 bu. 1 pk.

37. William is 15 yrs. 9 mo. and 15 d. old, Henry is 12 yrs. 10 mo. and 25 d. old; what is the difference of their ages? Ans. 2 yrs. 10 mo. 20 days.

When time is expressed in years, calendar months, and days, the difference of time between any two dates may be found by subtracting the former date from the latter, observing to number the months in their order.

38. A gave B his note, dated August 21st, 1844; the note was paid May 4th, 1845. How long was the note on interest?

	yrs.	mo.	d.
Latter date,	1845	4	4
Former date,	1844	7	21
		8	14 Ans.

39. The United States declared war against Great Britain, June 18th, 1812, a treaty of peace was signed December 24th, 1814; how long did the war continue? Ans. 2 yrs. 6 mo. 6 days.

40. The latitude of London is 51° 32′ north, the latitude of Boston is 42° 23′ north; what is the difference of latitude between the two places? Ans. 9° 9′.

41. The longitude of Washington is 77° 43′ west from Greenwich, the longitude of Boston is 71° 3′ west; what is the difference of longitude between the two cities? Ans. 6° 40′.

MULTIPLICATION OF COMPOUND NUMBERS.

Art. 63. Multiplication of Compound Numbers is the method of repeating any given sum or quantity, which is composed of two or more denominations, any required number of times.

Multiplication of compound numbers is performed by multiplying the number of each denomination in succession, beginning with the lowest, and changing each product to the next higher denomination.

1. If 1 yard of cloth cost 2s. 6d., what will 3 yards cost?

3. What will 5 bushels of apples cost, at 4s. 9d. a bushel?

5. If 1 yard of broadcloth is worth £1. 15s., what are 8 yards worth?

7. What is the weight of 9 silver cups, each weighing 3 oz. 10 pwts.?

9. If a piece of cloth measures 10 yards 3 qrs., how many yards are there in 11 such pieces?

11. What number of acres are there in 4 fields, each containing 12 a. 3 roods?

13. Purchased 6 kegs of molasses, each keg containing 10 gals. 2 qts.; how many gallons did I purchase?

15. In 9 barrels of apples, each containing 2 bu. 1 pk., how many bushels?

2. If 1 pair of shoes cost 4s. 3d., what will 4 pair cost?

4. What will 6 bushels of wheat cost, at 7s. 6d. a bushel?

6. If 1 cwt. of sugar is worth £2. 5s., what is the value of 7 cwt.?

8. What is the weight of 10 barrels of flour, each weighing 1 cwt. 3 qrs.?

10. In 1 rod there are 16 ft. 6 in.; what number of feet are there in 12 rods?

12. In 5 piles of wood, each containing 10 cords 4 feet, how many cords?

14. Purchased 8 cans of milk, each can containing 2 gals. 2 qts.; what number of gallons did I purchase?

16. If a man walks a mile in 12 m. 10 sec., how long will it take him to walk 12 miles?

Art. 64. What will five yards of broadcloth cost, at £1. 3s. 6d. 3qrs. a yard?

	£	s.	d.	qrs.
	1	3	6	3
				5
Ans.	5	17	9	3

If one yard is worth £1. 3s. 6d. 3 qrs., 5 yards are worth 5 times £1. 3s. 6d. 3 qrs.

We first write down the price of 1 yard; we then write 5, the number of yards, under the lowest denomination. We begin with the lowest denomination, and multiply the 3 farthings by 5, the product is 15 farthings, which we change to pence by dividing by 4; we write down the remainder, 3 farthings, and reserve the quotient, 3 pence. We then multiply the 6 pence by 5, the product is 30 pence, and 3 pence which we reserved are 33 pence, which we change to shillings by dividing by 12; we write down the remainder, 9 pence, and reserve the quotient, 2 shillings. We then multiply the 3 shillings by 5, the product is 15 shillings, and 2 shillings which we reserved are 17 shillings; as 17 shillings are not equal to 1 pound, we write them under the 3 shillings. Lastly, we multiply the 1 pound by 5, the product is £5, which we write underneath. The total product, or answer, is £5. 17s. 6d. 3qrs. Hence, the following rule for multiplication of compound numbers.

RULE. *Write down the given sum or quantity for a multiplicand; then write the multiplier under the lowest denomination of the multiplicand.*

Multiply the number of the lowest denomination by the multiplier; then change the product to the next higher denomination by dividing it by so many of the same denomination as are equal to a unit of the next higher; write down the remainder, and add the quotient to the product of the next higher denomination.

Multiply the number of each succeeding denomination in the same manner, and write down the whole product of the highest denomination.

17. What is the value of 9 yards of linen, at 5s. 3d. a yard? Ans. £2. 7s. 3d.

18. What will 12 barrels of apples cost, at 8s. 9½d. a barrel? Ans. £5. 5s. 6d.

19. Purchased 6 silver cups, each weighing 2 oz. 15 pwt. 10 gr.; what was the weight of the whole?
Ans. 16 oz. 12 pwts. 12 grs.

20. Bought 5 loads of hay, each weighing 18 cwt. 3 qrs. 14 lbs.; what was the whole weight of the 5 loads?
Ans. 4 t. 14 cwt. 1 qr. 14 lbs.

21. Purchased 6 pieces of cloth, each measuring 25 yds. 1 qr. 2 n.; what was the whole number of yards purchased?
Ans. 152 yds. 1 qr.

22. If a railroad car runs uniformly at the rate of 18 m. 4 fur. 20 rods in an hour, what distance will it run in 8 hours? Ans. 148 m. 4 fur.

23. There are 9 fields, each of which contains 12 a. 2 r. 25 rods; how many acres are there in the whole?
Ans. 113 a. 3 r. 25 rods.

24. In 10 piles of wood, each containing 15 cords 64 cubic feet, how many cords? Ans. 155 cords.

25. In 9 hogsheads of molasses, each containing 115 gallons 2 quarts, how many gallons? Ans. 1039 gals. 2 qts.

26. If a milkman sells 40 gals. 2 qts. of milk each day, how much will he sell in 7 days? Ans. 283 gals. 2 qts.

27. If an acre of land produces 40 bu. 2 pks. of Indian corn, how many bushels will 11 acres produce?
Ans. 445 bu. 2 pks.

28. The earth performs its annual revolution round the sun in 365 days, 5 h. 48 m. 51 sec.; in what time will it perform 5 revolutions? Ans. 1826 d. 5 h. 4m. 15 sec.

29. What will 25 bushels of wheat come to, at 7s. 6d. a bushel? Ans. £9. 7s. 6d.

30. What will 35 tons of iron cost, at £24. 15s. 6d. a ton?
Ans. £867. 2s. 6d.

31. If it take 3 yds. 3 qrs. of broadcloth to make 1 suit of clothes, how many yards will be sufficient to make 40 suits?
Ans. 150 yds.

32. How much wheat will be sufficient to make 96 barrels of flour, allowing 4 bu. 3 pks. to make a barrel?
Ans. 456 bushels.

33. A ship-builder purchased 12 sticks of timber, each stick containing 75 feet 144 inches; what was the whole amount purchased? Ans. 901 cubic feet.

34. A house-wright purchased 36 pine boards, each measuring 17 feet 36 inches; what number of feet did he purchase?
Ans. 621 square feet.

35. A trader purchased 4 bales of shirting; each bale contained 25 pieces, and each piece measured 32 yds. 3 qrs. What number of yards did he purchase?
Ans. 3275 yards.

36. A grocer purchased 56 casks of raisins, each weighing 1 cwt. 2 qrs. 12 lbs.; what is the whole weight of the 56 casks? Ans. 90 cwt.

37. A planter sold 72 hogsheads of tobacco, each containing 12 cwt. 3 qrs.; what was the weight of the whole quantity? Ans. 918 cwt.

38. A southern planter sold 108 bales of cotton, each weighing 4 cwt. 3 qrs. 14 lbs.; what was the weight of the whole quantity? Ans. 526 cwt. 2 qrs.

39. A merchant purchased 144 casks of rice, each cask containing 4 cwt. 2 qrs. 14 lbs.; what number of hundred weight did he purchase? Ans. 666 cwt.

40. If an acre of wheat produce 24 bushels 3 pks., what number of bushels will 132 acres produce? Ans. 3267 bu.

DIVISION OF COMPOUND NUMBERS.

Art. 65. Division of Compound Numbers is the method of finding any required part of any given sum or quantity, which is composed of two or more denominations.

Division of compound numbers is performed by dividing the number of each denomination in succession, beginning with the highest, and reducing each remainder to the next lower denomination.

1. If 3 yards of cloth cost 7s. 6d., what will 1 yard cost?

2. If 4 pair of shoes be worth 17s., what is 1 pair worth?

3. If 5 bushels of apples are worth £1. 3s. 9d., what is 1 bushel worth?

4. If 6 bushels of wheat are worth £2. 15s., what is 1 bushel worth?

5. If 8 yards of broadcloth are worth £14, what is 1 yard worth?

6. If 7 cwt. of sugar is worth £15. 15s., what is 1 cwt. worth?

7. Purchased 9 silver cups, weighing 31 oz. 10 pwt.; what was the weight of each cup?

8. Bought 10 barrels of flour, containing 17 cwt. 2 qrs.; how much did each barrel contain?

9. Purchased 11 pieces of cloth, measuring 118 yards 1 qr.; what was the length of each piece?

10. In 12 rods there are 198 feet; what number of feet are there in 1 rod?

11. In 4 pieces of land of equal size there are 51 acres; how many acres are there in each piece?

12. In 5 piles of wood, each of the same size, there are 50 cords 4 feet; how many cords are there in each pile?

13. Purchased 63 gallons of molasses, and had it put into 6 kegs of equal size; what quantity did each keg contain?

14. Bought 20 gallons of milk, and put it into 8 cans of equal size; what quantity did each can contain?

15. Purchased 20 bu. 1 pk. of apples, and put them into 9 barrels; what quantity did each barrel contain?

16. If a man walks 12 miles in 2 hours 26 minutes, in what time does he walk one mile?

Art. 66. If 5 yards of broadcloth are worth £8. 1s. 10½d., what is the value of 1 yard?

One yard is worth one fifth as much as 5 yards; therefore, dividing the price of 5 yards by 5, the quotient is the value of 1 yard.

	£	s.	d.	qr.
	5)8	1	10	2
Ans.	1	12	4	2

We first write down the price of 5 yards, £8. 1s. 10d. 2qrs., for a dividend; and then write 5, the number of yards, at the left, for a divisor. Then one fifth of £8 is £1, which we write under the £8, and

£3 remain. £3 are equal to 60s., and 1s. are 61s.; one fifth of 61s. is 12s., which we write under the 1s., and 1s. remains. 1s. is equal to 12d., and 10d. are 22d.; one fifth of 22d. is 4d., which we write under the 10d., and 2d. remain. 2d. are equal to 8qrs., and 2qrs. are 10qrs.; one fifth of 10qrs. is 2qrs., which we write under the 2qrs. The value of one yard is £1. 12s. 4d. 2qrs.

From the above illustration we derive the following rule for the division of compound numbers.

RULE. *Divide the number of each denomination in succession, beginning with the highest, and write each quotient under its respective denomination.*

Whenever there is a remainder, reduce it to the next lower denomination, by multiplying it by so many of the next lower denomination as are equal to a unit of the higher; to the product add the number of the lower denomination, if any; then divide their sum as before directed. Whenever the divisor is more than 12, *and not the product of two or more factors, divide as directed in long division, and write the several quotients in their order, at the right of the dividend.*

17. If 9 yards of linen are worth £2. 7s. 3d., what is the value of one yard? Ans. 5s. 3d.

18. Purchased 12 barrels of apples for £5. 5s. 6d.; what was the cost of each barrel? Ans. 8s. 9½d.

19. Bought 6 silver cups, weighing 16 oz. 12 pwt. 12 gr.; what was the weight of each cup?
Ans. 2 oz. 15 pwt. 10 grs.

20. Purchased 5 loads of hay, weighing 4 t. 14 cwt. 1 qr. 14 lbs.; what was the weight of each load?
Ans. 18 cwt. 3 qrs. 14 lbs.

21. Bought 6 pieces of cloth, measuring 152 yds. 1 qr.; what was the length of each piece?
Ans. 25 yds. 1 qr. 2 na.

22. If a railroad car runs 148 m. 4 fur. in 8 hours, what distance does it run each hour?
Ans. 18 m. 4, fur. 20 rods.

23. A farmer purchased 113 a. 3 r. 25 rods of land, and divided the whole quantity into 9 fields of equal size; what quantity did each field contain? Ans. 12 a. 2 r. 25 rods.

24. Purchased 155 cords of wood, and had it corded in 10 ranges, each of the same length and height; what quantity of wood did each range contain? Ans. 15 cords 64 c. feet.

25. Purchased 9 hogsheads of molasses, containing 1039 gals. 2 qts.; what quantity did each hogshead contain?
Ans. 115 gals. 2 qts.

26. In 7 days, a milkman sold 283 gals. 2 qts, of milk; what number of gallons must he have sold each day?
Ans. 40 gals. 2 qts.

27. A farmer raised 445 bu. 2 pks. of Indian corn on 11 acres of land; what was the product of each acre?
Ans. 40 bu. 2 pks.

28. The earth performs 5 revolutions round the sun in 1826 d. 5 h. 4 m. 15 sec.; in what time does it perform one revolution? Ans. 365 d. 5 h. 48 m. 51 sec.

29. Purchased 25 bushels of wheat for £9. 7s. 6d.; what was the value of each bushel? Ans. 7s. 6d.

30. A merchant purchased 35 tons of iron, for which he paid £867. 2s. 6d.; what was the value of each ton?
Ans. £24. 15s. 6d.

31. A tailor purchased 150 yards of broadcloth, from which he made 40 suits of clothes; what number of yards did each suit contain? Ans. 3 yds. 3 qrs.

32. A miller manufactured 96 barrels of flour from 456 bushels of wheat; what number of bushels did it require to make each barrel? Ans. 4 bu. 3 pks.

33. A merchant purchased a cargo of coal, containing 56 tons, for which he paid £100. 16s.; what did it cost him a ton? Ans. £1. 16s.

34. A manufacturer sold 67 yards of broadcloth, for which he received £92. 2s. 6d.; at what price did he sell it a yard?
Ans. £1. 7s. 6d.

35. A ship-builder purchased 12 sticks of timber of equal size, measuring 901 cubic feet; what number of cubic feet did each stick contain? Ans. 75 feet 144 inches.

36. A house-wright purchased 36 pine boards of equal size, measuring 621 square feet; what number of square feet did each board contain? Ans. 17 feet 36 inches.

37. A trader purchased 4 bales of sheeting, each bale containing 25 pieces of equal length; the whole number of yards purchased was 3275. What number of yards did each piece contain? Ans. 32 yds. 3 qrs.

38. A planter sold 72 hogsheads of tobacco, each containing the same quantity; the whole quantity weighing 918 cwt. What quantity did each hogshead contain?
Ans. 12 cwt. 3 qrs.

PRACTICAL QUESTIONS IN COMPOUND NUMBERS.

Art. 67. 1. Purchased 12 yards of Irish linen, at 4s. 6d. a yard; and 5 yards of linen cambric, at 6s. 9d. a yard. What was the amount of my purchase ?

2. Bought 6 silver table-spoons, weighing 1 oz. 15 pwt. each; and 12 tea-spoons, weighing 18 pwt. 6 grains each. What was the whole weight of the two kinds of spoons ?

3. A grocer purchased of one dairyman 15 cwt. 3 qrs. 14 lbs. of cheese; of another, 12 cwt. 2 qrs. 21 lbs; of another, 10 cwt. 1 qr. What was the quantity purchased?

4. A merchant purchased 67 cwt. 3 qrs. 14 lbs. of sugar; he sold 24 cwt. 1 qr. 7 lbs. to A; and 15 cwt. 2 qrs. 21 lbs. to B. What quantity has he remaining unsold ?

5. A merchant purchased 25 pieces of cotton shirting, each piece containing 32 yds. 3 qrs.; he has since sold 12 pieces. What number of yards has he remaining unsold ?

6. A gentleman purchased a farm, containing 325 acres; he has since sold a wood-lot, containing 17 acres 25 rods, to A; and has given his son 125 acres 2 roods. What number of acres has he remaining ?

7. A tailor purchased 4 pieces of broadcloth; each piece measured 42 yards 3 qrs.; what number of suits of clothes can there be made from the 4 pieces, allowing 3 yds. and 3 qrs. to make a suit ?

8. A wood-wharfinger purchased a cargo of wood, and had it piled in 8 equal ranges; each range contained 6 cords, 4 feet of wood, and 8 cubic feet. What was the whole number of cords purchased ?

9. A silversmith melted 21 oz. 3 pennyweights of old silver, which he manufactured into 12 spoons of equal size; what was the weight of each spoon ?

10. A farmer raised 457 bushels 2 pecks of potatoes, which he carried to market in 15 loads of equal size; what number of bushels did each load contain ?

11. A farmer picked 312 bushels 2 pecks of winter apples, which he put into barrels holding 2 bushels 2 pecks each; what number of barrels were required ?

12. A merchant purchased 25 boxes of sugar, each weighing 4 cwt. 2 qrs. 14 lbs.; what was the quantity purchased?

13. A speculator purchased a tract of land, containing 25 acres 3 roods and 5 rods, which he has divided into 125 house-lots of equal size; what quantity of land does each lot contain ?

DEFINITIONS OF ARITHMETICAL TERMS.

Art. 68. An *integer* is any whole number; as 1, 5.

A *measure* of a number is any number that will divide it without a remainder; thus, 3 is a measure of 9.

A *common measure* of two or more numbers is any number that will divide each of them without a remainder; thus, 4 is a common measure of 12, 24, and 36.

The *greatest common measure* of two or more numbers is the greatest number that will divide each of them without a remainder; thus, 12 is the greatest common measure of 12, 24, and 36.

A *multiple* of a number is a number that can be measured by it; thus, 9 is a multiple of 3.

A *common multiple* of two or more numbers is a number that can be measured by each of them; thus, 16 is a common multiple of 2, 4, and 8.

The *least common multiple* of two or more numbers is the least number that can be measured by each of them; thus, 8 is the least common multiple of 2, 4, and 8.

Even numbers are those numbers which can be measured by 2; thus, 4, 6, and 8, are even numbers.

Odd numbers are those numbers which cannot be measured by 2; thus, 3, 5, and 7, are odd numbers.

A *prime number* is a number which can only be measured by a unit, or by the number itself; thus, 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, &c., are prime numbers, each of which can only be measured by a unit, or by itself. Two numbers are *prime* to each other when a unit is the only number that will measure both of them; thus, 4 and 5 are prime to each other.

A *composite number* is a number that can be measured by two or more numbers, each of which is greater than a unit; thus, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, &c., are composite numbers, each of which can be measured by two or more numbers greater than a unit.

A *prime factor* of a number is a prime number that will measure it; and all the prime factors of a number are all the prime numbers that will measure it; thus, 3 is a prime factor of 15, and 3 and 5 are all the prime factors of 15.

A *component factor* of a number is a composite number that will measure it, and all the component factors of a number are all the composite numbers that will measure it; thus, 4 is a component factor of 24, and all the component factors of 24 are 4, 6, 8, 12, and 24.

All the different factors of a number are all the different prime and composite numbers that will measure it; thus, all the different factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

The *aliquot parts* of a number are all the numbers that will measure it; thus, the aliquot parts of 12 are 1, 2, 3, 4, 6, and 12.

Every composite number is equal to the product of all its *prime* factors; thus, all the prime factors of 30 are 1, 2, 3, 5; and $1 \times 2 \times 3 \times 5 = 30$.

All prime numbers, except 2 and 5, have 1, 3, 7, or 9, in the place of units.

ANALYSIS OF NUMBERS AND THEIR FACTORS.

Art. 69. Every number is either a *prime* number, or it is a *composite* number.

The number 1 is an original *prime* number, and is a factor or measure of every other number, and its corresponding factor is the number itself.

The number 2 is the only *even prime* number, because it is a factor of every second or even number, and its corresponding factor must be the half of the number; hence, we are sure that one half of the natural series of numbers are composite numbers.

Whenever we divide an odd number by 2, there is always an odd 1 remaining; but if we divide ever so many even numbers by 2, there is not an odd 1 remaining; therefore, the sum of any number of even numbers is an even number; so also is the sum of any *even* number of odd numbers, because the odd ones make an even number. Hence it follows that if 2, or any other number, is a factor of the sum of the remainders, it must be a factor of the sum of the numbers. Also, a factor of two numbers must be a factor of their sum, their difference, their product, or any multiple of either or both of them.

It is evident that 2 is a factor of every second number; 3, of every third; 4, of every fourth; 5, of every fifth; and so on.

If we can show that any factor will measure a number, except a certain number of figures on the right, whatever the figures at the left of these may be; and if we perceive that this factor will measure those right hand figures, then it will follow, on the principle that a factor of all the particular numbers is a factor of their sum, that the factor in question is a factor of the number.

Any number may be considered as consisting of as many numbers as there are figures in it; that it is all tens except the right hand figure; that it is all hundreds except the two right hand figures; that it is all thousands except the three right hand figures; therefore, if any factor of 10 measures the units, it must measure the number; if any factor of 100 measures the tens and units, it must measure the number; if any factor of 1000 measures the hundreds, tens, and units, it must measure the number.

If we examine every second or even number of the natural series of numbers, we shall find that the unit figure of each is either 0, 2, 4, 6, or 8. As 2 is a measure of each of these figures, and also of 10, it follows, that 2 is a factor of every number whose right hand figure is either 0, 2, 4, 6, or 8.

If we examine the figures of every third number of the natural series of numbers, we shall find that the sum of those figures can be measured by 3; hence, 3 is a factor of every number the sum of whose figures can be measured by 3.

4 is a factor of 100; hence it follows, that 4 is a factor of every number whose two right hand figures can be measured by 4.

5 is also a factor of 100; hence it follows, that 5 is a factor of every number whose two right hand figures can be measured by 5, or whose unit figure is either 0, or 5.

6 is a factor of every number of which 2 and 3 are both factors.

7 is a factor of every number whose right hand figure is 7, and whose left hand figures are any multiple of 7; as 147, 217, 287, &c.

7 is also a factor of every number whose two or three right hand figures can be measured by 7, and whose left hand figures are any multiple of those right hand figures; as 2814, 6321, 294147, &c.

7 is a factor of every number whose two right hand figures are one third of the left hand figure or figures; as 301, 602, 3010, 3311, 3612, &c.

7 is also a factor of every number less than 1000 whose left hand figure is one fifth of its two right hand figures; as 210, 315, 420, 525, 630, 735, 840, 945. 7 is also a factor of every number consisting of 4 figures whose two left hand figures are one fifth of its two right hand figures; as 1050, 1155, 1260, 1365, 1470, 1575, &c.

8 is a factor of 1000; hence it is a factor of every number whose three right hand figures can be measured by 8; thus, 2936, in which 936 can be measured by 8.

If the sum of the figures in any number added horizontally can be measured by 9, then 9 is a factor of the number.

"This may be shown as follows:—If from any number of 10s we take an equal number of 9s, the same number of 1s must remain. Every figure in a number, except the right hand figure, is a number of 10s, and if every figure, except the right hand figure, be divided by 9, there will remain the same number of units as the figure expresses; hence, we may consider all the figures in every number, including the units, as remainders after dividing by 9, and it has been shown that, if the divisor measures the sum of the remainders, it will measure the sum of the numbers." Thus the sum of the figures in 75681 is 27, and as 27 can be measured by 9, it follows that 9 is a factor of 75681.

10 is a factor of every number whose right hand or unit figure is a cipher.

11 is a factor of every number of which the sum of the figures in the odd places is equal to the sum of the figures in the even places, or differs by a number which can be measured by 11. Thus, 926519, in which $9 + 5 + 2 = 1 + 6 + 9$; and 381909, in which $9 + 9 + 8$ differs from $0 + 1 + 3$ by 22, which can be measured by 11.

12 is a factor of every number of which 3 and 4 are both factors.

If any number can be measured by 3 and 11, then 33 must be a factor of the number.

If any number can be measured by 9 and 11, 99 must be a factor of the number.

If any number whatever is a measure of any other number, the quotient must be a factor of the number measured.

PRIME AND COMPONENT FACTORS OF NUMBERS.

Art. 70. In Art. 68, we have given a definition of *prime* and *component* factors; we will now explain the method of finding them.

Suppose we wish to find all the prime factors of 100. If we divide 100 by the prime number 2, the quotient is 50. As 50 is not a prime number, we divide 50 by the prime number 2; the quotient is 25. As 25 is not a prime number, we divide 25 by the prime number 5; the quotient is

5, which is also a prime number. Hence, the prime factors of 100 are 1, 2, 2, 5, 5, and 1 × 2 × 2 × 5 × 5 = 100. From the above illustration, we derive the following rule for finding all the prime factors of any composite number.

RULE. *Divide the given number by the least prime number that will measure it; if the quotient is a prime number, the divisor, quotient, and a unit, are all its prime factors. If the quotient is not a prime number, then divide this quotient by the least prime number that will measure it, and so continue dividing until a quotient is obtained which is a prime number. The several divisors, last quotient, and a unit, are all its prime factors.*

1. What are the prime factors of 4, 6, 8, 10, and 12?

2. What are the prime factors of 14, 15, 16, 18, and 20?

3. What are the prime factors of 84?
Ans. 1, 2, 2, 3, 7.

4. What are the prime factors of 144?
Ans. 1, 2, 2, 2, 2, 3, 3.

5. What are the prime factors of 500?
Ans. 1, 2, 2, 5, 5, 5.

6. What are the prime factors of 864?
Ans. 1, 2, 2, 2, 2, 2, 3, 3, 3.

7. What are the prime factors of 2310?
Ans. 1, 2, 3, 5, 7, 11.

8. What are the prime factors of 29925?
Ans. 1, 3, 3, 5, 5, 7, 19.

Art. 71. Suppose we wish to find all the different factors of 36. We first resolve 36 into its prime factors, which are 1, 2, 2, 3, 3. We have before shown that every composite number is equal to the product of all its prime factors; hence, it follows that the several products resulting from all the different combinations of 1, 2, 2, 3, 3, must be all the component factors of 36. These *component* factors, and all the different *prime* factors, must be all the different factors of 36. The *different prime* factors of 36 are 1, 2, 3.—1 × 2 × 2 = 4.—1 × 2 × 3 = 6.—1 × 3 × 3 = 9.—1 × 2 × 2 × 3 = 12.—1 × 2 × 3 × 3 = 18.—1 × 2 × 2 × 3 × 3 = 36. Hence, all the different factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36. From the above illustration we derive the following rule for finding all the different factors of any composite number.

RULE. *First, find all the prime factors of the given number. Then find the product of each different combination* *which can be made of the prime factors; these products are the different component factors. The different prime factors and the different component factors are all the different factors of the number.*

If a composite number has only two prime factors greater than a unit, those factors, together with a unit and the number itself, are all its different factors.

1. What are all the different factors of 4, 6, 8, 10, and 12?

2. What are all the different factors of 14, 16, 18, and 20?

3. What are all the different factors of 48?

Ans. 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.

4. What are all the different factors of 100?

Ans. 1, 2, 4, 5, 10, 20, 25, 50, and 100.

5. What are all the different factors of 360?

Ans. 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, and 360.

Art. 72. TABLE OF PRIME NUMBERS.

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GREATEST COMMON FACTOR OR MEASURE OF TWO OR MORE NUMBERS.

Art. 73. Suppose we wish to find the greatest common factor or measure of 675, 1125, and 3375. The prime factors of 675 are 3, 3, 3, 5, 5. The prime factors of 1125 are 3, 3, 5, 5, 5. The prime factors of 3375 are 3, 3, 3, 5, 5, 5.

By examining the prime factors of each of the numbers, we find that the prime factors 3, 3, 5, 5, occur in each of them. We have shown, in Art. 71, that the product of any combination of the prime factors of a number is a measure of it; hence, it is evident that the product of all the common prime factors of two or more numbers must be their greatest common factor or measure. Therefore, $3 \times 3 \times 5 \times 5 = 225$, the greatest common factor or measure of 675, 1125, and 3375. Hence the following rule for finding the greatest common factor or measure of two or more numbers.

RULE. *Find all the prime factors of each of the given numbers. Then select those prime factors which are common to all of the numbers, and find their product; this product will be their greatest common factor or measure.*

If the several numbers have no common prime factor greater than a unit, then a unit is their greatest common measure. If they have only one common prime factor greater than a unit, this common factor is their greatest common measure.

1. What is the greatest common factor of 8, 12, 16, and 20?

2. What is the greatest common measure of 18, 24, 36, and 48?

3. What is the greatest common measure of 612 and 540? Ans. 36.

4. What is the greatest common measure of 1080 and 1224? Ans. 72.

5. What is the greatest common measure of 56, 280, and 840? Ans. 56.

6. What is the greatest common measure of 1326, 3094, and 4420? Ans. 442.

Art. 74. The greatest common factor or measure of two numbers may also be found by the following rule.

RULE. *Divide the greater number by the less, then divide the divisor by the remainder, and continue dividing the preceding divisor by the last remainder until nothing remains; the last divisor will be their greatest common measure.*

1. What is the greatest common measure of 36 and 96?

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                           36 ) 96 ( 2
                                72
                                ——
                                24 ) 36 ( 1
                                     24
                                     ——
Greatest common measure,        12 ) 24 ( 2
                                     24
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The truth of this rule may be shown as follows: — Since 12 divides 24, it also divides 24 + 12, or 36. Again, since 12 divides 24 and 36, it must also divide 36 × 2 + 24, or 96.

2. What is the greatest common measure of 296 and 407? Ans. 37.

3. What is the greatest common measure of 1825 and 2555? Ans. 365.

LEAST COMMON MULTIPLE OF TWO OR MORE NUMBERS.

Art. 75. In Art. 68, we have defined the least common multiple of two or more numbers to be the least number that can be measured by each of them. The least common multiple of two or more prime numbers, or of several numbers all of which are prime to each other, is, evidently, the product of all of them.

Suppose we wish to find the least common multiple of the composite numbers 56, 280, and 840. The prime factors of 56 are 2, 2, 2, 7. The prime factors of 280 are 2, 2, 2, 5, 7. The prime factors of 840 are 2, 2, 2, 3, 5, 7.

By examining the prime factors of each of these numbers, we find the different prime factors are 2, 3, 5, and 7; also, that 2 occurs three times in each of the numbers; 3, only once, and in one of the numbers only; 5, once in each of two of the numbers; and 7, once in each of the numbers.

We have shown, in Art. 68, that the product of all the prime factors of any composite number is equal to the number itself. We have now to show that the product of all the different prime factors of several composite numbers, using each factor, in finding the product, the greatest number of times it occurs in any of the given numbers, must be the least common multiple of those numbers.

The product of all the different prime factors of several numbers, using each factor, in finding the product, the greatest number of times it occurs in any of the given numbers, must include all the prime factors of each of them; and we have shown, in Art. 70, that the product of all the prime factors of any number can be measured by the product of any combination of those factors; consequently, the product of *all the different* prime factors of several numbers must be their least common multiple. Therefore, $2 \times 2 \times 2 \times 3 \times 5 \times 7 = 840$, the least common multiple of 56, 280, and 840.

From the preceding illustration we derive the following rule for finding the least common multiple of two or more numbers.

RULE. *Find all the prime factors of each of the given numbers. Then find the product of all their different prime factors, using each factor the greatest number of times it occurs in any of the given numbers; this product will be their least common multiple.*

1. What is the least common multiple of 4, 6, 8, and 12?

2. What is the least common multiple of 6, 8, 12, 16, and 24?

3. What is the least common multiple of 5, 15, 36, and 72? Ans. 360.

4. What is the least common multiple of 28, 35, 42, 70, and 77? Ans. 4620.

5. What is the least common multiple of 2, 3, 4, 5, 6, 7, 8, and 9? Ans. 2520.

6. What is the least common multiple of 23, 46, 92, and 374? Ans. 17204.

7. What is the least common multiple of 5, 7, 11, 13, 17, and 19? Ans. 1616615.

Art. 76. We may also find the least common multiple of two or more numbers by the following

RULE. *Write the numbers in a horizontal line; divide them by the least prime number that will measure two or more of them; write the quotients and undivided numbers in a second horizontal line; divide the numbers in this second line in the same manner, and thus continue dividing until the quotients and undivided numbers are prime to each other. Find the product of the several divisors and numbers in the last horizontal line, this product will be the least common multiple.*

1. What is the least common multiple of 28, 35, 42, 70, and 77?

OPERATION.

$$\begin{array}{r|l} 2 & 28,\ 35,\ 42,\ 70,\ 77 \\ \hline 5 & 14,\ 35,\ 21,\ 35,\ 77 \\ \hline 7 & 14,\ \ 7,\ 21,\ \ 7,\ 77 \\ \hline & \ \ 2,\ \ 1,\ \ 3,\ \ 1,\ 11 \end{array}$$

It may be observed that the several divisors and numbers in the last horizontal line comprise all the different prime factors of the given numbers; hence the rule.

$2 \times 5 \times 7 \times 2 \times 3 \times 11 = 4620$, the least common multiple required.

FRACTIONS.

Art. 77. Fractions, or *fractional numbers*, are the expressions or measures of the parts of a unit, or integer.

There are two kinds of fractions, viz., *vulgar* fractions, and *decimal* fractions.

VULGAR FRACTIONS.

Art. 78. A *vulgar fraction* is any number of the equal parts of a unit, or integer.

When a unit, or any whole number, is divided into any number of equal parts, those parts are called fractions; and the number of parts into which it is divided always indicates the name of the parts, and their name is the *ordinal number* of the number of parts.

Thus, if any whole number is divided into two equal parts, one of the parts is called *one half;* if it is divided into three equal parts, one of the parts is called *one third*, two of them, *two thirds;* if it is divided into four equal parts, one of the parts is called *one fourth*, two of them, *two fourths*, three of them, *three fourths;* if it is divided into five equal parts, the parts are called *fifths;* if into ten equal parts, *tenths:* &c.

NOTATION AND NUMERATION OF VULGAR FRACTIONS.

Art. 79. Notation of vulgar fractions is writing or expressing them by figures, and Numeration is the reading of those figures.

A vulgar fraction is expressed by two numbers written one above the other, with a short line between them; thus, $\frac{2}{3}$ $\frac{\text{two}}{\text{thirds}}$; $\frac{3}{4}$ $\frac{\text{three}}{\text{fourths}}$; $\frac{4}{5}$ $\frac{\text{four}}{\text{fifths}}$.

The number below the line is called the *denominator*, because it indicates the name of the number of equal parts into which the unit, or integer, is supposed to be divided.

The number above the line is called the *numerator*, because it indicates the number of equal parts expressed by the fraction.

The numerator and denominator are called the *terms* of the fraction.

Suppose we wish to write the fraction fifteen twenty-fifths. We first write the numerator, or number of twenty-fifths, *over a short* line; we then write the denominator, or *cardinal* number of twenty-fifths, under it; thus, $\frac{14}{25}$. Hence the following rule for writing vulgar fractions.

RULE. *Write the numerator, or number of parts, over a short line, then write the denominator, or cardinal number of the name of the parts into which the unit or integer is supposed to be divided, under it.*

1. What figures will express each of the fractions two fifths, three sevenths, five eighths, and seven ninths?	2. Write the fractions twelve nineteenths, seventeen twenty-fourths, nineteen thirty-fifths, and forty-seven sixtieths.

RULE FOR READING VULGAR FRACTIONS. *First read the numerator, or number of parts, then read the ordinal number of the denominator, or name of those parts.*

1. Read the fractions $\frac{17}{19}$, $\frac{12}{21}$, $\frac{41}{57}$, and $\frac{57}{90}$.	2. Read the fractions $\frac{101}{301}$, $\frac{416}{751}$, $\frac{711}{1000}$, and $\frac{5000}{10000}$.

ILLUSTRATION OF VULGAR FRACTIONS.

ART. 80. The origin of fractions has been shown in division; hence, any fraction may be viewed as an unexecuted operation in division, the numerator being the dividend, and the denominator the divisor.

Every fraction expresses a quotient, and it is the only way we have of expressing the quotient of a less number divided by a greater; thus, $3 \div 4 = \frac{3}{4}$.

Any whole number may be expressed in a fractional form by writing the number itself for a numerator, and 1 for a denominator; the whole number 5 is expressed in a fractional form thus, $\frac{5}{1}$.

There are *six* kinds of vulgar fractions, viz., *proper*, *improper*, *simple*, *compound*, *mixed*, and *complex*.

A *proper* fraction is a fraction whose numerator is less than its denominator; thus, $\frac{4}{5}$ is a proper fraction.

An *improper* fraction is a fraction whose numerator is equal to or greater than its denominator; thus, $\frac{5}{5}$ and $\frac{9}{7}$ are improper fractions.

A *simple* fraction is a single fraction, in which there is but one numerator and one denominator. A simple fraction may be either proper, or improper; thus, $\frac{3}{4}$ and $\frac{4}{3}$ are simple fractions.

A *compound* fraction is a fraction of a fraction, or it is several fractions connected by the word *of*; thus, $\frac{3}{4}$ of $\frac{4}{5}$, and $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{3}{8}$, are compound fractions.

A *mixed number* is a fractional number composed of a whole number and a fraction; thus, $15\frac{3}{4}$ is a mixed number.

A *complex* fraction is a fractional number which has a fraction either in its numerator, or in its denominator, or in both; thus, $\frac{\frac{3}{4}}{5}$, $\frac{5}{7\frac{1}{2}}$, and $\frac{\frac{3}{8}}{9\frac{5}{7}}$ are complex fractions.

REDUCTION OF VULGAR FRACTIONS.

Art. 81. Reduction of vulgar fractions is changing them from one form to another, to prepare them for the operations of addition, subtraction, multiplication, and division.

TO REDUCE A FRACTION TO ITS LOWEST TERMS.

Art. 82. When the numerator and denominator of a fraction are prime to each other, that is, when they have no common measure greater than a unit, the fraction is in its lowest terms.

As the numerator of every fraction is some proportional part of its denominator, and since the size or value of a fraction depends upon the proportional part which the numerator is of its denominator, it follows that dividing both terms of any fraction by the same number does not alter the size or value of the fraction; thus, if we divide both terms of the fraction $\frac{6}{12}$ by 6, we obtain the quotient $\frac{1}{2}$, 1 being the same proportional part of 2 that 6 is of 12. Hence the following rule for reducing a fraction to its lowest terms.

RULE. *Divide both terms of the fraction by their greatest common measure; the quotients will express the fraction in its lowest terms.*

1. Reduce $\frac{4}{6}$, $\frac{6}{8}$, $\frac{4}{10}$, $\frac{3}{12}$, $\frac{9}{15}$, $\frac{12}{16}$, $\frac{15}{18}$, and $\frac{16}{20}$, to their lowest terms.

2. Reduce $\frac{12}{15}$; $\frac{13}{26}$, $\frac{21}{28}$, $\frac{25}{30}$, $\frac{24}{32}$, $\frac{35}{49}$, $\frac{44}{55}$, and $\frac{45}{99}$, to their lowest terms.

3. Reduce $\frac{126}{210}$ to its lowest terms. Ans. $\frac{3}{5}$.
4. Reduce $\frac{176}{192}$ to its lowest terms. Ans. $\frac{11}{12}$.
5. Reduce $\frac{1296}{3024}$ to its lowest terms. Ans. $\frac{3}{7}$.
6. Reduce $\frac{1727}{2355}$ to its lowest terms. Ans. $\frac{11}{15}$.

TO CHANGE AN IMPROPER FRACTION TO A WHOLE OR MIXED NUMBER.

Art. 83. As the denominator of a fraction always expresses the number of parts which make a unit, there must be as many units in any improper fraction as the denominator is contained times in the numerator. Hence, the following rule for reducing an improper fraction to a whole or mixed number.

Rule. *Divide the numerator by the denominator; the quotient will be the whole or mixed number.*

1. Change $\frac{12}{3}$, $\frac{15}{4}$, $\frac{18}{5}$, $\frac{24}{4}$, $\frac{35}{5}$, $\frac{42}{6}$, and $\frac{49}{7}$, to whole or mixed numbers.

2. Change $\frac{56}{8}$, $\frac{63}{9}$, $\frac{72}{10}$, $\frac{84}{12}$, $\frac{98}{15}$, and $\frac{112}{16}$, to whole or mixed numbers.

3. Change $\frac{379}{15}$ to a whole or mixed number.
Ans. $25\frac{4}{15}$.

4. Change $\frac{49364}{39}$ to a whole or mixed number.
Ans. $1265\frac{29}{39}$.

5. Change $\frac{317925}{45}$ to a whole or mixed number.
Ans. 7065.

TO CHANGE A MIXED NUMBER TO AN IMPROPER FRACTION.

Art. 84. Since multiplying any whole number by the denominator of any fraction reduces the whole number to fractional parts of the same kind with those expressed by the denominator, we have the following rule.

Rule. *Multiply the whole number by the denominator of the fraction; add the numerator to the product, and write the sum over the denominator.*

1. Change $3\frac{1}{2}$, $4\frac{2}{3}$, $5\frac{3}{4}$, $6\frac{5}{6}$, $7\frac{6}{7}$, and $8\frac{7}{8}$, to improper fractions.

2. Change $9\frac{3}{5}$, $10\frac{4}{7}$, $11\frac{5}{8}$, $12\frac{6}{7}$, $15\frac{3}{4}$, and $20\frac{4}{5}$, to improper fractions.

3. Change $25\frac{4}{15}$ to an improper fraction. Ans. $\frac{379}{15}$.

4. Change $1265\frac{29}{39}$ to an improper fraction.
Ans. $\frac{49364}{39}$.

5. Change 7065 to an improper fraction whose denominaor shall be 45. Ans. $\frac{317925}{45}$.

TO REDUCE A COMPOUND FRACTION TO A SIMPLE FRACTION.

Art. 85. Let it be required to reduce the compound fraction $\frac{3}{4}$ of $\frac{5}{6}$ to a simple fraction. $\frac{1}{4}$ of $\frac{1}{6}$ is $\frac{1}{24}$, and $\frac{1}{4}$ of $\frac{5}{6}$ is $\frac{5}{24}$, and $\frac{3}{4}$ of $\frac{5}{6}$ is three times $\frac{5}{24}$, which is $\frac{15}{24}=\frac{5}{8}$.

If the above analysis and results be carefully examined, it will be seen that $\frac{1}{24}$ is obtained by multiplying $\frac{1}{6}$ by $\frac{1}{4}$, thus, $\frac{1}{6}\times\frac{1}{4}=\frac{1}{24}$. The second result is obtained by multiplying

$\frac{5}{6}$ by $\frac{1}{4}$, thus, $\frac{5}{6}\times\frac{1}{4}=\frac{5}{24}$. And the final result, $\frac{15}{24}$, is obtained by multiplying $\frac{5}{6}$ by $\frac{3}{4}$, thus, $\frac{5}{6}\times\frac{3}{4}=\frac{15}{24}=\frac{5}{8}$.

Hence we have the following rule for reducing a compound fraction to a simple fraction.

RULE. *Find the product of all the numerators for a new numerator, and of all the denominators for a new denominator; then reduce this new fraction to its lowest terms.*

If any part of the compound fraction be a whole or mixed number, it must be reduced to an improper fraction.

All equal numerators and denominators may be cancelled, that is crossed, and not used as factors; the simple fraction will thus be obtained in its lowest terms, and the operation very much shortened.

ILLUSTRATION. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ is reduced to a simple fraction thus, $\frac{2}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{4}}{5}=\frac{2}{5}$.

If any numerator and denominator have a common measure, divide them both by this common measure, and use the quotients as factors.

ILLUSTRATION. $\frac{4}{5}$ of $\frac{10}{21}$ of $\frac{7}{9}$ is reduced to a simple fraction

$$\text{thus, }\frac{4}{\underset{1}{\cancel{5}}}\times\frac{\overset{2}{\cancel{10}}}{\underset{3}{\cancel{21}}}\times\frac{\overset{1}{\cancel{7}}}{9}=\frac{8}{27}.$$

1. Reduce $\frac{1}{2}$ of $\frac{3}{5}$, $\frac{2}{3}$ of $\frac{4}{7}$, $\frac{1}{4}$ of $\frac{7}{8}$, and $\frac{3}{4}$ of $\frac{5}{11}$, to simple fractions.

2. Reduce $\frac{2}{3}$ of $\frac{3}{5}$, $\frac{3}{4}$ of $\frac{5}{8}$, $\frac{4}{5}$ of $\frac{7}{8}$, and $\frac{5}{6}$ of $\frac{11}{12}$, to simple fractions.

3. Reduce $\frac{5}{8}$ of $\frac{9}{7}$ of $\frac{10}{19}$ to a simple fraction.
Ans. $\frac{450}{1064}=\frac{225}{532}$.

4. Reduce $\frac{3}{12}$ of $\frac{13}{15}$ of $\frac{8}{17}$ of 20 to a simple fraction.
Ans. $\frac{6240}{3060}=\frac{104}{51}=2\frac{2}{51}$.

5. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{1}{4}$ of $12\frac{1}{2}$ to a simple fraction.
Ans. $\frac{75}{64}=1\frac{11}{64}$.

TO CHANGE ANY GIVEN FRACTION TO AN EQUIVALENT FRACTION WHICH SHALL HAVE ANY REQUIRED DENOMINATOR.

Art. 85. Let it be required to change the fraction $\frac{4}{5}$ to an equivalent fraction whose denominator shall be 8.

It is plain that there will be $\frac{8}{5}$ as many eighths as there are fifths, because eighths are only $\frac{5}{8}$ as large as fifths. 8 times $\frac{4}{5}$ are $\frac{32}{5}=\frac{6\frac{2}{5}}{8}$, the number of eighths required. Hence the *following rule.*

Rule. *Multiply the numerator of the given fraction by the required denominator; then divide the product by the denominator of the given fraction; the quotient will be the numerator of the required denominator.*

1. Change $\frac{1}{2}$ and $\frac{3}{4}$ to eighths.
2. Change $\frac{2}{3}$ and $\frac{5}{6}$ to twelfths.
3. Change $\frac{5}{6}$ and $\frac{7}{9}$ to eighteenths.
4. Change $\frac{5}{6}$ and $\frac{9}{12}$ to twenty-fourths.

5. Change $\frac{9}{11}$ to a fraction whose denominator is 45.

Ans. $\frac{36\frac{9}{11}}{45}$.

6. Change $\frac{5}{9}$ of a shilling to twelfths of a shilling.

Ans. $\frac{6\frac{6}{9}}{12}=\frac{6\frac{2}{3}}{12}$s.

7. Change $\frac{4}{7}$ of a pound to twentieths of a pound.

Ans. $\frac{£11\frac{3}{7}}{20}$.

TO REDUCE A COMPLEX FRACTION TO A SIMPLE FRACTION.

Art. 86. Let it be required to reduce $\frac{\frac{3}{4}}{\frac{5}{6}}$ to a simple fraction. If we multiply both terms of any fraction by the same number, the size or value of the fraction is not altered.

Multiplying both terms of the fraction in the numerator by the denominator of the other, thus, $\frac{3}{4}\times 6=\frac{18}{24}$, we find that $\frac{3}{4}=\frac{18}{24}$. Multiplying both terms of the fraction in the denominator by the denominator of the other, thus, $\frac{5}{6}\times 4=\frac{20}{24}$, we find that $\frac{5}{6}=\frac{20}{24}$. As the new fractions have the same denominator, it is plain that the numerators will express the required simple fraction; thus, $\frac{18}{20}=\frac{9}{10}$. Hence we have the following rule.

Rule. *If the numerator and denominator be whole or mixed numbers, reduce them to improper fractions.*

Then multiply the numerator of each fraction by the denominator of the other; the product will express the required simple fraction.

1. Reduce $\frac{3\frac{3}{4}}{5}$ to a simple fraction.
2. Reduce $\frac{6\frac{2}{5}}{8}$ to a simple fraction.

3. Reduce $\frac{6\frac{2}{3}}{12}$ to a simple fraction. Ans. $\frac{5}{9}$.

4. Reduce $\frac{13\frac{2}{3}}{20}$ to a simple fraction. Ans. $\frac{41}{60}$.

5. Reduce $\frac{26\frac{8}{11}}{45}$ to a simple fraction. Ans. $\frac{8}{11}$.

TO CHANGE FRACTIONS HAVING DIFFERENT DENOMINATORS TO EQUIVALENT FRACTIONS HAVING A COMMON DENOMINATOR.

Art. 87. Let it be required to change $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$, to equivalent fractions which shall have a common denominator.

If we multiply the numerator and denominator of each of these fractions by the same numbers, the value of each fraction will not be changed, because the numerator of each will still be the same proportional part of its denominator as before.

Multiplying both terms of the fraction $\frac{3}{4}$ by the denominators of the other two, we have $\frac{3\times5\times6}{4\times5\times6}=\frac{90}{120}$.

Multiplying both terms of the fraction $\frac{4}{5}$ by the denominators of the other two, we have $\frac{4\times4\times6}{5\times4\times6}=\frac{96}{120}$.

Multiplying both terms of the fraction $\frac{5}{6}$ by the denominators of the other two, we have $\frac{5\times4\times5}{6\times4\times5}=\frac{100}{120}$.

Therefore, the fractions $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$, changed to equivalent fractions, having a common denominator, are $\frac{90}{120}$, $\frac{96}{120}$, and $\frac{100}{120}$.

From the above illustration we derive the following rule.

RULE. *Reduce all compound and complex fractions to simple fractions, and each fraction to its lowest terms.*

Multiply each numerator by all the denominators except its own, for a new numerator; then find the product of all the denominators, for a common denominator, and write it under each new numerator.

1. Change $\frac{2}{3}$ and $\frac{4}{5}$ to equivalent fractions having a common denominator.

2. Change $\frac{4}{7}$ and $\frac{5}{9}$ to equivalent fractions having a common denominator.

3. Change $\frac{3}{5}$ of $\frac{5}{7}$, and $\frac{8}{13}$, to equivalent fractions which shall have a common denominator. Ans. $\frac{39}{91}$, and $\frac{56}{91}$.

4. Change $\frac{5}{11}$, $\frac{8}{15}$, and $\frac{4\frac{3}{5}}{12}$, to fractions of equal value which shall have a common denominator.

Ans. $\frac{4500}{9900}$, $\frac{5280}{9900}$, and $\frac{3795}{9900}$.

5. Change $\frac{8}{12}$, $\frac{9}{15}$, $\frac{15}{18}$, and $\frac{18}{24}$, to equivalent fractions which shall have a common denominator.

Ans. $\frac{240}{360}$, $\frac{216}{360}$, $\frac{300}{360}$, and $\frac{270}{360}$.

TO CHANGE FRACTIONS WHICH HAVE DIFFERENT DENOMINATORS TO FRACTIONS OF EQUAL VALUE WHICH SHALL HAVE THE LEAST COMMON DENOMINATOR.

Art. 88. It is obvious that the product of all the denominators of several fractions is a common multiple or denominator of all of them, and it is also obvious that their *least* common multiple must be their *least* common denominator. Hence, the following rule.

Rule. *Reduce all compound and complex fractions to simple ones, and each fraction to its lowest terms.*

Find the least common multiple of all the given denominators, which will be the least common denominator.

Then divide the least common denominator by each given denominator, and multiply each quotient by its given numerator; the several products will be the several new numerators.

1. Change $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$, to equivalent fractions having the least common denominator.

2. Change $\frac{2}{5}$, $\frac{7}{10}$, and $\frac{8}{15}$, to fractions of equal value having the least common denominator.

3. Change $\frac{4}{5}$, $\frac{5}{7}$, and $\frac{12}{35}$, to fractions of equal value having the least common denominator. Ans. $\frac{28}{35}$, $\frac{25}{35}$, and $\frac{12}{35}$.

4. Change $\frac{4}{5}$ of $\frac{5}{6}$, and $\dfrac{9\frac{2}{7}}{15}$, to equivalent fractions having the least common denominator. Ans. $\frac{14}{21}$ and $\frac{13}{21}$.

5. Change $\frac{9}{11}$, $\frac{3}{7}$ of $\frac{7}{9}$, and $\dfrac{8\frac{4}{7}}{45}$, to fractions of equal value which shall have the least common denominator.
Ans. $\frac{189}{231}$, $\frac{77}{231}$, and $\frac{44}{231}$.

TO REDUCE THE FRACTION OF ANY HIGHER DENOMINATION OF MONEY, WEIGHT OR MEASURE, TO THE FRACTION OF ANY LOWER DENOMINATION, OF EQUAL VALUE.

Art. 89. Let it be required to reduce $\frac{5}{6}$ of a dollar to the fraction of a cent.

A cent is 100 times less valuable than a dollar; hence it follows that the fraction of a cent, equal in value to any given fraction of a dollar, must be 100 times greater than the fraction of a dollar. As the size or value of a fraction depends upon the proportional part which the numerator is of the denominator, it follows that if we make the numerator 100 times greater, the fraction becomes 100 times greater; thus, $\$\frac{5}{6}\times100=\frac{500}{6}=\frac{250}{3}$ of a cent.

From the above illustration we deduce the following rule.

RULE. *Multiply the numerator of the given fraction by that number of the lower denomination which is equal to a unit of the higher, and write the product over its denominator.*

1. Reduce $\frac{4}{5}$ of a pound to the fraction of a shilling.

2. Reduce $\frac{3}{7}$ of a shilling to the fraction of a penny.

3. Reduce $\frac{7}{9}$ of a ton to the fraction of a pound.
Ans. $\frac{15680}{9}$ of a lb.

4. Reduce $\frac{11}{12}$ of an acre to the fraction of a rod.
Ans. $\frac{1760}{12}=\frac{440}{3}$ of a rod.

5. Reduce $\frac{3}{4}$ of an hour to the fraction of a minute.
Ans. $\frac{180}{4}=\frac{45}{1}$ of a min.

TO CHANGE THE FRACTION OF ANY LOWER DENOMINATION OF MONEY, WEIGHT OF MEASURE, TO THE FRACTION OF ANY HIGHER DENOMINATION, OF EQUAL VALUE.

Art. 90. Let it be required to change $\frac{250}{3}$ of a cent to the fraction of dollar.

A dollar being 100 times greater in value than a cent, the fraction of a dollar, equal in value to any given fraction of a cent, must be 100 times less than the fraction of a cent. As the size or value of a fraction depends upon the proportional part which the numerator is of the denominator, if we make the denominator 100 times greater, the fraction becomes 100 times less; thus, $\frac{250^{\text{ct.}}}{3\times100}=\frac{250}{300}=\frac{5}{6}$ of a dollar. Hence we derive the following

RULE. *Multiply the denominator of the given fraction by that number of the lower denomination which is equal to a unit of the higher, and write the product under the numerator.*

1. Change $\frac{80}{5}$ of a shilling to the fraction of a pound.

2. Change $\frac{36}{7}$ of a penny to the fraction of a shilling.

3. Change $\frac{15680}{9}$ of a pound to a fraction of a ton.
Ans. $\frac{7}{9}$ of a ton.

4. Change $\frac{440}{3}$ of a rod to the fraction of an acre.
Ans. $\frac{11}{12}$ of an acre.

5. Change $\frac{45}{1}$ of a minute to the fraction of an hour.
Ans. $\frac{3}{4}$ of an hour.

6. Change $\frac{5}{9}$ of a pound to the fraction of a cwt.
Ans. $\frac{5}{1008}$ of a cwt.

TO REDUCE ANY SIMPLE OR COMPOUND QUANTITY TO THE FRACTIONAL PART OF ANY OTHER SIMPLE OR COMPOUND QUANTITY OF A LIKE KIND.

Art. 91. Suppose we wish to express what part 7 inches is of 9 inches.

As every fraction expresses the part which its numerator is of its denominator, we have only to write the number which the question requires to be made the fractional part of the other for the numerator of a fraction, and the other for the denominator; the fraction thus formed will express the fractional part required. Then writing 7 inches for the numerator of a fraction, and 9 inches for the denominator, thus $\frac{7}{9}$, we find that 7 inches is 7 ninths of 9 inches.

Again; suppose we wish to reduce 13s. 4d. to the fractional part of £1.

13s. 4d. are equal to 160 pence. £1 is equal to 240 pence. Writing 160 pence for the numerator of a fraction, and 240 pence for the denominator, thus, $\frac{160}{240}=\frac{2}{3}$, we find that 13s. 4d. is 160 two hundred fortieths, or 2 thirds, of £1. From these illustrations we derive the following

RULE. *Reduce the given quantities to the lowest denomination in either of them; then write the number which the question requires to be made the fractional part of the other for the numerator of a fraction, and the other number for the denominator; the fraction thus formed will express the required part.*

1. Reduce 12 cents and 5 mills to the fraction of a dollar.
2. Reduce 4s. 6d. to the fraction of a dollar.
3. What fractional part of 1 yard is 1 foot 9 inches?
4. What fractional part of 5 acres is 3 acres 2 roods?

5. Reduce 7 oz. 4 pwts. to the fraction of a pound.
Ans. $\frac{3}{5}$ of a pound.

6. What fractional part of a cwt. are 3 qrs. 14 lbs.?
Ans. $\frac{7}{8}$ of a cwt.

7. Reduce 6 fur. 26 rds. 3 yds. 2 ft. to the fraction of a mile.
Ans. $\frac{5}{6}$ of a mile.

8. What fractional part of an acre are 1 rood and 30 rods?
Ans. $\frac{7}{16}$ of an acre.

9. Reduce 3 hhds. 31 gals. 2 qts. to the fraction of a ton.
Ans. $\frac{7}{8}$ of a ton.

10. What fractional part of a month are 3 w. 1 d. 9 h. 36 m.?
Ans. $\frac{4}{5}$ of a month.

11. What fractional part of 10 dollars is \$6.47?
Ans. $\frac{647}{1000}$ of 10 dollars.

12. What fractional part of £5. 16s. is 18s. 6d.?
Ans. $\frac{37}{232}$ of £5. 16s.

13. What fractional part of 15 acres is 24 acres 3 roods?
Ans. $\frac{33}{20}$ of 15 acres.

14. What fractional part of 12 cwt. is 7 cwt. 3 qrs. 14 lbs.?
Ans. $\frac{21}{32}$ of 12 cwt.

15. What fractional part of 25 m. 5 fur. is 17 m. 3 fur. 20 rods?
Ans. $\frac{279}{410}$ of 25 m. 5 fur.

TO REDUCE THE FRACTION OF ANY HIGHER DENOMINATION OF MONEY, WEIGHT, OR MEASURE, TO ITS VALUE IN WHOLE NUMBERS OF LOWER DENOMINATIONS.

Art. 92. Suppose we wish to find the value of $\frac{2}{3}$ of £1. in shillings and pence.

$\frac{2}{3}$ of a pound is equal to 20 times as great a fraction of a shilling, or $\frac{40}{3}$ of a shilling, and $\frac{40}{3}$ of a shilling is equal to $13\frac{1}{3}$ shillings.

Again, $\frac{1}{3}$ of a shilling is equal to 12 times as great a fraction of a penny, or $\frac{12}{3}$ of a penny, and $\frac{12}{3}$ of a penny is equal to 4 pence. We have thus found that $\frac{2}{3}$ of £1. is equal to 13s. 4d. Hence the following

Rule. *Reduce the given fraction to a fraction of the next lower denomination by multiplying its numerator by that number of the lower denomination which is equal to a unit of the higher, and write the product over the denominator; if this product be an improper fraction, reduce it to a whole or mixed number by dividing the numerator by the denominator. If the quotient be a mixed number, reduce the fractional part to a fraction of the next lower denomination, as before, and thus proceed through all the lower denominations; the several quotients will express the value of the given fraction in the lower denominations.*

1. Reduce $\frac{1}{8}$ of a dollar to its value in cents and mills.

3. What is the length of $\frac{7}{12}$ of a yard, in feet and inches?

2. Reduce $\frac{3}{4}$ of a dollar to its value in shillings and pence.

4. What number of roods and rods in $\frac{7}{8}$ of an acre?

5. Reduce $\frac{3}{5}$ of a lb. troy to its value in lower denominations.
Ans. 7 oz. 4 pwts.

6. In $\frac{7}{8}$ of a cwt., how many quarters and pounds?
Ans. 3 qrs. 14 lbs.

7. Reduce $\frac{5}{6}$ of a mile to its value in whole numbers of lower denominations. Ans. 6 fur. 26 rds. 3 yds. 2 ft.

8. In $\frac{7}{16}$ of an acre, how many roods and rods?
Ans. 1 rood 30 rods.

9. Reduce $\frac{7}{8}$ of a tun of wine to its value in whole numbers of lower denominations. Ans. 3 hhds. 31 gals. 2 qts.

10. In $\frac{4}{5}$ of a month, how many weeks, days, hours, and minutes? Ans. 3 w. 1 d. 9 h. 36 m.

11. What is the value of $\frac{7}{9}$ of a £. in shillings, pence, and farthings? Ans. 15s. 6d. $2\frac{2}{3}$qrs.

12. What is the value of $\frac{2}{3}$ of a lb. avoirdupois, in ounces and drams? Ans. 10 oz. $10\frac{2}{3}$ drs.

13. What is the value of $\frac{7}{9}$ of a cwt., in quarters, pounds, ounces, and drams? Ans. 3 qrs. 3 lbs. 1 oz. $12\frac{4}{9}$ drs.

14. What is the value of $\frac{4}{7}$ of a mile, in furlongs, rods, yards, feet, and inches?
Ans. 4 fur. 22 rds. 4 yds. 2 ft. $1\frac{5}{7}$ in.

ADDITION OF VULGAR FRACTIONS.

Art. 93. Addition of vulgar fractions is the method of finding the sum of two or more fractional numbers or quantities.

Suppose we wish to find the sum of $\frac{3}{12}$, $\frac{5}{12}$, and $\frac{7}{12}$, of a yard.

When fractions have a common denominator, and are of the same denomination, we can find the sum of them by adding the numerators and writing their sum over the common denominator; thus, $\frac{3}{12}+\frac{5}{12}+\frac{7}{12}=\frac{15}{12}=1\frac{1}{4}$ yards.

When fractions have not a common denominator, they must be changed to fractions having a common denominator before we can add them.

When fractions are of different denominations, they must either be reduced to the same denomination, or to their value in whole numbers of lower denominations, before they can be added.

Compound and complex fractions must be reduced to simple fractions. Mixed numbers may either be reduced to improper fractions, or the whole numbers and fractional parts may be added separately.

From the preceding remarks we derive the following

RULE. *Reduce compound and complex fractions to simple fractions, mixed numbers to improper fractions, fractions of*

different denominations to the same, and all of them to their lowest terms.

If the fractions have different denominators, change them to fractions having the least common denominator, then add the numerators and write their sum over the common denominator. If the result be an improper fraction, reduce it to a whole or mixed number.

1. What is the sum of $\frac{2}{3}$ and $\frac{3}{4}$?	2. What is the sum of $\frac{5}{6}$ and $\frac{5}{7}$?
3. What is the sum of $3\frac{1}{2}$ and $4\frac{2}{5}$?	4. What is the sum of $9\frac{5}{6}$ and $12\frac{6}{7}$?
5. What is the sum of $\frac{3}{4}$ of $\frac{4}{5}$ and $\frac{2}{5}$ of $\frac{5}{8}$?	6. What is the sum of $\frac{3}{4}$ of a pound and $\frac{5}{6}$ of a shilling?

7. What is the sum of $\frac{3}{4}$, $\frac{1}{2}$ of $\frac{2}{3}$, and $9\frac{1}{2}$? Ans. $10\frac{7}{12}$.

8. What is the sum of $\frac{3}{5}$, $\frac{1}{8}$, $\frac{1}{9}$, and $\frac{7}{11}$? Ans. $1\frac{1871}{3960}$.

9. What is the amount of $\frac{3}{7}$ of $\frac{11}{15}$ and $\frac{1}{9}$ of $4\frac{1}{2}$?
Ans. $\frac{57}{70}$.

10. What is the sum of $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{1}{4}$, $4\frac{1}{5}$, and $5\frac{1}{6}$?
Ans. $16\frac{9}{20}$.

11. What is the amount of $\frac{4\frac{1}{2}}{5}$, $\frac{5\frac{2}{3}}{6}$, and $\frac{8\frac{3}{4}}{9}$?
Ans. $2\frac{49}{60}$.

12. What is the sum of $\frac{1}{8}$ of a dollar, $\frac{2}{3}$ of a cent, and $\frac{7}{8}$ of a mill?
Ans. 13 cts. $2\frac{4}{5}$ mills.

13. What is the sum of $\frac{2}{5}$ of a pound and $\frac{4}{7}$ of a shilling?
Ans. 8s. 0d. $3\frac{3}{7}$ qrs.

14. What is the sum of $\frac{3}{4}$ of a mile and $\frac{7}{10}$ of a furlong?
Ans. 6 fur. 28 rods.

15. What is the amount of $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, $\frac{1}{2}$ of an hour, and $\frac{3}{4}$ of a minute?
Ans. 2 days 2 hours 30 min. 45 sec.

16. What is the sum of $\frac{5}{8}$, $\frac{1}{2}$ of $4\frac{1}{6}$, and $\frac{2}{22}$ of $6\frac{3}{7}$?
Ans. $3\frac{1853}{3080}$.

17. What is the amount of $\frac{1}{5}$ of $\frac{5}{8}$, $\frac{1}{2}$ of $5\frac{1}{2}$, $409\frac{3}{8}$, and $819\frac{3}{4}$? Ans. 1232.

18. What is the sum of $\frac{4}{7}$ of $18\frac{3}{11}$ and $\frac{11}{12}$ of $\frac{4}{9}$ of $16\frac{4}{13}$?
Ans. $17\frac{2309}{27027}$.

19. What is the sum of $\frac{4}{5}$ of a ton, $\frac{5}{8}$ of a cwt., $\frac{4}{7}$ of a qr., and $\frac{7}{8}$ of a lb.? Ans. 16 cwt. 3 qrs. 2 lbs. 14 oz.

20. A trader purchased 4 pieces of silk; the first measured $35\frac{3}{8}$ yds.; the second, $36\frac{1}{4}$ yds.; the third, $37\frac{1}{2}$ yds.; and the fourth, $38\frac{7}{8}$ yds. What number of yards did he *purchase?* Ans. 148 yards.

SUBTRACTION OF VULGAR FRACTIONS.

Art. 94. Subtraction of vulgar fractions is the method of finding the difference between any two fractional numbers or quantities.

When two fractions have a common denominator, and are of the same denomination, we can find their difference by taking the less numerator from the greater, and writing their difference over the common denominator; thus, the difference between $\frac{5}{8}$ of a yard and $\frac{3}{8}$ of a yard is $\frac{2}{8}$ or $\frac{1}{4}$ of a yard.

When the two fractions have not a common denominator, they must be reduced to a common denominator before subtraction can be performed.

When the two fractions are of different denominations, they must either be reduced to the same denomination, or to their value in whole numbers of lower denominations.

Compound and complex fractions must be reduced to simple fractions. Mixed numbers may be reduced to improper fractions, or the fractional parts and whole numbers may be subtracted separately. Hence the following

Rule. *Reduce compound and complex fractions to simple fractions, mixed numbers to improper fractions, fractions of different denominations to the same, and both of them to their lowest terms.*

If the two fractions have different denominators, change them to fractions having the least common denominator, then subtract the less numerator from the greater, and write the remainder over the common denominator.

1. What is the difference between $\frac{1}{3}$ and $\frac{1}{2}$?

2. What is the difference between $\frac{2}{3}$ and $\frac{3}{4}$?

3. What is the difference between $\frac{3}{4}$ and $\frac{7}{8}$?

4. What is the difference between $\frac{3}{5}$ and $\frac{5}{7}$?

5. What is the difference between $\frac{2}{3}$ of $\frac{3}{4}$ and $\frac{3}{4}$ of $\frac{4}{5}$?

6. What is the difference between $\frac{3}{5}$ of $\frac{4}{5}$ and $\frac{5}{6}$ of $\frac{7}{8}$?

7. What is the difference between $12\frac{3}{4}$ yards and $15\frac{7}{8}$ yards?

8. What is the difference between $15\frac{4}{5}$ lbs. and $21\frac{5}{8}$ lbs.?

9. What is the difference between $4\frac{1}{2}$ and $1\frac{4}{5}$?
Ans. $2\frac{7}{10}$.

10. What is the difference between $\frac{1}{3}$ of $2\frac{1}{2}$ and $\frac{1}{6}$ of $1\frac{3}{5}$?
Ans. $\frac{17}{30}$.

11. What is the difference between $9\frac{3}{5}$ and $\frac{2}{5}$ of $6\frac{1}{4}$?
Ans. $7\frac{1}{10}$.

12. What is the difference between $45\frac{7}{12}$ and $36\frac{7}{8}$?
Ans. $8\frac{17}{24}$.

MULTIPLICATION OF VULGAR FRACTIONS.

TO MULTIPLY A FRACTION BY A WHOLE NUMBER.

Art. 95. To multiply a fraction by a whole number is to repeat the size or value of the fraction as many times as there are units in the multiplier or whole number.

The size or value of every fraction depends upon the proportion which its numerator bears to its denominator. The denominator of a fraction not only expresses the number of parts into which a unit or integer is supposed to be divided, but it also indicates the size of those parts. As the numerator indicates the number of parts expressed by the fraction, if the numerator be made any number of times greater, while the denominator remains the same, the fraction is made the same number of times greater.

If the denominator be made any number of times smaller, while the numerator remains the same, the fraction is made the same number of times greater.

Let it be required to multiply $\frac{4}{15}$ by 3. Multiplying the numerator, 4 fifteenths, by 3, the product is 12 fifteenths, which we write over the denominator, thus: $\frac{4\times3}{15}=\frac{12}{15}=\frac{4}{5}$. Dividing the denominator, 15 fifteenths, by 3, the quotient is 5 fifths, which we write under the numerator, thus: $\frac{4}{15\div3}=\frac{4}{5}$, as above.

From the above remarks and illustration we deduce the following rules for multiplying a fraction by a whole number.

RULE I. *Multiply the numerator by the whole number, and write the product over the denominator.*

RULE II. *Divide the denominator by the whole number, and write the quotient under the numerator.*

If the given fraction be compound or complex, reduce it to a simple fraction. Mixed numbers may be reduced to improper fractions, and then multiplied, or the whole number and fraction may be multiplied separately.

1. If $\frac{3}{4}$ of a yard of cloth will make 1 vest, how many yards will make 4 vests?

2. If 1 pound of tea be worth $\frac{2}{3}$ of a dollar, how many dollars are 6 pounds worth?

3. If 1 yard of silk be worth $\frac{5}{6}$ of a dollar, what are 8 yards worth?

4. Suppose $\frac{5}{8}$ of a yard of silk will make a bonnet, how much will make 9 bonnets?

5. What will be the product of $\frac{3}{4}$ of $\frac{4}{5}$ multiplied by 12?
Ans. $7\frac{1}{5}$.

6. What will be the product of $\frac{37}{64}$ multiplied by 32?
Ans. $18\frac{1}{2}$.

7. What will be the product of $125\frac{17}{25}$ multiplied by 25?
Ans. 3142.

TO MULTIPLY A WHOLE NUMBER BY A FRACTION.

Art. 96. To multiply a whole number by a fraction is to find such a part or parts of the whole number, for a product, as the fraction indicates.

To multiply any whole number by $\frac{1}{2}$, we find one half of the number for the product; to multiply any whole number by $\frac{3}{4}$, we find three fourths of the number for the product.

Suppose we wish to multiply 24 by $\frac{5}{8}$. If we multiply 24 by the numerator, 5, the product is 120; if we divide 120 by the denominator, 8, the quotient is 15, which is 5 eighths of 24. Again, if we divide 24 by the denominator, 8, the quotient is 3; if we multiply 3 by the numerator, 5, the product is 15, which is 5 eighths of 24.

The reason why these two operations produce the same result is, that 1 eighth of 5 times any given number is equal to 5 times 1 eighth of the same number. Hence we derive the following rules for multiplying a whole number by a fraction.

Rule I. *Multiply the whole number by the numerator of the fraction, and divide the product by the denominator.*

Rule II. *Divide the whole number by the denominator of the fraction, and multiply the quotient by the numerator.*

1. If a quire of paper is worth 24 cents, what is $\frac{3}{4}$ of a quire worth?

3. If you multiply 50 by $\frac{3}{5}$, what will be the product?

5. If a yard of silk is worth 56 cents, what will be the value of $\frac{7}{8}$ of a yard?

7. If 63 be multiplied by $\frac{7}{9}$, what will be the product?

2. If linen is worth 40 cents a yard, what is $\frac{5}{8}$ of a yard worth?

4. If 60 be multiplied by $\frac{5}{6}$, what will be the product?

6. If an acre of land is worth 30 dollars, what is the value of $\frac{7}{10}$ of an acre?

8. What will be the product of 144 multiplied by $\frac{7}{12}$?

9. What will be the product if 1728 be multiplied by $\frac{17}{48}$? Ans. 612.

10. If a ton of hay is worth 16 dollars, what is $\frac{3}{4}$ of $\frac{4}{5}$ of a ton worth? Ans. $9\frac{3}{5}$ dollars.

11. What will be the product if 3456 be multiplied by $\frac{3}{4}$ of $\frac{4\frac{7}{8}}{5}$? Ans. $2527\frac{1}{5}$.

TO MULTIPLY A FRACTION BY A FRACTION.

Art. 97. To multiply a fraction by a fraction is to find such a part or parts of any given fraction as is indicated by the fractional multiplier.

Let it be required to multiply $\frac{4}{5}$ by $\frac{3}{4}$.

One fourth of $\frac{4}{5}$ is $\frac{4}{20}$, which is obtained by multiplying the denominator of the multiplicand by the denominator of the multiplier; thus, $\frac{4}{5 \times 4} = \frac{4}{20}$. Three fourths of $\frac{4}{5}$ is 3 times $\frac{4}{20}$, or $\frac{12}{20}$, which is obtained by multiplying the numerator of the multiplicand by the numerator of the multiplier; thus, $\frac{4 \times 3}{20} = \frac{12}{20} = \frac{3}{5}$.

From the preceding illustration we derive the following rule for multiplying a fraction by a fraction.

Rule. *Reduce compound and complex fractions to simple fractions, and mixed numbers to improper fractions.*

Multiply the numerator of the multiplicand by the numerator of the multiplier for the numerator of the product, and the denominator of the multiplicand by the denominator of the multiplier for the denominator of the product.

1. What is $\frac{1}{3}$ of $\frac{1}{2}$ of a dollar?

3. What is the product of $\frac{1}{2}$ multiplied by $\frac{1}{4}$?

5. What is $\frac{2}{3}$ of $\frac{3}{4}$ of an orange?

7. What is the product of $\frac{3}{5}$ multiplied by $\frac{5}{6}$?

9. A merchant bought $\frac{3}{4}$ of a ship, and sold $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of what he purchased; what part of the whole ship did he sell?

2. What is $\frac{1}{4}$ of $\frac{2}{3}$ of a dollar?

4. What is the product of $\frac{1}{2}$ multiplied by $\frac{3}{4}$?

6. What is $\frac{3}{4}$ of $\frac{4}{5}$ of a melon?

8. What is the product of $\frac{4}{7}$ multiplied by $\frac{3}{5}$?

10. A manufacturer owned $\frac{7}{8}$ of a flour mill, and has sold $\frac{3}{4}$ of what he owned; what part of the whole mill did he sell?

11. If a yard of silk is worth $\frac{7}{8}$ of a dollar, what is $\frac{4}{5}$ of a yard worth? Ans. $\frac{7}{10}$ of a dollar.

12. If corn is worth $\frac{5}{8}$ of a dollar a bushel, what will $\frac{3}{4}$ of a bushel cost? Ans. $\frac{15}{32}$ of a dollar.

13. When sugar is worth $8\frac{3}{4}$ dollars a cwt., what is the value of $\frac{1}{2}$ of $\frac{3}{4}$ of a cwt.? Ans. $3\frac{9}{32}$ dollars.

14. If a yard of broadcloth is worth $3\frac{7}{8}$ dollars, what will $15\frac{3}{4}$ yards cost? Ans. $61\frac{1}{32}$ dollars.

DIVISION OF VULGAR FRACTIONS.

TO DIVIDE A FRACTION BY A WHOLE NUMBER.

Art. 98. To divide a fraction by a whole number is to find such a part of the fraction as is indicated by the whole number, or it is to find what part of one time the fraction will contain the whole number.

The operations in dividing a fraction by a whole number must be the reverse of those in multiplying a fraction by a whole number; hence it follows that as many times as the numerator is made smaller, while the denominator remains the same, so many times is the fraction made smaller; also, that as many times as the denominator is made greater, while the numerator remains the same, so many times is the fraction made smaller, because the number of parts in the denominator having been made any number of times greater, each of the parts is the same number of times smaller.

Let it be required to divide the fraction $\frac{3}{5}$ by the whole number 3; that is, to find one third of $\frac{3}{5}$, or to find what part of 1 time the fraction $\frac{3}{5}$ will contain the whole number 3.

One third of $\frac{3}{5}$ is $\frac{1}{5}$, which is obtained by dividing the numerator of the fraction $\frac{3}{5}$ by the whole number 3; thus, $\frac{3\div3}{5}=\frac{1}{5}$. The same result may be obtained by multiplying the denominator of the fraction $\frac{3}{5}$ by the whole number 3; thus, $\frac{3}{5\times3}=\frac{3}{15}=\frac{1}{5}$.

From the above illustration we derive the following rules for dividing a fraction by a whole number.

Rule I. *Divide the numerator by the whole number, and write the quotient over the denominator.*

Rule II. *Multiply the denominator by the whole number, and write the product under the numerator.*

1. If $\frac{6}{7}$ of a bushel of apples be equally divided among 3 boys, what part of a bushel will each boy receive?

2. If $\frac{8}{9}$ a dollar be equally divided among 4 girls, what part of a dollar will each of them receive?

3. If $\frac{15}{17}$ be divided by 5, what will be the quotient?

4. If $\frac{24}{37}$ be divided by 6, what will be the quotient?

5. If $\frac{7}{8}$ of a yard of silk be cut into 7 pieces of equal length, what will be the length of each piece?

6. If $\frac{8}{9}$ of a dollar be equally divided among 8 boys, what part of a dollar will each of them receive?

7. What part of 1 time is 9 contained in $\frac{5}{6}$?

8. What part of 1 time is 11 contained in $\frac{3}{4}$?

9. If you have $\frac{3}{5}$ of a ton of hay, and distribute it equally among 75 horses, what part of a ton will each horse receive? Ans. $\frac{1}{125}$ of a ton.

10. If $\frac{11}{15}$ be divided by 25, what will be the quotient? Ans. $\frac{11}{375}$.

11. If $\frac{15}{16}$ of a cwt. of sugar be equally divided among 64 soldiers, what part of a cwt. will each receive? Ans. $\frac{15}{1024}$ of a cwt.

12. What will be the quotient if the fraction $\frac{17}{19}$ be divided by 19? Ans. $\frac{17}{361}$.

13. What part of 1 time will the fraction $\frac{7}{12}$ contain the whole number 144? Ans. $\frac{7}{1728}$.

14. If $\frac{3}{4}$ of $\frac{4}{5}$ be divided by 12, what will be the quotient? Ans. $\frac{1}{20}$.

15. If 5 yards of cloth be worth $\frac{5}{6}$ of $\frac{7}{8}$ of a dollar, what is the value of 1 yard? Ans. $\frac{7}{48}$ of a dollar.

16. If 15 yards of silk be worth $12\frac{7}{8}$ dollars, what is the value of 1 yard? Ans. $\frac{103}{120}$ of a dollar.

TO DIVIDE A WHOLE NUMBER BY A FRACTION.

Art. 99. To divide a whole number by a fraction is to find the number of times, or part of time, a fraction is contained in a whole number.

It is obvious that the operations in dividing a whole number by a fraction must be the reverse of those in multiplying a whole number by a fraction.

Let it be required to divide the whole number 15 by the fraction $\frac{5}{8}$.

The fraction $\frac{5}{8}$ being only 5 eighths of a unit, it is plain that $\frac{5}{8}$ is contained $\frac{8}{5}$ as many times in any whole number as there are units in that number. If we multiply the whole number 15 by the denominator 8, the product will be eighths; if we divide this product by the numerator, 5 eighths, the quotient will be the number of times that $\frac{5}{8}$ is contained in 15; thus, $15 \times 8 = 120$, and $120 \div 5 = 24$.

Again, if we divide 15 by the numerator 5, and then multiply the quotient by the denominator 8, the product will be the number of times that $\frac{5}{8}$ is contained in 15; thus, $15 \div 5 = 3$, and $3 \times 8 = 24$.

From these illustrations we deduce the following rules for dividing a whole number by a fraction.

Rule I. *Multiply the whole number by the denominator of the fraction, and divide the product by the numerator.*

Rule II. *Divide the whole number by the numerator of the fraction, and multiply the quotient by the denominator.*

1. If a pair of shoes be worth $\frac{3}{4}$ of a dollar, how many pair can you buy with 6 dollars?

2. If a yard of linen be worth $\frac{5}{8}$ of a dollar, how many yards can you buy with 5 dollars?

3. How many times is $\frac{3}{5}$ contained in 6?

4. How many times is $\frac{5}{8}$ contained in 10?

5. When apples are worth $\frac{7}{8}$ of a dollar a bushel, how many bushels can you purchase with 7 dollars?

6. If $\frac{5}{7}$ of a barrel of flour last a family 1 month, how long will 5 barrels last the same family?

7. How many times is $\frac{7}{10}$ contained in 7?

8. How many times is $\frac{11}{12}$ contained in 11?

9. If $\frac{3}{4}$ of a cord of wood be worth 6 dollars, what is the value of a whole cord? Ans. 8 dollars.

10. How many times is $\frac{7}{15}$ contained in 25? Ans. $53\frac{4}{7}$ times.

11. If $\frac{4}{5}$ of an acre of land be worth 80 dollars, what is the value of an acre? Ans. 100 dollars.

12. If 75 be $\frac{5}{12}$ of some number, what is the whole of that number? Ans. 180.

13. If $\frac{17}{45}$ of a cotton manufactory be worth 8500 dollars, what is the whole of it worth? Ans. 22500 dollars.

14. How many times is $16\frac{1}{2}$, or $\frac{33}{2}$, contained in 132? Ans. 8 times.

15. If $\frac{3}{5}$ of $\frac{7}{8}$ of a ship is worth 2100 dollars, what is the value of the whole ship? Ans. 4000 dollars.

TO DIVIDE A FRACTION BY A FRACTION.

Art. 100. Dividing a fraction by a fraction is the method of finding what part of 1 time, or what number of times, one fraction is contained in another.

As the operations in dividing one fraction by another are the reverse of those in multiplying one fraction by another, we may divide the numerator of the dividend by the numerator of the divisor for the numerator of the quotient, and the denominator of the dividend by the denominator of the divisor for the denominator of the quotient; thus, $\frac{12}{25} \div \frac{3}{5} = \frac{4}{5}$.

One fraction may be divided by another by dividing the numerator of the dividend by the fractional divisor, and writing the quotient over the denominator of the dividend; thus, $\frac{12 \div \frac{3}{5}}{25} = \frac{20}{25} = \frac{4}{5}$.

Again, one fraction may be divided by another by multiplying the denominator of the dividend by the fractional divisor, and writing the product under the numerator of the dividend; thus, $\frac{12}{25 \times \frac{3}{5}} = \frac{12}{15} = \frac{4}{5}$.

As the result of each of the above methods of operation will often be a complex fraction, the following methods of operation will be more convenient.

Let it be required to divide $\frac{7}{8}$ by $\frac{3}{8}$. As these fractions have a common denominator, it is plain that $\frac{3}{8}$ is contained in $\frac{7}{8}$ as many times as 3, the numerator of the divisor, is contained in 7, the numerator of the dividend, which is $2\frac{1}{3}$ times.

Again, let it be required to divide $\frac{2}{3}$ by $\frac{3}{4}$. $\frac{2}{3} = \frac{8}{12}$, and $\frac{3}{4} = \frac{9}{12}$. Then, dividing 8, the numerator of the dividend, by 9, the numerator of the divisor; thus, $\frac{8}{9}$; we find that $\frac{3}{4}$ is contained in $\frac{2}{3}$ $\frac{8}{9}$ of 1 time.

Hence it is plain, that one fraction can be divided by another, when they have a common denominator, by dividing the numerator of the dividend by the numerator of the divisor.

When the two fractions have not a common denominator, and as the common denominator is not used in the operation, it is only necessary to find the new numerators; these new numerators will express the quotient.

From the above illustration we derive the following rule for dividing one fraction by another.

RULE. *Reduce compound and complex fractions to simple fractions, mixed numbers to improper fractions, fractions of different denominations to the same. If the two fractions have the same denominator, divide the numerator of the dividend by the numerator of the divisor.*

If the two fractions have different denominators, multiply the numerator of the dividend by the denominator of the divisor, the product will be the numerator of the quotient; then multiply the numerator of the divisor by the denominator of the dividend, the product will be the denominator of the quotient.

1. How many pounds of raisins, at $\frac{1}{8}$ of a dollar a pound, can you buy with $\frac{7}{8}$ of a dollar?

2. How many pounds of tea, at $\frac{3}{4}$ of a dollar a pound, can you purchase with $5\frac{1}{4}$ dollars?

3. How many times is $\frac{1}{3}$ contained in $\frac{5}{6}$?

5. How many yards of cloth, at $\frac{3}{5}$ of a dollar a yard, can be purchased with $4\frac{4}{5}$ dollars?

7. How many times is $1\frac{3}{5}$ contained in $9\frac{3}{5}$?

9. When figs are worth $\frac{1}{8}$ of a dollar a pound, how many pounds can you purchase with $\frac{3}{4}$ of a dollar?

11. How many times is $\frac{4}{7}$ contained in $\frac{7}{8}$?

4. How many times is $\frac{2}{5}$ contained in $\frac{5}{7}$?

6. How many boxes of oranges, at $3\frac{3}{8}$ dollars a box, can be purchased with $12\frac{7}{8}$ dollars?

8. How many times is $\frac{7}{8}$ contained in $9\frac{1}{8}$?

10. If a bushel of potatoes cost $\frac{3}{5}$ of a dollar, how many bushels can you buy with $\frac{7}{8}$ of a dollar?

12. How many times is $\frac{3}{8}$ contained in $\frac{5}{6}$?

13. At $3\frac{3}{8}$ dollars a yard, how many yards of broadcloth can be purchased with $25\frac{3}{4}$ dollars? Ans. $7\frac{17}{27}$ yards.

14. If $\frac{3}{4}$ of a yard of silk cost $\frac{7}{8}$ of a dollar, what will 1 yard cost? Ans. $1\frac{1}{6}$ dollars.

15. A man purchased $\frac{2}{7}$ of an acre of land for $24\frac{4}{5}$ dollars; what is the value of an acre at the same rate?

Ans. $86\frac{4}{5}$ dollars.

16. A baker purchased $15\frac{1}{2}$ barrels of flour for $85\frac{1}{4}$ dollars; how much did the flour cost him a barrel?

Ans. $5\frac{1}{2}$ dollars.

17. If $\frac{3}{4}$ of $\frac{7}{8}$ of a yard of linen cost $\frac{3}{5}$ of $\frac{7}{9}$ of a dollar, what will be the value of 1 yard? Ans. $\frac{32}{45}$ of a dollar.

18. How many tons of coal, at $5\frac{3}{8}$ dollars a ton, can be purchased with $125\frac{3}{4}$ dollars? Ans. $23\frac{17}{43}$ tons.

19. How many barrels of apples can you purchase with $48\frac{4}{5}$ dollars, at $2\frac{1}{8}$ dollars a barrel? Ans. $22\frac{82}{85}$ barrels.

20. If $\frac{2}{3}$ of $\frac{3}{4}$ of a ship is worth $1225\frac{3}{4}$ dollars, what is the value of the whole of the ship? Ans. $2451\frac{1}{2}$ dollars.

21. How many times is $\frac{5}{6}$ of a rod contained in $\frac{7}{8}$ of a mile? Ans. 336 times.

22. How many bottles, containing $1\frac{1}{2}$ pints each, can be filled with $27\frac{3}{4}$ gallons of beer? Ans. 148 bottles.

23. If a yard of ribbon cost $\frac{7}{8}$ of a shilling, how many yards can you purchase with $\frac{5}{8}$ of a pound? Ans. $14\frac{2}{7}$ yards.

24. If the wheels of a railroad car are $11\frac{7}{12}$ feet in circumference, how many times will each of them turn round in running $42\frac{4}{5}$ miles? Ans. $19509\frac{57}{139}$ times.

PRACTICAL QUESTIONS IN VULGAR FRACTIONS.

Art. 101. 1. What is the sum of $\frac{7}{8}$ and $\frac{6}{15}$? What is their difference?

2. What is the sum of $\frac{4}{5}$ of $\frac{5}{6}$ and $\frac{2}{7}$ of $\frac{5}{8}$? What is their difference?

3. What is the sum of $17\frac{3}{8}$ and $12\frac{4}{5}$? What is their difference?

4. What is the sum of $\frac{5}{8}$ of a pound and $\frac{6}{7}$ of a shilling? What is their difference?

5. What is the amount of $\frac{3}{4}$ of an English ell and $\frac{4}{5}$ of a yard? What is their difference?

6. What is the amount of $15\frac{4}{5}$ acres and $3\frac{2}{3}$ roods? What is their difference?

7. What is the amount of $12\frac{3}{5}$ tons and $15\frac{4}{5}$ pounds? What is their difference?

8. What is the amount of $\frac{5}{6}$ of a day and $\frac{4}{5}$ of an hour? What is their difference?

9. What will be the product of $\frac{5}{6}$ of $\frac{7}{8}$ multiplied by $\frac{3}{4}$ of $\frac{4}{5}$?

10. What will be the quotient of $\frac{5}{6}$ of $\frac{7}{8}$ divided by $\frac{3}{4}$ of $\frac{4}{5}$?

11. What will be the product of $25\frac{7}{8}$ multiplied by $12\frac{3}{4}$?

12. What will be the quotient of $25\frac{7}{8}$ divided by $12\frac{3}{4}$?

13. If a yard of silk is worth $\frac{7}{8}$ of a dollar, what is $\frac{4}{5}$ of a yard worth?

14. If $\frac{4}{5}$ of a yard of silk is worth $\frac{7}{10}$ of a dollar, what is the value of a yard?

15. If a yard of broadcloth is worth $3\frac{7}{8}$ dollars, what are $3\frac{3}{4}$ yards of the same cloth worth?

16. If $3\frac{3}{4}$ yards of broadcloth are worth $14\frac{17}{32}$ dollars, what is the value of one yard of the same kind of cloth?

17. If $\frac{3}{4}$ of a cord of wood is worth $5\frac{3}{8}$ dollars, what is the value of a cord? What is the value of $12\frac{4}{5}$ cords?

18. If $31\frac{1}{2}$ gallons of molasses are worth $9\frac{3}{5}$ dollars, what is one gallon worth? What are $5\frac{3}{4}$ gallons worth?

19. When coal is worth $7\frac{1}{2}$ dollars a ton, what is $\frac{7}{8}$ of a ton worth? What is the value of $12\frac{4}{5}$ tons?

20. A merchant purchased $\frac{7}{16}$ of a ship; he has since sold $\frac{3}{4}$ of his share. What part of the whole ship did he sell? What part does he still own?

21. If $\frac{3}{4}$ of $\frac{7}{8}$ of an acre of land is worth $15\frac{4}{5}$ dollars, what is the value of an acre? What is the value of a farm containing $325\frac{3}{4}$ acres?

DECIMAL FRACTIONS.

Art. 102. A *decimal fraction* is any number of tenths, hundredths, thousandths, &c., of a unit or integer; hence, the denominator of a decimal fraction is a unit with so many ciphers annexed as there are figures in the numerator.

A decimal fraction is expressed by writing only the numerator, or number of parts, and then placing a point at its left to distinguish it from a whole number. If the figures of the numerator do not equal in number the ciphers of the denominator, they must be made equal by prefixing ciphers.

$\frac{5}{10}$ are written thus : .5 tenths.
$\frac{25}{100}$ are written " .25 hundredths.
$\frac{375}{1000}$ are written " .375 thousandths.
$\frac{5}{100}$ are written " .05 hundredths.
$\frac{5}{1000}$ are written " .005 thousandths.

It is a principle of whole numbers that they *increase* in a tenfold proportion from the place of units towards the left; so it is a principle of decimal fractions that they *decrease* in a tenfold proportion from the place of units towards the right; which is shown in the following table.

NOTATION AND NUMERATION TABLE.

Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten Thousandths.	Hundred Thousandths.	Millionths.	Ten Millionths.	Hundred Millionths.	Billionths.
5	5	5	5	5	5	5	5	5.	5	5	5	5	5	5	5	5	5
9th place	8th place	7th place	6th place	5th place	4th place	3rd place	2nd place	1st place	1st place	2nd place	3rd place	4th place	5th place	6th place	7th place	8th place	9th place
Whole numbers.									Decimal parts.								

The figures at the left of the decimal point, in the table, express whole numbers. The 5 in the 1st place at the right of the point expresses 5 tenths; 5 in the 2nd place, 5 hundredths; 5 in the 3rd place, 5 thousandths; 5 in the 4th place, 5 ten thousandths; 5 in the 5th place, 5 hundred thousandths; 5 in the 6th place, 5 millionths; 5 in the 7th place, 5 ten millionths; 5 in the 8th place, 5 hundred millionths; 5 in the 9th place, 5 billionths; and all of them taken together are read thus: five hundred fifty-five million, five hundred fifty-five thousand, five hundred fifty-five billionths.

NOTATION AND NUMERATION OF DECIMAL FRACTIONS.

Since decimal fractions decrease in a tenfold proportion from the left towards the right, they must increase in the same proportion from the right towards the left; hence it follows that they are subject to the same law of notation, and consequently to the same modes of operation, that whole numbers are.

NUMERATION OF DECIMAL FRACTIONS.

Art. 103. Numeration of decimal fractions is the method of reading them. It will be perceived that the name of the parts expressed by the numerator is the ordinal number of its denominator, and the denominator is always known to be a unit with as many ciphers annexed as there are figures in the numerator. Since the numerator expresses the number of parts, and the ordinal number of its denominator expresses the name of those parts, we have the following rule for reading decimal fractions.

RULE. *Read the numerator, or number of parts, as if it were a whole number, and then read the ordinal number of its denominator, or name of those parts.*

Pupils should be required to read the following decimal numbers.

.375
.406
.017
.008
.049

.1234
.2053
.0405
.0046
.0509

.23456
.04063
.00567
.50608
.06079

17.406504
40.064056
51.140864
75.000046
97.800009

NOTATION OF DECIMAL FRACTIONS.

Art. 104. Notation of decimal fractions is the method of writing or expressing them by their proper figures.

By examining the table, it will be perceived that a figure in the first place at the right of the decimal point expresses *tenths*, in the second place, hundredths, in the third place, thousandths, &c. Hence we have the following rule for writing decimal fractions.

RULE. *Write a figure expressing the number of tenths, hundredths, thousandths, &c., of the given decimal, each in its proper place, observing to write a cipher in the place of each of the parts omitted.*

Pupils should be required to write the following decimal numbers.

1. Five hundred and five thousandths.
2. Seven thousand five hundred and fifteen ten thousandths.
3. Twenty-five thousand and seventy-five hundred thousandths.
4. Forty-eight thousand and eighty-nine hundred thousandths.
5. One hundred twenty thousand two hundred and five millionths.
6. Six hundred seventy-six thousand and forty-five ten millionths.
7. Five hundred thousand and forty-five hundred millionths.
8. Nine hundred eighty-seven thousand and twenty-five billionths.
9. Eight hundred twenty thousand and twenty-five ten billionths.
10. Seven hundred and fifteen thousand and forty-five trillionths.

Art. 105. Annexing a cipher to a decimal fraction reduces it to the next lower order or denomination, but does not alter its size or value, since every significant figure continues to occupy the same place; thus, .5, .50, and .500, are all of the same size or value, each being equal to $\frac{5}{10}$ or $\frac{1}{2}$ of a unit.

Prefixing a cipher to a decimal fraction reduces it to the next lower order or denomination, and also diminishes its size or value in a tenfold proportion by removing the significant figures further from the decimal point; thus, .5, .05, and .005, differ in size or value, .5 being equal to $\frac{5}{10}$ or $\frac{1}{2}$, .05 being equal $\frac{5}{100}$ or $\frac{1}{20}$, and .005 being equal to $\frac{5}{1000}$ or $\frac{1}{200}$.

Two or more decimal fractions, each containing the same number of figures, have a common denominator; thus, .005, .050, and .500, have 1000 for a common denominator.

Two or more decimal fractions, each containing a different number of figures, may be changed to fractions having a common denominator, by annexing so many ciphers as are necessary to make the number of figures in each fraction the same; thus, .5, .05, and .500, may be changed to .500, .050, and .500, which have 1000 for a common denominator.

A mixed number, that is, a whole number and decimal annexed, is equal to an improper fraction whose numerator is all the figures of the mixed number, taken as a whole number, and whose denominator is that of the decimal part of the mixed number. Thus, 45.75 is equal to $\frac{4575}{100}$, which is obvious from the method of changing a mixed number to an improper fraction.

The denominations of federal money are purely decimal, dollars being units or whole numbers, dimes tenths of a dollar, cents hundredths of a dollar, and mills thousandths of a dollar; consequently, federal money and decimal fractions are subject to the same methods of operation.

ADDITION OF DECIMAL FRACTIONS.

Art. 106. Since decimal fractions increase in a tenfold proportion from right to left, like whole numbers, we have the following rule for addition of decimal fractions.

Rule. *Write the numbers to be added, whether mixed or pure decimals, placing whole numbers under whole numbers, tenths under tenths, hundredths under hundredths, &c.*

Find their sum as in addition of simple numbers; and, from the right, point off so many figures for decimals as are equal to the greatest number of decimal figures in any of the given numbers.

1. What is the sum of .5, .7, and .8?

2. What is the amount of .25, .75, and .95?

3. What is the sum of 5.5, 10.7, and 25.8?

4. What is the amount of 25.25 and 75.75?

5. What is the sum of 85.385, 848.25, .3085, 28.75, 3.4867, and .835? Ans. 967.0152.

6. What is the amount of 538.25, 375.025, 211.0025, and 34.75? Ans. 1159.0275.

7. What is the sum of .85745, .68379, .53048, and .00025? Ans. 2.07197.

8. What is the amount of 25.000007, 145.643, 175.89, and 17.00348? Ans. 363.536487.

9. What is the sum of $20.25, $12.5, $10.125, and $5.05?

10. What is the amount of $45.375, $75.125, $125.625, and $475.875?

SUBTRACTION OF DECIMAL FRACTIONS.

Art. 107. Since decimal fractions increase in a tenfold proportion from right to left, we have the following

RULE. *Write the less number under the greater, placing whole numbers under whole numbers, tenths under tenths, hundredths under hundredths, &c.*

Then proceed as in subtraction of simple numbers, and from the right of the remainder point off so many figures for decimals as are equal to the greatest number of decimal figures in either of the given numbers.

1. From .5 take .25.
2. From .75 take .5.
3. Subtract .5 from 10.
4. Subtract .75 from 25.
5. From 3.75 take 1.8975. Ans. 1.8525.
6. Subtract .3785 from 1.5. Ans. 1.1215.
7. From $236 take .125 of a dollar. Ans. $235.875.
8. From seventy-five take twenty-five millionths. Ans. 74.999975.
9. Subtract twenty thousandths from twenty thousand. Ans. 19999.980.
10. From one million take one millionth. Ans. 999999.999999.
11. Subtract $1.375 from 10 dollars and 10 cents.
12. From 100 dollars take 75 dollars and 25 cents.

MULTIPLICATION OF DECIMAL FRACTIONS.

Art. 108. When a whole number, or a fraction, is multiplied by a whole number, the product will be greater than the multiplicand in the same proportion that the multiplier is greater than a unit; thus, $5 \times 5 = 25$; here the product is 5 times as great as the multiplicand; and $.5 \times 5 = 25$ tenths, or 2.5; here, also, the product is 5 times as great as the multiplicand.

When a whole number, or a fraction, is multiplied by a fraction, the product will be less than the multiplicand in the same proportion that the multiplier is less than a unit; thus, $12 \times .5 = 60$ tenths, or 6 units; here the product is only .5 tenths or 1 half of the multiplicand; and $.5 \times .5 = .25$ hundredths; here, also, the product is only .5 tenths or 1 half of the multiplicand.

From the above illustrations it is plain, that when a whole number is multiplied by a decimal fraction, or when a decimal fraction is multiplied by a whole number, the product will be in the lowest denomination, or order of parts, named in the given decimal; thus, 175 × .005 thousandths = .875 thousandths.

It is also obvious, that when a decimal fraction is multiplied by a decimal fraction, the product will be in the denomination produced by multiplying the lowest denomination, or order of parts, in the multiplicand by the lowest in the multiplier; thus, .375 thousandths × .45 hundredths = .16875 hundred thousandths. Hence the following general rule for the multiplication of decimal fractions.

RULE. *Multiply as in whole numbers, then point off so many figures in the product for decimals, counting from the right, as there are decimal figures in both factors. If there are not as many figures in the product, prefix so many ciphers as are necessary to make up the required number.*

1. What is the product of .15 multiplied by 5?

2. What is the product of 50 multiplied by .5?

3. What is the product of .75 multiplied by 6?

4. What is the product of 12.5 multiplied by .12?

5. What is the value of 10 yards of cloth, at .125 of a dollar a yard?

6. What is the value of .7 of an acre of land, at 30 dollars an acre?

7. Multiply 200 by .75. Product, 150.
8. Multiply 5000.5 by .05. Product, 250.025.
9. Multiply 12.386 by .354. Product, 4.384644.
10. Multiply .3785 by .003. Product, .0011355.
11. What is the value of 12.5 tons of coal, at $6.75 a ton?
12. A merchant purchased a ship, for which he paid 12500 dollars; what is .625 of the ship worth?
13. A gentleman purchased a lot of land for 750 dollars; what is .875 of it worth?
14. If an insurance office charge .0125 of the value of a house for insuring it against fire during one year, what will be the expense of insuring a house valued at $4675?

Decimal fractions are multiplied by 10, 100, 1000, *&c., by removing the decimal point so many figures towards the right as there are ciphers in the multiplier.*

Thus, .475 × 10 = 4.75.
.475 × 100 = 47.5.
.475 × 1000 = 475.

DIVISION OF DECIMAL FRACTIONS.

Art. 109. When a whole number or a fraction is divided by a whole number, the quotient will be less than the dividend in the same proportion that the divisor is greater than a unit; thus, $25 \div 5 = 5$; here the divisor is 5 times as great as a unit, and the quotient is only 1 fifth as great as the dividend; and $.8 \div 4 = .2$; here the divisor is 4 times as great as a unit, and the quotient is only 1 fourth as great as the dividend.

When a whole number is divided by a fraction, the quotient will be greater than the dividend in the same proportion that the divisor is less than a unit; thus, $5 \div .5 = 10$; here the divisor is 5 tenths or 1 half of a unit, and the quotient is 2 times as great as the dividend.

When a greater fraction is divided by a less one, the quotient will be greater than the dividend in the same proportion that the divisor is less than a unit; thus, $.4 \div .2 = 2$; here the divisor is 2 tenths or 1 fifth of a unit, and the quotient is 5 times as great as the dividend.

When a less fraction is divided by a greater one, the quotient will be less than a unit in the same proportion that the divisor is greater than the dividend; thus, $.2 \div .4 = .5$; here the divisor is 2 times as great as the dividend, and the quotient is only 5 tenths or 1 half of a unit.

From the above remarks and illustrations it is plain, that when the divisor is a whole number, there must be as many decimal figures in the quotient as there are in the dividend.

It is obvious, that when the divisor only contains decimal figures, or when there are more decimal figures in the divisor than there are in the dividend, if we make the number of decimal figures in the dividend equal to the number of decimal figures in the divisor, by annexing decimal ciphers to the dividend, the quotient will be a whole number, except when the divisor is greater than the dividend.

It is also obvious, that when there are more decimal figures in the dividend than there are in the divisor, there will be as many decimal figures in the quotient as the number of decimal figures in the dividend exceeds the number of decimal figures in the divisor. Hence the following general rule for the division of decimal fractions.

RULE. *Divide as in whole numbers, then point off so many of the quotient figures for decimals, counting from the right, as the number of decimal figures in the dividend exceeds the* *number of decimal figures in the divisor. If there are not as many figures in the quotient, prefix so many ciphers as are necessary to make up the required number.*

When there are more decimal figures in the divisor than there are in the dividend, or when the divisor exceeds the dividend, annex so many decimal ciphers to the dividend as are necessary to obtain any required number of decimal figures in the quotient.

When there is a remainder, ciphers may be annexed to it, and the division continued until the required number of decimal figures is obtained.

1. What is the quotient of .12 divided by 6?
2. What is the quotient of 6 divided by .12?
3. What is the quotient of 1.2 divided by 8?
4. What is the quotient of 1.2 divided by .15?
5. How many times is .8 of a dollar contained in 16 dollars?
6. What part of 1 time is 16 dollars contained in .8 of a dollar?

7. Divide 22.1275 by 5.3. Quotient, 4.175.
8. Divide .11875 by 4.75. Quotient, .025.
9. Divide 1.87 by 3.5. Quotient, .53428+.

+. *This character shows that the quotient is not complete.*

10. Divide 14.4375 by 8.25. Quotient, 1.75.
11. Divide 8.38 by .666. Quotient, 12.58258+.
12. Divide .0011355 by .3785. Quotient, .003.
13. Divide 4.543944 by .354. Quotient, 12.836.
14. Divide .25 by 400. Quotient, .000625.
15. Divide 400 by .25. Quotient, 1600.
16. What will be the quotient of 150 divided by .75? Ans. 200.
17. What will be the quotient of .75 divided by 200? Ans. .00375.
18. What will be the quotient of 500 divided by .05? Ans. 10000.
19. What will be the quotient of .05 divided by 500? Ans. .0001.
20. When wood is worth $6.75 a cord, how many cords can you buy with 81 dollars?
21. If 1.875 yard of cloth is sufficient to make a coat, how many coats can be made of 37.5 yards?

When the divisor is 10, 100, 1000, *&c., division is performed by removing the decimal point in the dividend so many figures towards the left as there are ciphers in the divisor.*

Thus, $4.75 \div 10 = .475.$

$47.5 \div 100 = .475$

$475 \div 1000 = .475.$

REDUCTION OF DECIMAL FRACTIONS.

TO CHANGE A VULGAR FRACTION TO AN EQUIVALENT DECIMAL FRACTION.

Art. 110. Suppose we wish to change the vulgar fraction $\frac{7}{8}$ to an equivalent decimal fraction.

OPERATION.

```
8 ) 7.0 ( .875 Ans.
    64
    —
     60
     56
     —
      40
      40
```

We write the numerator 7, and place a point at the right, for a dividend, and the denominator 8 for a divisor. We annex a cipher to the dividend, which reduces it to tenths; we then divide the 70 tenths by 8, the quotient is 8 tenths, and 6 tenths remain. We annex a cipher to the remainder, which reduces it to hundredths; we then divide the 60 hundredths by 8, the quotient is 7 hundredths, and 4 hundredths remain. We annex a cipher to the remainder, which reduces it to thousandths; we then divide the 40 thousandths by 8, the quotient is 5 thousandths.

From the above illustration, we obtain the following general rule for changing a vulgar fraction to an equivalent decimal fraction.

RULE. *Annex a cipher to the numerator which reduces it to tenths, then divide the number of tenths by the denominator; if there is a remainder, annex a cipher which reduces it to hundredths, then divide the number of hundredths; and thus continue until there is no remainder, or until a sufficient number of quotient figures is obtained.*

There are some vulgar fractions which cannot be expressed exactly in decimals. Since we multiply each successive remainder by 10, and as 10 is composed of the prime factors 2 and 5, it follows, that whenever the denominator of a vulgar fraction is a prime number other than 2 or 5, or when it contains any prime factor except 2 and 5, the vulgar fraction cannot be accurately expressed in decimals.

1. What decimal is equal to $\frac{1}{2}$? | 2. What decimal is equal to $\frac{1}{4}$?
3. What decimal is equal to $\frac{3}{4}$? | 4. What decimal is equal to $\frac{1}{8}$?
5. What decimal is equal to $\frac{3}{8}$? | 6. What decimal is equal to $\frac{5}{8}$?
7. What decimal is equal to $\frac{1}{5}$? | 8. What decimal is equal to $\frac{7}{8}$?

9. Change $\frac{15}{16}$ to an equivalent decimal. Ans. .9375.
10. Change $\frac{1}{1250}$ to an equivalent decimal. Ans. .0008.
11. What decimal is equivalent to $\frac{11}{32}$? Ans. .34375.
12. What decimal is equivalent to $\frac{19}{128}$?
Ans. .1484375.
13. Change $\frac{41}{256}$ to its equivalent decimal.
Ans. .16015625.

14. Change $\frac{2}{3}$ to its approximate decimal.
Ans. .66666+.

15. Change $\frac{6}{11}$ to its approximate decimal.
Ans. .54545+.

TO CHANGE A DECIMAL FRACTION TO AN EQUIVALENT VULGAR FRACTION.

Art. 111. Since the denominator of a decimal fraction is a unit with so many ciphers annexed as there are figures in the given decimal, we have the following

RULE. *Change the decimal fraction to a vulgar fraction by writing its denominator under it, then reduce this vulgar fraction to its lowest terms.*

1. What vulgar fraction is equal to .5?

2. What vulgar fraction is equal to .25?

3. What vulgar fraction is equal to .75?

4. What vulgar fraction is equal to .125?

5. What vulgar fraction is equal to .375?

6. What vulgar fraction is equal to .625?

7. Change .9375 to an equivalent vulgar fraction.
Ans. $\frac{15}{16}$.

8. Change .0008 to an equivalent vulgar fraction.
Ans. $\frac{1}{1250}$.

9. Change .34375 to an equivalent vulgar fraction.
Ans. $\frac{11}{32}$.

10. Change .1484375 to an equivalent vulgar fraction.
Ans. $\frac{19}{128}$.

TO CHANGE THE LOWER DENOMINATIONS OF ANY COMPOUND NUMBER TO AN EQUIVALENT DECIMAL FRACTION OF ANY HIGHER DENOMINATION.

Art. 112. Suppose we wish to change 7s. 6d. to the decimal of a pound. Sixpence is $\frac{6}{12}$s., and $\frac{6}{12}$s., changed to a decimal, is equal to .5s. If we divide .5s. by 20, we shall change it to the decimal of a pound, because any fraction of a shilling is only 1 twentieth as great a fraction of a pound; thus, .5s. ÷ 20 = .025£. Seven shillings is $\frac{7}{20}$£., and $\frac{7}{20}$£., changed to a decimal, is equal to .35£.; and .35£. + .025£. = .375£., the decimal required. Instead of changing .5s. and $\frac{7}{20}$£. separately to decimals, we may annex the .5s. to the 7s.; thus, 7.5s.; and 7.5s. ÷ 20 = .375£., as above. Hence the following

RULE. *Write the given numbers under each other for dividends, proceeding in order from the lowest denomination to* *the highest. Then begin with the lowest denomination, and divide the number of each denomination, in succession, (annexing as many decimal ciphers as may be necessary,) by that number which is equal to a unit of the next higher, and write each quotient at the right of the next lower dividend; the last quotient will be the decimal required.*

Change 15s. 9d. 3qrs. to an equivalent decimal of a pound.

OPERATION.

4) 3.00
12) 9.7500
20) 15.812500
Ans. .790625£.

Three farthings are ¾d.; dividing 3.00qrs. by 4, changes ¾d. to the decimal .75d., which we place at the right of the 9d.; dividing 9.75d. by 12, changes 9.75d. to the decimal of a shilling, which we place at the right of the 15s.; dividing 15.8125s. by 20, changes 15.8125s. to the decimal of a pound.

A compound number may be changed to the decimal of a higher denomination, by reducing the given compound number to a vulgar fraction, (Art. 91,) *and then reducing the vulgar fraction to a decimal.* (Art. 110.) In the above question, 15s. 9d. 3qrs. = 759qrs. 1£. = 960qrs.; then 15s. 9d. 3qrs. is equal to $\frac{759}{960}$£., and $\frac{759}{960}$£. = .790625£.

1. Change 1d. 2qrs. to the decimal of a shilling.

2. Change 2s. 6d. to the decimal of a pound.

3. Change 4s. 6d. to the decimal of a dollar.

4. Change 2qrs. 2n. to the decimal of a yard.

5. Change 12s. 6d. 3qrs. to the decimal of a pound.
Ans. .628125£.

6. Change 6d. 1qr. to the decimal of a shilling.
Ans. .52083+s.

7. Change 10 oz. 13 pwts. 9 grs. to the decimal of a pound.
Ans. .8890625 lb.

8. Change 7 cwt. 3 qrs. 17 lbs. 10 oz. 12 drs. to the decimal of a ton.
Ans. .39538+ ton.

9. Change 3 pks. 1 qt. 1 pt. to the decimal of a bushel.
Ans. .796875 bushel.

10. Change 5 fur. 12 rods to the decimal of a mile.
Ans. .6625 mile.

11. Change 2 roods 16 rods to the decimal of an acre.
Ans. .6 acre.

12. Change 13 hours 30 minutes to the decimal of a day.
Ans. .5625 day.

13. Change 3 quarters 2 nails to the decimal of a yard.
Ans. .875 yard.

14. Change 15 gallons 3 quarts to the decimal of a barrel.
Ans. .5 barrel.

TO CHANGE A DECIMAL FRACTION OF ANY HIGHER DENOMINATION TO ITS EQUIVALENT VALUE EXPRESSED IN WHOLE NUMBERS OF LOWER DENOMINATIONS.

Art. 113. Suppose we wish to change .790625£. to its equivalent value in shillings, pence, and farthings.

OPERATION.
.790625
20
s. 15.812500
12
d. 9.750000
4
qrs. 3.000000

Multiplying the given decimal of a pound by 20, the product must be the numerator of a decimal fraction 20 times as great, or the fraction of a shilling equal in value to the given fraction of a pound; this product being an improper fraction, we divide it by the denominator 1000000, by pointing off six figures, counting from the right; the figures at the left of the point express the quotient, or number of shillings, and the figures at the right of the point express the remainder, or decimal of a shilling. Multiplying this decimal of a shilling by 12, the product is the decimal of a penny of equal value; pointing off six figures as before, the figure at the left of the point expresses the quotient, or number of pence, and the figures at the right express the remainder, or decimal of a penny. Multiplying this decimal of a penny by 4, the product is the decimal of a farthing of equal value; pointing off six figures, the figure at the left of the point expresses the quotient, or number of farthings. Therefore, .790625£. = 15s. 9d. 3qrs.

From the above example and its illustration we obtain the following

RULE. *Multiply the given decimal by that number of the next lower denomination which is equal to a unit of the higher, and, from the right of the product, point off so many figures for decimals as there are figures in the given decimal.*

Multiply the decimal of each lower denomination in the same manner, and the several numbers at the left of the decimal points will express the equivalent value of the given decimal in the proper denominations.

1. Change .125 of a shilling to its value in pence and farthings.

2. Change .125 of a pound to its value in shillings and pence.

3. Change .75 of a dollar to its value in shillings and pence.

4. Change .625 of a yard to its value in quarters and nails.

5. Change .628125 of a pound to its equivalent value in shillings, pence, and farthings. Ans. 12s. 6d. 3qrs.

6. Change .8890625 of a troy pound to its equivalent value in ounces, pennyweights, and grains.
Ans. 10 oz. 13 pwts. 9 grs.

7. Change .796875 of a bushel to its equivalent value in pecks, quarts, and pints. Ans. 3 pks. 1 qt. 1 pt.

8. Change .6625 of a mile to its equivalent value in furlongs and rods. Ans. 5 fur. 12 rods.

9. Change .6 of an acre to its equivalent value in roods and rods. Ans. 2 roods 16 rods.

10. Change .5625 of a day to its equivalent value in hours and minutes. Ans. 13 hours 30 minutes.

TO CHANGE ANY NUMBER OF SHILLINGS, PENCE, AND FARTHINGS, TO THE DECIMAL OF A POUND, THE DECIMAL TERMINATING AT THOUSANDTHS.

Art. 114. Suppose we wish to change 17s. 10d. 2qrs. to the decimal of a pound.

OPERATION.

.8 = half of 16s.
.05 = the odd shilling.
.044 = the number of farthings.

.894 in 10½d. increased by 2.

As shillings are twentieths of a pound, half their number must be tenths of a pound; consequently, we write a figure expressing half the even number in the place of tenths. One shilling is $\frac{1}{20}$£. = .05£.; therefore, when there is an odd shilling, we write 5 in the place of hundredths. Again, farthings are 960ths of a pound, and if there were 1000 instead of 960 farthings in a pound, it is plain that any number of farthings would be so many thousandths of a pound. But 960 plus $\frac{1}{24}$ of 960 or 40 = 1000; consequently, any number of farthings, plus $\frac{1}{24}$ of the number, will be so many thousandths of a pound. Whenever the number of farthings is more than 12, $\frac{1}{24}$ of the number is more than $\frac{1}{2}$ of a farthing; therefore 1 must be added to the number; and when the number of farthings is more than 36, $\frac{1}{24}$ of the number is more than 1½ farthings; therefore, 2 must be added to the number; and the figures expressing the number thus increased we write in the second and third places. Hence we have the following

RULE. *Write half the greatest even number of shillings in the place of tenths, and 5 in the place of hundredths, when the number of shillings is odd.*

Write the number of farthings in the given pence and farthings in the second and third places; increasing their number by 1 when it exceeds 12, and by 2 when it exceeds 36.

When there are no shillings, or only one shilling, in the given sum, a cipher must be written in the place of tenths.

When the number of farthings in the given pence and farthings does not exceed 9, a cipher must be written in the second place, or place of hundredths.

1. Change 4s. 6d. 1qr. to the decimal of a pound.

2. Change 7s. 7d. 2qrs. to the decimal of a pound.

3. Change 1s. 1d. 3qrs. to the decimal of a pound.

4. Change 9s. 9d. 2qrs. to the decimal of a pound.

5. Change 5s. 5d. 1qr.; 6s. 6d. 2qrs.; 7s. 7d. 3qrs.; 8s. 8d. 1qr.; 9s. 9d. 2qrs., and 10s. 10d. 3qrs., each to the decimal of a pound.

Ans. .272; .327; .382; .434; .490, and .545.

6. Change 1d. 1qr.; .1d. 3qrs.; 2d. 1qr.; .2d. 3qrs.; 5d. 2qrs.; 1s. 1d.; 1s. 6d., and 1s. 11d., each to the decimal of a pound.

Ans. .005; .007; .009; .011; .023; .054; .075; .096.

TO CHANGE ANY DECIMAL OF A POUND, TERMINATING AT THOUSANDTHS, TO ITS VALUE IN SHILLINGS, PENCE AND FARTHINGS.

Art 115. Suppose we wish to change .894 of a pound to its value in shillings, pence and farthings.

OPERATION.
£.894
£.8 × 2 = 16s.
£.05 = 1s.
£.044 — .002 = 42 qrs. = 10½d.
Ans. 17s. : 10½.d.

It is evident that the operation in this Art. must be the reverse of the operation in Art. 114. As 1 tenth of a pound is equal to 2s., there must be twice as many shillings as there are tenths of a pound, and .8 × 2 = 16s. — 5 hundredths of a pound being equal to 1s. — we deduct 5 hundredths from the 9 hundredths, and add its equal, 1s., to 16s., and there remain 44 thousandths of a pound; and 44 thousandths, less 2 thousandths, equal 42 farthings, or 10½ pence. Hence we obtain the following

RULE. *Write twice the number of tenths of a pound for shillings; and if the number of hundredths is* 5, *or more than* 5, *add another shilling; then deduct the* 5 *hundredths, and call the remaining number of thousandths so many farthings; deducting* 1 *when the number exceeds* 12, *and* 2 *when it exceeds* 36.

1. Change .226 of a pound to its value in shillings, pence and farthings.

2. Change .381 of a pound to its value in shillings, pence and farthings.

3. Change .057 of a pound to its equivalent value.

4. Change .490 of a pound to its equivalent value.

5. Change .542 of a pound to its equivalent value.

6. Change .999 of a pound to its equivalent value.

7. Change .272; .327; .382; .434; .490, and .545, of a pound to their equivalent value in shillings, pence and farthings.

Ans. 5s. 5d. 1qr.; 6s. 6d. 2qrs.; 7s. 7d. 3qrs.; 8s. 8d. 1qr.; 9s. 9d. 2qrs., and 10s. 10d. 3qrs.

REPEATING OR CIRCULATING DECIMALS.

Art. 116. We have seen, (Art. 110,) that a vulgar fraction whose denominator is any prime number other than 2 or 5, or when it contains any prime factor except 2 and 5, cannot be exactly expressed in decimals.

Thus, if we reduce $\frac{1}{9}$ to a decimal we shall obtain the repeating figure $.\dot{1}$, which we distinguish by placing a point over it. Since the repeating decimal figure $.\dot{1}$ is equal to the vulgar fraction $\frac{1}{9}$, $.\dot{2}$ must equal $\frac{2}{9}$, $.\dot{3}$ must equal $\frac{3}{9}$, &c.; therefore, every single repeating decimal figure must be equal to a vulgar fraction whose numerator is the repeating figure, and whose denominator is 9.

If we reduce $\frac{1}{99}$ to a decimal, we shall obtain the repeating figures $.\dot{0}\dot{1}$, which we distinguish by placing a point over each figure. Since the repeating decimal figures $.\dot{0}\dot{1}$ are equal to the vulgar fraction $\frac{1}{99}$, $.\dot{0}\dot{2}$ must be equal to $\frac{2}{99}$, $.\dot{0}\dot{3}$ must be equal to $\frac{3}{99}$, &c.

Again, if we reduce $\frac{1}{999}$ to a decimal, we shall obtain the repeating figures $.\dot{0}0\dot{1}$, which we distinguish by placing a point over the first and last figure. Since the repeating decimal figures $.\dot{0}0\dot{1}$ are equal to $\frac{1}{999}$, $.\dot{0}0\dot{2}$ must be equal to $\frac{2}{999}$, $.\dot{0}0\dot{3}$ must be equal to $\frac{3}{999}$, &c. This correspondence exists universally; therefore, any repeating or circulating decimal is equal to a vulgar fraction whose numerator is the repeating figure or figures, and whose denominator is a number of 9s equal to the number of repeating figures.

TO CHANGE ANY REPEATING OR CIRCULATING DECIMAL TO ITS EQUIVALENT VULGAR FRACTION.

RULE. *Make the repeating figure or figures the numerator, and a number of 9s equal to the number of repeating figures the denominator; this fraction, reduced to its lowest terms, will be the equivalent vulgar fraction required.*

1. Change $.\dot{4}$ to its equivalent vulgar fraction.

2. Change $.\dot{6}$ to its equivalent vulgar fraction.

3. Change $.\dot{7}$ to its equivalent vulgar fraction.

4. Change $.\dot{8}$ to its equivalent vulgar fraction.

5. Change $.\dot{3}\dot{6}$ to an equivalent vulgar fraction. Ans. $\frac{4}{11}$.

6. What is the equivalent vulgar fraction of $.\dot{8}\dot{1}$? Ans. $\frac{9}{11}$.

TO CHANGE A REPEATING OR CIRCULATING DECIMAL TO AN EQUIVALENT VULGAR FRACTION WHICH DOES NOT BEGIN TO REPEAT AT THE FIRST PLACE.

Art. 117. If we change $\frac{8}{15}$ to a decimal, we shall obtain $.5\dot{3}$. This decimal begins to repeat at the second place, or hundredths. The first figure, 5, is $\frac{5}{10}$, and the repeating figure, $\dot{3}$, is $\frac{3}{9}$ of $\frac{1}{10}$ or $\frac{3}{90}=\frac{1}{30}$. $\frac{5}{10}=\frac{15}{30}+\frac{1}{30}=\frac{16}{30}$, or $\frac{8}{15}$, the original vulgar fraction.

Again, if we change $\frac{261}{550}$ to a decimal, we shall obtain $.47\dot{4}\dot{5}$. This decimal begins to repeat at the third place or thousandths. The first two figures are $\frac{47}{100}$, and the two repeating figures $\frac{45}{99}$ of $\frac{1}{100}$, or $\frac{45}{9900}=\frac{5}{1100}$. $\frac{47}{100}=\frac{517}{1100}$, and $\frac{517}{1100}+\frac{5}{1100}=\frac{522}{1100}$, or $\frac{261}{550}$, the original vulgar fraction.

From the above illustrations we obtain the following

RULE. *Change the finite part, or figures which do not repeat, to a vulgar fraction by writing its decimal denominator; then change the repeating figure or figures to a vulgar fraction, (Art. 116,) which will be a fraction of the finite part; reduce this compound fraction to a simple one; then reduce the two fractions to their least common denominator and find their sum; this fraction, reduced to its lowest terms, will be the equivalent vulgar fraction required.*

1. Change $.4\dot{6}$ to its equivalent vulgar fraction. Ans. $\frac{7}{15}$.

2. Change $.3\dot{8}$ to an equivalent vulgar fraction, and prove the result to be correct.

3. Change $.37\dot{4}$ to an equivalent vulgar fraction.
Ans. $\frac{337}{900}$.

4. Change $.463\dot{5}$ to an equivalent vulgar fraction, and prove the answer to be right.

5. Change $.47\dot{6}4\dot{7}$ to an equivalent vulgar fraction, and then change the result to a circulating decimal.

6. Change $.5\dot{9}2\dot{5}$ to an equivalent vulgar fraction, and then change the vulgar fraction to a decimal.

7. If 1 pound sterling of English currency be equal to $\$4.\dot{4}$, what is the value of 500 pounds sterling?

8. If 1 yard of linen is worth $\$.\dot{6}$, what is the value of 12 yards?

9. If 1 yard of silk is worth $\$.8\dot{3}$, what is the value of 20 yards?

EXCHANGE OF CURRENCIES.

Art. 118. Exchange of Currencies is the process of changing the denominations of the money of one country to the denominations of the money of another country.

We have already shown (Art. 35) that federal money is the currency of the United States. The currency of a country includes its gold, silver, and copper coins, also those coins of other countries which are in circulation, together with such bank notes as are redeemable in specie.

Coins are pieces of metal composed of gold, silver, and copper, each piece having some device or legend stamped upon it with a die.

TABLE OF FOREIGN COINS WHOSE VALUES ARE FIXED BY LAW.

Pound Sterling of Great Britain,	$4.84
Pound of the British Provinces of Nova Scotia, New Brunswick, Newfoundland and Canada,	4.00
Pagoda of India,	1.84
Tale of China,	1.48
Milrea of Portugal,	1.12
Specie Dollar of Sweden and Norway,	1.06
Specie Dollar of Denmark,	1.05
Rix Dollar of Denmark,	1.00
Milrea of Madeira,	1.00
Milrea of the Azores,	.83⅓
Ducat of Naples,	.80
Rix Dollar of Bremen,	.78¾
Rouble, Silver, of Russia,	.75
Thaler of Prussia and Northern States of Germany,	.69
Florin of Austria and the city of Augsburg,	.48½
Rupee of British India,	.44½
Rupee, Company,	.44½
Florin of the Southern States of Germany,	.40
Florin of the Netherlands,	.40
Guilder of the Netherlands,	.40
Marc Banco of Hamburg,	.35
Livre Tournois of France,	.18½
Franc of France,	.18⅗
Franc of Belgium,	.18⅗
Livre of Sardinia,	.18⅗
Lira of the Lombardo-Venetian Kingdom,	.16
Lira of Tuscany,	.16
Real Plate of Spain,	.10
Real Vellon of Spain,	.05

Any given number of either of the above coins may be changed to federal money, by multiplying the value of one by the given number.

Art. 119. The legal value of the pound sterling of Great Britain, which is represented by the gold sovereign, was \$4.44$\frac{4}{9}$ previous to the change in our standard. By an act of Congress, passed January, 1837, the eagle is to be 900 thousandths fine, and to weigh 258 grains; the half and quarter in proportion. The dollar is to be of the same fineness, and to weigh 412$\frac{1}{2}$ grains, the parts in proportion. The relative value, therefore, of silver to gold, is 15.9884 to 1. This change raised the legal value of the pound to \$4.84.

1. Suppose that a merchant of Boston receives an invoice of merchandise from Liverpool, amounting to £560. 17s. 10d. sterling. What is the amount of the invoice in federal money, allowing the pound to be equal to \$4.44$\frac{4}{9}$? What is the amount, allowing the pound to be equal to \$4.84?

As 1 dollar is equal to 4s. 6d. sterling, and 4s. 6d. sterling is equal to $\frac{9}{40}$ of 1 pound sterling, there must be $\frac{40}{9}$ as many dollars as pounds.

Hence, the following rule for changing sterling money to federal money, allowing the pound to be equal to \$4.44$\frac{4}{9}$.

RULE. *Change the shillings, pence, and farthings, if any, to the decimal of a pound,* (*Art.* 114,) *and annex it to the pounds. Then divide the given number of pounds and decimals of a pound by* $\frac{9}{40}$; *the quotient will be dollars and decimals of a dollar.*

£560. 17s. 10d. = £560.892, and £560.892 ÷ $\frac{9}{40}$ = \$2492.853$\frac{1}{3}$, Ans.

And \$4.84 × £560.892 = \$2714.71728, Ans.

2. Change 1000 pounds sterling to federal money, allowing the pound to be equal to \$4.44$\frac{4}{9}$.

3. Change £1250. 6s. 8d. sterling to federal money, allowing the pound to be equal to \$4.84.

4. Change 975 francs of France to federal money. 1 franc = \$.186, and \$.186 × 975 = \$181.35, Ans.

5. Change 5000 francs to federal money.

6. Change £75 Canada currency to federal money. \$4 × £75 = \$300, Ans.

7. Change £125. 15s. Canada currency to federal money.

8. Change 1750 tales of China to federal money.

9. Change 4650 specie dollars of Sweden to federal money.

Art. 130. Previous to the adoption of federal money in the United States, all accounts were kept in pounds, shillings, pence, and farthings.

A Table, showing the number of shillings at which a dollar is valued in each State; also, the value of a dollar expressed in the fractional part of a pound, with the legal rate of interest in each of the States.

	States.	Number of shillings in a dollar.	Value of a dollar in the fraction of a pound.	Legal rate of interest.
1	Maine	6s.	$\$1 = £\frac{72}{240} = £\frac{3}{10}$	6 pr. ct.
2	N. Hampshire	6s.	$\$1 = £\frac{72}{240} = £\frac{3}{10}$	6 pr. ct.
3	Vermont	6s.	$\$1 = £\frac{72}{240} = £\frac{3}{10}$	6 pr. ct.
4	Massachusetts	6s.	$\$1 = £\frac{72}{240} = £\frac{3}{10}$	6 pr. ct.
5	Rhode Island	6s.	$\$1 = £\frac{72}{240} = £\frac{3}{10}$	6 pr. ct.
6	Connecticut	6s.	$\$1 = £\frac{72}{240} = £\frac{3}{10}$	6 pr. ct.
7	New York	8s.	$\$1 = £\frac{96}{240} = £\frac{2}{5}$	7 pr. ct.
8	Ohio	8s.	$\$1 = £\frac{96}{240} = £\frac{2}{5}$	6 pr. ct.
9	New Jersey	7s. 6d.	$\$1 = £\frac{90}{240} = £\frac{3}{8}$	6 pr. ct.
10	Pennsylvania	7s. 6d.	$\$1 = £\frac{90}{240} = £\frac{3}{8}$	6 pr. ct.
11	Delaware	7s. 6d.	$\$1 = £\frac{90}{240} = £\frac{3}{8}$	6 pr. ct.
12	Maryland	7s. 6d.	$\$1 = £\frac{90}{240} = £\frac{3}{8}$	6 pr. ct.
13	Michigan	8s.	$\$1 = £\frac{96}{240} = £\frac{2}{5}$	7 pr. ct.
14	Indiana	6s.	$\$1 = £\frac{72}{240} = £\frac{3}{10}$	6 pr. ct.
15	Illinois	6s.	$\$1 = £\frac{72}{240} = £\frac{3}{10}$	6 pr. ct.
16	Missouri	6s.	$\$1 = £\frac{72}{240} = £\frac{3}{10}$	6 pr. ct.
17	Virginia	6s.	$\$1 = £\frac{72}{240} = £\frac{3}{10}$	6 pr. ct.
18	Kentucky	6s.	$\$1 = £\frac{72}{240} = £\frac{3}{10}$	6 pr. ct.
19	Tennessee	6s.	$\$1 = £\frac{72}{240} = £\frac{3}{10}$	6 pr. ct.
20	N. Carolina	10s.	$\$1 = £\frac{120}{240} = £\frac{1}{2}$	6 pr. ct.
21	S. Carolina	4s. 8d.	$\$1 = £\frac{56}{240} = £\frac{7}{30}$	7 pr. ct.
22	Georgia	4s. 8d.	$\$1 = £\frac{56}{240} = £\frac{7}{30}$	7 pr. ct.
23	Alabama	Federal Money.		8 pr. ct.
24	Mississippi	6s.	$\$1 = £\frac{72}{240} = £\frac{3}{10}$	6 pr. ct.
25	Louisiana	Federal Money.		6 pr. ct.
26	Arkansas	Federal Money.		6 pr. ct.
27	Florida	Federal Money.		6 pr. ct.
28	Texas	6s.	$\$1 = £\frac{72}{240} = £\frac{3}{10}$	8 pr. ct.
29	Nova Scotia and Canada	5s.	$\$1 = £\frac{60}{240} = £\frac{1}{4}$	6 pr. ct.

Notwithstanding that all accounts are usually kept in federal money, still, many traders name the prices of articles of merchandise in the old currency of the State in which they reside; therefore, we shall give some general rules for changing the different currencies to federal money.

Art. 191. In order to change any of the old currencies to federal money, the shillings, pence, and farthings, if there be any, must first be changed to the decimal of a pound, (Art. 114,) and annexed to the pounds.

TO CHANGE THE OLD CURRENCY OF NEW ENGLAND, AND OTHER STATES, IN WHICH THE DOLLAR IS EQUAL TO 6 SHILLINGS, TO FEDERAL MONEY.

As 6 shillings is equal to $\frac{3}{10}$ of a pound, there must be $\frac{10}{3}$ as many dollars and decimals of a dollar as there are pounds and decimals of a pound. Hence the following rule.

Rule. *Divide the given number of pounds and decimals of a pound by* $\frac{3}{10}$; *the quotient will be dollars and decimals of a dollar.*

1. Change £75. 15s. 9½d., New England currency, to federal money.

£75. 15s. 9½d. = £75.79, and £75.79 ÷ $\frac{3}{10}$ = $252.633⅓, Ans.

2. Change 7s. 6d., New England currency, to federal money.

3. Change £100. 10s. 7½d., Virginia currency, to federal money.

4. Change 1s., 2s., 3s., 4s., 5s., 6s., 7s., 8s., and 9s., New England currency, to federal money.

5. Change 10s. 2¼d., 11s. 3½d., 12s. 4¾d., 17s. 11¼d., and 19s. 11¾d., New England currency, to federal money.

TO CHANGE THE OLD CURRENCY OF NEW YORK, AND OTHER STATES, IN WHICH THE DOLLAR IS EQUAL TO 8 SHILLINGS, TO FEDERAL MONEY.

As 8 shillings is equal to $\frac{4}{10}$ of a pound, there must be $\frac{10}{4}$ as many dollars as there are pounds.

Rule. *Divide the given number of pounds and decimals of a pound by* $\frac{4}{10}$; *the quotient will be dollars and decimals of a dollar.*

6. Change £500, New York currency, to federal money.

7. Change £1275. 17s. 6d., Ohio and Michigan currency, to federal money.

8. Change 1s., 2s., 3s., 4s., 5s., 6s., and 7s., New York currency, to federal money.

8. Change £175. 15s., New York currency, to federal money.

TO CHANGE THE OLD CURRENCY OF NEW JERSEY, AND OTHER STATES, IN WHICH THE DOLLAR IS EQUAL TO 7s. 6d., TO FEDERAL MONEY.

As 7s. 6d. is equal to $\frac{3}{8}$ of a pound, there must be $\frac{8}{3}$ as many dollars as there are pounds.

RULE. *Divide the given number of pounds and decimals of a pound by* $\frac{3}{8}$; *the quotient will be dollars and decimals of a dollar.*

9. Change £350, New Jersey currency, to federal money.
10. Change £475. 11s. 10½d., Pennsylvania currency, to federal money.
11. Change 1s., 2s., 3s., 4s., 5s., 6s., and 7s., Pennsylvania currency, to federal money.
12. Change 8s. 6d., 9s. 6d., 10s. 6d., 11s. 6d., 12s. 6d., 13s. 6d., 14s. 6d., and 15s. 6d., New Jersey currency, to federal money.
13. Change £575. 17s. 6d., Pennsylvania currency, to federal money.

TO CHANGE THE OLD CURRENCY OF SOUTH CAROLINA AND GEORGIA, IN WHICH THE DOLLAR IS EQUAL TO 4s. 8d., TO FEDERAL MONEY.

As 4s. 8d. is equal to $\frac{7}{30}$ of a pound, there must be $\frac{30}{7}$ as many dollars as there are pounds.

RULE. *Divide the given number of pounds and decimals of a pound by* $\frac{7}{30}$; *the quotient will be dollars and decimals of a dollar.*

14. Change £1000, South Carolina currency, to federal money.
15. Change £25. 6s. 8d., Georgia currency, to federal money.
16. Change £525. 17s. 9d., South Carolina currency, to federal money.
17. Change £750. 10s. 6d., Georgia currency, to federal money.
18. Change 1s., 2s., 3s., 4s., 5s., 6s., 7s., 8s., 9s., and 10s., South Carolina currency, to federal money.
19. Change 11s., 12s., 13s., 14s., 15s., 16s., 17s., 18s., and 19s., Georgia currency, to federal money.

DUODECIMALS.

Art. 123. Duodecimals are fractions of a square or cubic foot, and have their origin from the multiplication of one compound number by another, in finding the area of surfaces, and the cubical contents of solid bodies.

In measuring surfaces and solids, the dimensions are usually taken in feet and inches, feet being integers or whole numbers, and inches 12ths of a foot; hence it is plain that the multiplication of feet and inches by feet and inches is multiplying integers and 12ths by integers and 12ths. Then it follows, that, when feet in length are multiplied by feet in width, the product is square feet. When feet in length are multiplied by inches in width, or when inches in width are multiplied by feet in length, the product is 12ths of a square foot. When inches in length are multiplied by inches in width, the product is 144ths of a square foot or square inches.

It is also obvious, that, when square feet are multiplied by feet in length, the product is cubic feet. When square feet are multiplied by inches in length, or when 12ths of a square foot are multiplied by feet in length, the product is 12ths of a cubic foot. When 12ths of a square foot are multiplied by inches or 12ths of a foot in length, the product is 144ths of a cubic foot. When 144ths of a square foot or square inches, are multiplied by inches or 12ths of a foot in length, the product is 1728ths of a cubic foot or cubic inches.

Illustration. What number of cubic feet are there in a granite pillar 3 feet 9 inches in width, 2 feet 3 inches in thickness, and 12 feet 6 inches in length?

$$\begin{array}{r}
\text{ft. in.} \quad \text{ft.} \qquad\quad \\
3\ \ 9 = 3 + \frac{9}{12} \qquad\qquad \\
2\ \ 3 = 2 + \frac{3}{12} \qquad\qquad \\
\hline
\frac{11}{12} + \frac{3}{144} \\
7 + \frac{6}{12} \qquad\qquad \\
\hline
8 + \frac{5}{12} + \frac{3}{144} = \text{square feet in the base or end of the pillar.} \\
12 + \frac{6}{12} \qquad\qquad \\
\hline
4 + \frac{2}{12} + \frac{7}{144} + \frac{6}{1728} \\
101 + \frac{3}{12} + \frac{0}{144} \qquad\qquad \\
\hline
\text{Ans. } 105 + \frac{5}{12} + \frac{7}{144} + \frac{6}{1728} = 105\tfrac{15}{32} \text{ cubic feet.}
\end{array}$$

Since the fractions of a foot decrease uniformly in a twelvefold ratio, we may write the numerators only, if we distinguish each of them by some mark. 12ths of a foot are usually distinguished by an accent, thus, (′); 144ths, thus, (″); 1728ths, thus, (‴). 12ths of a foot are called *primes;* 144ths, *seconds;* and 1728ths, *thirds.*

Omitting the denominators, the operation will appear as follows.

$$\begin{array}{rrrrl}
 & & \text{ft.} & ' & \\
 & & 3 & 9 & \\
 & & 2 & 3' & \\ \cline{3-4}
 & & 11' & 3'' & \\
 & 7 & 6' & & \\ \cline{2-4}
 & 8 & 5' & 3'' & = \text{square feet in the base or end of the pillar.} \\
 & 12 & 6' & & \\ \cline{2-4}
 & 4 & 2' & 7'' & 6''' \\
101 & 3' & 0'' & & \\ \cline{1-5}
\end{array}$$

Ans. Ft. 105 5′ 7″ 6‴ = $105\frac{15}{32}$ cubic feet.

From the foregoing remarks and illustrations we derive this

Rule. *Write the several denominations of the multiplier under the corresponding denominations of the multiplicand. Multiply the several denominations of the multiplicand by each of the denominations in the multiplier, in succession, beginning with the lowest, and write the first term or lowest denomination of each partial product directly under its multiplier. Find the sum of the partial products as in addition of compound numbers, their sum will be the total product.*

1. What number of square feet are there in a board 12 feet in length, and 6 inches in width?

2. How many square feet are there in a floor 20 feet in length, and 15 feet 6 inches in width?

3. How many cubic feet are there in a box 5 feet in length, 4 feet in width, and 3 feet 6 inches in depth?

4. What number of cubic feet are there in a stick of timber 12 feet 9 inches in length, and 2 feet square?

5. What number of square feet are there in a floor 16 feet 6 inches long, and 12 feet 8 inches wide?

Ans. 209 square feet.

6. How many square feet are there in a board 17 feet 6 inches in length, and 1 foot 7 inches in width?

Ans. 27 sq. ft. 8′ 6″ = $27\frac{17}{24}$ sq. ft.

PROBLEMS.

Art. 128. A PROBLEM is a question proposed, in which two or more numbers or terms are given, to find one or more numbers or terms answering the conditions of the question.

PROBLEM I. THE SUM OF TWO NUMBERS AND ONE OF THEM BEING GIVEN, TO FIND THE OTHER.

The sum of two numbers is 20, and one of them is 12; what is the other?

If we take the given number 12 from the given sum 20, the number left must be the other; thus, 20 — 12 = 8, the other number. Hence, the following rule.

RULE. *Subtract the given number from the given sum, the remainder will be the number required.*

1. James gave 75 cents for an arithmetic and slate; the price of the arithmetic was 50 cents. What was the price of the slate?

2. A farmer paid 110 dollars for a yoke of oxen and a cow; the cow was worth 30 dollars. What was the value of the oxen?

3. Bell-metal is composed of copper and tin; the great bell at Moscow weighs 432000 pounds, and contains 345600 pounds of copper. What number of pounds of tin does it contain? Ans. 86400 lbs. of tin.

PROBLEM II. THE DIFFERENCE BETWEEN TWO NUMBERS AND THE SMALLER NUMBER BEING GIVEN, TO FIND THE GREATER NUMBER.

Suppose the difference between two numbers to be 25, and the smaller number to be 50; what is the greater number?

Adding the difference 25 to the smaller number 50, their sum is 75, the greater number. Hence, the following rule.

RULE. *Add the difference to the smaller number, their sum will be the greater number.*

1. Henry paid 25 cents for his breakfast, which was 15 cents less than he paid for his dinner. How many cents did he pay for his dinner?

2. William paid 35 cents more for his cap than he paid for his shoes; his shoes cost 90 cents. What number of cents did he pay for his cap?

3. Suppose the difference between two numbers to be 4750, and the smaller number to be 7250; what is the greater number? Ans. 12000.

PROBLEM III. THE DIFFERENCE BETWEEN TWO NUMBERS AND THE GREATER NUMBER BEING GIVEN, TO FIND THE SMALLER NUMBER.

Suppose the difference between two numbers to be 30, and the greater number to be 120; what is the smaller number?

If we take the difference 30 from the greater number 120, the remainder is 90, the smaller number. Hence, the following rule.

RULE. *Subtract the difference from the greater number, the remainder will be the smaller number.*

1. Sarah performs 350 questions in arithmetic each week; Caroline performs 75 questions less than Sarah. How many questions does Caroline perform each week?

2. Mary has studied 500 pages of history; Eliza has studied 125 pages less than Mary. What number of pages of history has Eliza studied?

3. Suppose the difference between two numbers to be 12650, and the greater number to be 24225; what is the smaller number? Ans. 11575.

PROBLEM IV. THE SUM AND DIFFERENCE OF TWO NUMBERS BEING GIVEN, TO FIND THE NUMBERS.

If, to the sum of two numbers, we add their difference, the amount will be twice the greater number. Take the numbers 60 and 40; their sum is 100, and their difference is 20, and $100+20=120$, which is twice 60, the greater number.

Again; if, from the sum of two numbers, we subtract their difference, the remainder will be twice the smaller number; thus, $100-20=80$, which is twice 40, the smaller number. Hence, the following rules.

RULE I. *Add the difference of the two numbers to their sum, divide the amount by 2, the quotient will be the greater number; then subtract the difference from the greater number, the remainder will be the smaller number.*

RULE II. *Subtract the difference of the two numbers from their sum, divide the remainder by 2, the quotient will be the smaller number; then add the difference to the smaller number, their sum will be the greater number.*

1. A gentleman paid 325 dollars for a horse and chaise; the chaise cost 75 dollars more than the horse. What was the cost of each?

2. The salaries of two teachers, A and B, amount to 2700 dollars a year; A receives 300 dollars more than B. What is the salary of each?

3. A gentleman left an estate amounting to 18500 dollars, to be divided between his wife and daughter; the wife was to have 3750 dollars more than the daughter. What number of dollars did each receive?

Ans. Wife received $11125. Daughter received $7375.

PROBLEM V. THE PRICE OF A UNIT OF ANY QUANTITY BEING GIVEN, TO FIND THE VALUE OF THE GIVEN QUANTITY. ALSO, TO FIND THE VALUE OF ANY PART OF A UNIT OF THAT QUANTITY.

Suppose the price of a barrel of flour to be 7 dollars, what is the value of 5 barrels? The value of 5 barrels must be 5 times 7 dollars, which is 35 dollars. What is the value of $8\frac{1}{2}$ barrels? $8\frac{1}{2}$ times 7 dollars, which is $59\frac{1}{2}$ dollars. What is the value of $\frac{3}{4}$ of a barrel? $\frac{3}{4}$ of 7 dollars, which is $5\frac{1}{4}$ dollars. Hence, the following rule.

RULE. *Multiply the price of a unit of the given quantity by the number expressing the quantity, the product will be the value of the quantity required.*

1. If the price of a bushel of potatoes is 45 cents, what is the value of 40 bushels?

2. What is the value of a piece of cloth measuring $12\frac{3}{4}$ yards, at 20 cents a yard?

3. If an acre of land is worth $32\frac{3}{4}$ dollars, what is the value of $325\frac{1}{2}$ acres? Ans. $10660\frac{1}{8}$ dollars.

4. If the price of an acre of land is $32\frac{3}{4}$ dollars, what is the value of $\frac{7}{8}$ of an acre? Ans. $28\frac{21}{32}$ dollars.

PROBLEM VI. THE VALUE OF ANY QUANTITY BEING GIVEN, TO FIND THE PRICE OF A UNIT OF THAT QUANTITY.

Suppose the value of 5 barrels of flour to be 35 dollars, what is the price of 1 barrel? The price of 1 barrel must be $\frac{1}{5}$ of 35 dollars, which is 7 dollars. If $8\frac{1}{2}$ barrels be worth $59\frac{1}{2}$ dollars, what is 1 barrel worth? 1 barrel must be worth $\frac{2}{17}$ of $59\frac{1}{2}$ dollars, which is 7 dollars. If $\frac{3}{4}$ of a barrel be worth $5\frac{1}{4}$ dollars, what is a barrel worth? A barrel must be worth ⅛ of 56 dollars, which is 7 dollars. Hence, the following rule.

Rule. *Divide the value of the given quantity by the number expressing the quantity, the quotient will be the value of a unit of that quantity.*

1. If 40 bushels of potatoes are worth 18 dollars, what is the value of 1 bushel?	2. If 12¾ yards of cloth are worth 2⅛ dollars, what is the value of 1 yard?

3. A gentleman purchased a farm containing 326½ acres, for which he paid 10660⅛ dollars; what did it cost per acre? Ans. 32¾ dollars.

4. A gentleman purchased a lot of land containing ⅞ of an acre, for which he paid 28 21/32 dollars; what is the value of an acre? Ans. 32¾ dollars.

PROBLEM VII. THE PRODUCT OF TWO NUMBERS OR FACTORS AND EITHER OF THEM BEING GIVEN, TO FIND THE OTHER.

If we analyse the questions in Problem 6, we shall perceive that the value of any given quantity is the product of the price of a unit multiplied by the number expressing the quantity. The area or superficial contents of a square or parallelogram is the product of its length multiplied by its breadth. Hence the following rule.

Rule. *Divide the given product by the given factor, the quotient will be the other factor.*

1. The product of two numbers is 63, and one of them is 7; what is the other?	2. The product of two factors is 132, and one of them is 11; what is the other?

3. The floor of a school-room contains 2400 square feet; its length is 60 feet. What is its width? Ans. 40 feet.

4. A certain lot of land contains 1650 square feet; its width is 22 feet. What is its length? Ans. 75 feet.

5. A gentleman purchased a farm containing 120 acres; its form is a right-angled parallelogram; its length is 160 rods. What is its width? Ans. 120 rods.

6. There is a board fence which is 5 feet in height, and contains 720 square feet; what is its length?

7. A farmer planted a piece of ground with potatoes in rows 3 feet apart; the hills in each row were also 3 feet apart; there were 120 rows and 18000 hills in the field. What was the number of hills in each row?

PROBLEM VIII. THE PRODUCT OF THREE NUMBERS OR FACTORS AND ANY TWO OF THEM BEING GIVEN, TO FIND THE OTHER.

Suppose a stone pillar to be 3 feet in width, 2 feet in thickness, and 10 feet in length; what number of cubic feet does it contain?

Multiplying its width, 3 feet, by its thickness, 2 feet, the product is 6 square feet, the area of its end or base. Multiplying this area 6 square feet, by its length 10 feet, the product is 60 cubic feet. Hence, it is plain, that if we divide the number of cubic feet in any regular solid body by the product of its width multiplied by its thickness, the quotient must be its length.

Also, if the product of any three numbers or factors be divided by the product of any two of them, the quotient must be the third number or factor. Hence, the following rule.

RULE. *Divide the given product by the product of the two given numbers or factors, the quotient will be the third number or factor required.*

1. Suppose a load of wood to be 8 feet long, and 4 feet wide; how high must it be to contain 1 cord or 128 cubic feet?

2. Suppose a pile of wood to be 8 feet high, and 4 feet wide; how long must it be to contain 4 cords or 512 cubic feet?

3. There is a stick of timber 1 foot 9 inches in width, 1 foot 6 inches in thickness; what must be its length to contain 1 ton? Ans. 15$\frac{5}{21}$ feet.

4. The product of three numbers is 8250; the first is 40, the second is 16.5. What is the third? Ans. 12.5.

5. Suppose a carpenter wishes to construct a cubical box of sufficient capacity to contain 360 cubic feet; he has been directed to make it 12 feet in length, and 6 feet in width. What must be its depth? Ans. 5 feet.

6. There were 57600 cubic feet of earth dug from the cellar of a church; its length is 90 feet, and its depth 8 feet. What is its width?

7. In excavating a canal 2640000 squares of earth were removed; its width is 30 feet, and its depth 12 feet. What is its length?

NOTE. A square of earth contains 216 cubic feet

PRACTICAL QUESTIONS IN DECIMAL FRACTIONS.

Art. 124. 1. What is the sum of seventy-five hundred thousandths and ninety-five millions? What is their difference?

2. The sum of two fractions is one hundred sixty-five ten-thousandths; the smaller fraction is five thousandths. What is the greater fraction?

3. The difference between two fractions is six hundred seventy-five thousandths; the smaller fraction is seventy-five thousandths. What is the greater fraction?

4. The difference between two fractions is three hundred and fifteen thousandths; the greater fraction is thirty-six hundredths. What is the smaller fraction?

5. The sum of two fractional numbers is 4.25; their difference is 2.75. What are the numbers?

6. The product of two fractional numbers is .015; one of the numbers is .06. What is the other?

7. If the price of a yard of broadcloth is $4.25, what is the value of .875 of a yard?

8. If .125 of a yard of broadcloth is worth .375 of a dollar, what is a yard of the same kind of cloth worth?

9. If 32.25 yards of shirting be worth $6.45, what is the value of 1 yard?

10. The floor of a school-room contains 983.125 square feet; its length is 32.5 feet. What is its width?

11. A gentleman purchased a lot of land containing 1868.625 square feet; it measures 24.75 feet in width. What is its length?

12. A gentleman purchased a farm for which he paid $3193.875; the price per acre was $25.50. What number of acres did the farm contain?

13. A log of mahogany contains 57.375 cubic feet; its length is 12.75 feet; its width is 2.25 feet. What is its depth?

14. There is a pile of wood which measures 5 cords; its length is 40 feet, its width is 3.75 feet. What is its height?

15. A carpenter was directed to make a bin of sufficient capacity to hold 75 bushels of grain. He was also directed to make it 8 feet in length inside, and 3.25 feet in width. What must be its depth?

COMPARISON OF NUMBERS AND QUANTITIES.

Art. 125. EVERY number is some proportional part of every other number, and every quantity is some proportional part of every other quantity of the same kind.

We compare a less number with a greater to ascertain what part the less number is of the greater, and we compare the greater with the less to ascertain what part the greater is of the less.

For a similar purpose, we compare a smaller quantity with a larger of the same kind, and the larger with the smaller.

As every fraction expresses the part which its numerator is of its denominator, we can express the part which one of two given numbers is of the other by making that number which is called the part the numerator of a fraction, and the other the denominator; the fraction thus formed will express the required part.

ILLUSTRATION. William has 5 apples and Henry has 10. It is plain that William has $\frac{5}{10}$ or $\frac{1}{2}$ as many as Henry, and it is equally plain that Henry has $\frac{10}{5}$ or 2 times as many as William.

What part of 8 shillings 3 pence is 2 shillings 9 pence? 8s. 3d. = 99 pence. 2s. 9d. = 33 pence; and 33 pence is $\frac{33}{99}$ or $\frac{1}{3}$ of 99 pence.

What part of \$1.625 is \$4? \$1.625 = 1625 mills. \$4 = 4000 mills. 4000 mills is $\frac{4000}{1625}$ or $\frac{32}{13}$ of 1625 mills.

What part of 4.5 miles is 2.25 miles? 4.5 = 450 hundredths. 2.25 = 225 hundredths. 225 hundredths is $\frac{225}{450}$ or $\frac{1}{2}$ of 450 hundredths.

What part of $4\frac{7}{9}$ is $2\frac{3}{5}$? $4\frac{7}{9} = \frac{43}{9}$. $2\frac{3}{5} = \frac{13}{5}$. $\frac{43}{9} = \frac{215}{45}$. $\frac{13}{5} = \frac{117}{45}$. And $\frac{117}{45}$ is $\frac{117}{215}$ of $\frac{215}{45}$.

From the above remarks and illustrations we derive the following

RULE. *If the two numbers or quantities are of different denominations, reduce them to the same. If the two fractions have different denominators, change them to fractions which shall have the least common denominator. Then write that number or numerator which the question requires to be a part of the other for the numerator of a fraction, and the other for the denominator; the fraction thus formed will express the part required.*

1. What part of 9 pounds is 7 pounds?

3. What part of 17 acres is 15 acres?

5. What part of 2 shillings is 5 pence?

7. What part of 5 gallons is 3 quarts?

9. What part of 5 yards is 2.75 yards?

2. What part of 5 yards is 13 yards?

4. What part of 9 dollars is 16 dollars?

6. What part of 17 shillings is 3 pounds?

8. What part of 2 h. 30 m. is 1 h. 15 m.?

10. What part of $\frac{1}{7}$ of a mile is $\frac{3}{5}$ of a mile?

11. What part of £12. 15s. 6d. is £9. 12s. 3d. 2qrs.? Ans. $\frac{4615}{6132}$.

12. What part of 500 dollars is 25 dollars and 15 cents? Ans. $\frac{503}{10000}$.

13. What part of 5 tons is 15 cwt. 3 qrs. 14 pounds? Ans. $\frac{127}{800}$.

14. What part of 35 yards 3 qrs. is 15 yds. 1 qr. 2 na.? Ans. $\frac{123}{286}$.

15. What part of 5 hhds. is 3 hhds. 17 gals. 3 qts. 1 pt.? Ans. $\frac{331}{504}$.

16. What part of 10 dollars is 15 dollars and 75 cents? Ans. $\frac{63}{40}$.

17. What part of 42 miles is 17 miles 5 furlongs? Ans. $\frac{47}{112}$.

18. What part of 125 acres is 175 acres 2 roods? Ans. $\frac{351}{250}$.

19. What part of $\frac{3}{4}$ of $\frac{4}{5}$ is $\frac{6}{7}$ of $\frac{7}{8}$? Ans. $\frac{5}{4}$.

20. What part of $25\frac{3}{4}$ is $\frac{3}{4}$ of $12\frac{1}{5}$? Ans. $\frac{183}{515}$.

21. What part of 17 hours 20 minutes is 21 hours? Ans. $\frac{63}{52}$.

22. What part of 75.25 acres is 25.5 acres? Ans. $\frac{102}{301}$.

23. What part of a barrel of flour can I purchase with 5 dollars, when the price is $7.375 a barrel? Ans. $\frac{40}{59}$.

24. When flour is worth 6 dollars a barrel, what part of a barrel can be purchased with 2.25 dollars? Ans. $\frac{3}{8}$ of a barrel.

25. When wheat is $\frac{5}{8}$ of a dollar a bushel, what part of a bushel can you buy with $\frac{4}{5}$ of a dollar? Ans. $\frac{32}{25}$.

26. What part of a ton of iron can you purchase with $27\frac{5}{8}$ dollars, when the price of a ton is 95 dollars? Ans. $\frac{221}{760}$ of a ton.

27. When coal is worth 7 dollars a ton, what part of a ton can you purchase with 2.125 dollars? Ans. $\frac{17}{56}$ of a ton.

28. A owns $425\frac{1}{4}$ acres of land, B owns $375\frac{3}{4}$ acres. What part as much land does B own as A? Ans. $\frac{1503}{1701}$.

SIMPLE RATIO AND SIMPLE PROPORTION.

Art. 126. The part which one number is of another is sometimes called the ratio of the one to the other; thus, 3 is $\frac{3}{5}$ of 5, and the ratio of 3 to 5 is $3 \div 5 = \frac{3}{5}$. Hence, the ratio of one number to another is expressed by the quotient obtained by dividing the one by the other.

The two given numbers are called the terms of the ratio; the first number or term is called the *antecedent*, and the second, the *consequent*.

The ratio of one number to another is expressed or written in two different forms.

The first is by a fraction, writing the antecedent for the numerator, and the consequent for the denominator; thus, the ratio of 8 *to* 12 *is* $\frac{8}{12}$.

The second is by placing two points indicating division between the two terms of the ratio; thus, the ratio of 8 *to* 12 *is written* 8 : 12.

1. What is the ratio of 4 to 8?
2. What is the ratio of 8 to 4?
3. What is the ratio of 7 to 9?
4. What is the ratio of 9 to 7?
5. What is the ratio of 15 cents to 17 cents?
6. What is the ratio of 19 pounds to 16 pounds?
7. What is the ratio of 17 shillings to £2?
8. What is the ratio of 2 gallons to 3 quarts?
9. What is the ratio of $1.25 to 5 dollars?
10. What is the ratio of $\frac{7}{8}$ of a yd. to $\frac{3}{4}$ of a yd.?

Art. 127. When we have four quantities or numbers given, two of them having the same name, the other two having also a like name, and the ratio of the first to the second being equal to the ratio of the third to the fourth, these four quantities or numbers are said to be in proportion. Hence, proportion is the equality of two ratios.

Illustration. There are two pieces of cloth: one measures 5 yards, and is worth 15 dollars; the other measures 8 yards, and is worth 24 dollars. Comparing 8 yards with 5 yards, we perceive that 8 yards is $\frac{8}{5}$ of 5 yards. Comparing 24 dollars with 15 dollars, we perceive that 24 dollars is $\frac{24}{15}$ of 15 dollars. Hence, it is plain that 24 dollars is the same proportional part of 15 dollars that 8 yards is of 5 yards; also, that the ratio of 5 yards to 8 yards is equal to the ratio of 15 dollars to 24 dollars; therefore, these four quantities or numbers constitute a proportion.

The usual method of indicating that the two ratios are equal, and that the four terms are in proportion, is, by placing two short parallel lines between the two ratios, and two points indicating division between the two terms of each ratio; thus, $5 : 8 = 15 : 24$. The whole expression is read thus: the ratio of 5 to 8 is equal to the ratio of 15 to 24.

A more convenient method of indicating the equality of two ratios is by expressing each of the ratios by a fraction, and placing the sign of equality between them; thus, $\frac{5}{8} = \frac{15}{24}$. This expression is read thus: 5 divided by 8 equals 15 divided by 24.

A third method is by placing four points between the two ratios, and two points between the two terms of each ratio; thus, $5 : 8 :: 15 : 24$; and this expression is usually read thus: as 5 is to 8, so is 15 to 24.

Art. 128. The first and fourth numbers or terms of a proportion are called the extremes, and the second and third numbers or terms are called the means. In the other method of writing the terms, the numerator of the first fraction and the denominator of the second are the extremes, and the denominator of the first fraction and the numerator of the second are the means.

Since the quotient of the first term divided by the second is equal to the quotient of the third divided by the fourth, it follows that the product of the extremes is equal to the product of the means.

Take the proportion $5 : 8 = 15 : 24$, or $\frac{5}{8} = \frac{15}{24}$. The product of the extremes $5 \times 24 = 120$. The product of the means $8 \times 15 = 120$.

The product of the extremes in every proportion being equal to the product of the means, it follows that if the product of the extremes be divided by either of the means, the quotient will be the other mean; also, if the product of the means be divided by either extreme, the quotient will be the other extreme.

Art. 129. In every practical question in simple proportion there are three numbers or terms given, to find a fourth term, or answer to the question; two of which have the same name, or are of a like kind, and the other has the same name, or is of the same kind, as the required fourth term or answer.

Illustration. Suppose 6 barrels of flour to be worth 36 dollars, how many dollars are 8 barrels worth?

In this question we have two quantities of flour given, 6 barrels and 8 barrels, which numbers are the first and second terms of a proportion; we have given also 36 dollars, the value of 6 barrels, which is the third term, and we are required to find the value of 8 barrels, or the fourth term.

Multiplying the third term $36, by the second term 8, the product is $288; dividing this product by the first term 6, the quotient is $48, the fourth term or value of 8 barrels.

Art. 130. Arranging the three given numbers or terms in proper form or order, is called stating the question.

From the preceding remarks and illustrations we obtain the following rule.

Rule. *Write that number for the third term which has the same name, or is of the same kind, as the required fourth term or answer.*

Then ascertain whether the fourth term or answer must be greater or less than the third term; if the fourth term must be greater than the third, write the greater of the two remaining numbers for the second term, or numerator of a fraction, and the smaller for the first term, or denominator; but if the fourth term or answer must be less than the third, write the smaller of the two remaining numbers for the second term, or numerator of a fraction, and the greater for the first term, or denominator.

Multiply the third term by the numerator, or second term, and divide the product by the denominator, or first term; the quotient will be the fourth term, or answer, in the same denomination as the third term.

When the first and second terms are compound numbers, or when they are of different denominations, reduce them to the lowest denomination mentioned in either of them; and when the third term is a compound number, reduce it to the lowest denomination mentioned in it.

1. The first term of a proportion is 6, the second 8, the third 12; what is the fourth?

3. If 6 yards of cloth cost 12 dollars, how many dollars will 8 yards cost?

5. If a man travel 12 miles in 3 hours, how many miles will he travel in 5 hours?

7. If 10 bushels of potatoes last a family 8 months, what number of bushels will last the same family 12 months?

2. The first term of a proportion is 8, the second 6, the third 16; what is the fourth?

4. If 8 yards of cloth cost 16 dollars, how many dollars will 6 yards cost?

6. If a man travel 20 miles in 5 hours, how many miles will he travel in 3 hours?

8. If a family consume 15 bushels of apples in 12 months, how many bushels will the same family consume in 8 months?

9. If 10 barrels of flour are worth 60 dollars, how many dollars are 15 barrels worth?

11. If 12 men can perform a piece of work in 15 days, how many men will perform the same work in 30 days?

13. If 8 men can dig a cellar in 10 days, what number of men will be required to dig the same in 5 days?

10. If 15 barrels of flour are worth 90 dollars, how many dollars are 10 barrels worth?

12. If 6 men can perform a piece of work in 30 days, how many men will perform the same work in 15 days?

14. If 16 men can build the walls of a house in 5 days, how many days will it take 8 men to build the same walls?

1. If 5 yards of broadcloth cost 30 dollars, how many dollars will 8 yards of the same cloth cost?

1ST STATEMENT. $\overset{\text{yds.}}{5} : \overset{\text{yds.}}{8} = \overset{\$}{30} : \overset{\$}{48}$. Ans.

2ND STATEMENT. $\frac{8}{5} \times \overset{\$}{30} = \frac{240}{5} = \overset{\$}{48}$. Ans.

In this question the required fourth term or answer is to be in money; hence we write 30 dollars for the third term. 8 yards being $\frac{8}{5}$ of 5 yards, 8 yards will cost $\frac{8}{5}$ of 30 dollars; therefore, we write 8 for the second term or numerator of a fraction, and 5 for the first term or denominator. We obtain the fourth term, or answer, by multiplying the third term by the second, or numerator, and then dividing the product by the first term, or denominator.

2. If 48 dollars will pay for 8 yards of broadcloth, how many yards of the same cloth can be purchased with 30 dollars?

1ST STATEMENT. $\overset{\$}{48} : \overset{\$}{30} = \overset{\text{yds.}}{8} : \overset{\text{yds.}}{5}$. Ans.

2ND STATEMENT. $\frac{30}{48} \times \overset{\text{yds.}}{8} = \frac{240}{48} = \overset{\text{yds.}}{5}$. Ans.

In this question, the required fourth term, or answer, is to be in yards; hence we write 8 yards for the third term. 30 dollars being only $\frac{30}{48}$ of 48 dollars, we can purchase only $\frac{30}{48}$ of 8 yards with 30 dollars; therefore, we write 30 for the second term, or numerator of a fraction, and 48 for the first term, or denominator.

3. If 20 men can perform a piece of work in 15 days, how many men will be required to perform the same work in 10 days?

1ST STATEMENT. $\overset{\text{da.}}{10} : \overset{\text{da.}}{15} = \overset{\text{m.}}{20} : \overset{\text{m.}}{30}$. Ans.

2ND STATEMENT. $\frac{15}{10} \times \overset{\text{m.}}{20} = \overset{\text{m.}}{30}$. Ans.

As the required fourth term, or answer, is to be a number of men, we write the 20 men for the third term. As the same work is to be performed in $\frac{10}{15}$ of the time in which 20 men can perform it, it will require $\frac{15}{10}$ of 20 men to perform it in 10 days; therefore, we write 15 days for the second term, or numerator of a fraction, and 10 days for the first term, or denominator.

4. If 30 men can perform a piece of work in 10 days, how many men will be required to perform the same work in 15 days?

1ST STATEMENT.	2ND STATEMENT.
$\overset{d.}{15} : \overset{d.}{10} = \overset{m.}{30} : \overset{m.}{20}$. Ans.	$\frac{10}{15} \times \overset{m.}{30} = \overset{m.}{20}$. Ans.

As the answer is to be a number of men, we write 30 men for the third term. As it will require only $\frac{10}{15}$ as many men to perform the same work in 15 days, we write 10 days for the second term, or numerator of a fraction, and 15 days for the first term, or denominator.

Art. 131. When the first and second terms, or numerator and denominator, have a common factor, the operation may be abridged by dividing each term by that common factor, or by reducing the fraction to its lowest terms.

5. If 40 sheep yield 120 pounds of wool at a shearing, how many pounds will be obtained from 320 sheep?

1ST STATEMENT.	2ND STATEMENT.
$\overset{s.}{40} : \overset{s.}{320} = \overset{lbs.}{120} : \overset{lbs.}{960}$. Ans.	$\frac{320}{40} \times \overset{lbs.}{120} = \overset{lbs.}{960}$. Ans.
Dividing the first and second terms by their greatest common factor 40, we have this statement.	Reducing $\frac{320}{40}$ to its lowest terms by dividing its terms by 40, we have this statement.
$\overset{s.}{1} : \overset{s.}{8} = \overset{lbs.}{120} : \overset{lbs.}{960}$. Ans.	$\frac{8}{1} \times \overset{lbs.}{120} = \overset{lbs.}{960}$. Ans.

6. If 75 acres of land are worth 1500 dollars, what is the value of a farm containing 300 acres? Ans. 6000 dollars.

7. If 500 men consume 50 barrels of provisions in 6 months, how many barrels will 350 men consume in the same time? Ans. 35 barrels.

8. If 200 dollars gain 12 dollars interest in one year, how much interest will 750 dollars gain in the same time?
Ans. 45 dollars.

9. Purchased 75 tons of coal, for which I paid 525 dollars; what must I pay for a cargo containing 325 tons?
Ans. 2275 dollars.

10. If a post 6 feet in height casts a shadow 8 feet in length, what must be the height of a tree that casts a shadow 90 feet in length at the same time? Ans. 67½ feet.

11. How many days will it take a ship to sail round the

globe, allowing it to be 25000 miles in circumference, if she sails at the rate of 2640 miles in 12 days?

Ans. $113\frac{7}{11}$ days.

12. If it will take 25 barrels of flour to pay a debt when flour is $5.25 a barrel, how many barrels will it take to pay the same debt when flour is worth $6.50 a barrel?

Ans. $20\frac{5}{26}$ barrels.

13. Borrowed of a friend 225 dollars for 30 days; I afterwards loaned him 450 dollars. What number of days must he keep the 450 dollars to balance the previous favor?

Ans. 15 days.

14. A ship's company of 12 persons is supposed to have bread sufficient to last their voyage, allowing each person 12 ounces a day. They pick up a crew of 6 persons in distress, whom they permit to share their daily allowance with them. What will be the daily allowance of each person?

Ans. 8 ounces.

15. If 2 cwt. 1 qr. 14 lbs. of sugar be worth $21.75, what will be the value of 42 cwt. 3 qrs. at the same rate?

$$\overset{\text{cwt.}}{2}.\ \overset{\text{qr.}}{1}.\ \overset{\text{lbs.}}{14} : \overset{\text{cwt.}}{42}.\ \overset{\text{qr.}}{3} = \overset{\$}{21}.\overset{\text{c.}}{75} : \overset{\$}{391}.\overset{\text{c.}}{50}.\ \text{Ans.}$$

In this question, the first and second terms being compound numbers, each of them must be reduced to pounds.

16. If a piece of linen containing 21 yards is worth £3. 5s. 3d. 1qr., what will be the value of 4 pieces of linen, containing 84 yards? Ans. £13. 1s. 1d.

17. If 7 yards of ribbon are worth 6s. 8d., what is the value of 42 yards? Ans. £2.

18. If 7 ounces of gold are worth £30, what is the value of 7 lbs. 11 ounces? Ans. £407. 2s. 10d. $1\frac{1}{7}$ qr.

19. Purchased 54 yds. of broadcloth, for which I paid £60. 15s.; how many yards of the same kind of cloth can I purchase with £10. 2s. 6d.? Ans. 9 yards.

20. Purchased 25 a. 2 r. 20 rods of land, for which I paid $639.0625; what is the value of a farm containing 205 acres, supposing the land to be of the same quality?

Ans. $5112.50.

21. If $\frac{3}{4}$ of an acre of land is worth 15 dollars, what is the value of $\frac{7}{8}$ of an acre?

1ST STATEMENT.

$$\overset{\text{a.}}{\tfrac{3}{4}} : \overset{\text{a.}}{\tfrac{7}{8}} = \overset{\$}{15} : \overset{\$}{17}.\overset{\text{cts.}}{50}.\ \text{Ans.}$$

2ND STATEMENT.

$$\frac{\frac{7}{8}}{\frac{3}{4}} = \frac{7}{6} \times \overset{\$}{15} = \overset{\$}{17}.\overset{\text{cts.}}{50}.\ \text{Ans.}$$

If $\frac{3}{4}$ of an acre of land is worth 15 dollars, $\frac{4}{4}$ or a whole acre must be worth $\frac{4}{3}$ of 15 dollars, and $\frac{7}{8}$ of an acre must be worth $\frac{1}{2}$ as much as a whole acre; hence we have the following 3rd statement.

$\frac{7}{3}$ of $\frac{1}{2}$ of $\overset{\$}{15} = \overset{\$\ \text{cts.}}{17.50}$. Ans.

Pupils should be required to analyze, state, and perform each of the following problems in a similar manner.

22. If $\frac{5}{7}$ of a yard of cloth is worth \$1.15, what is $\frac{4}{9}$ of a yard worth? Ans. 71$\frac{7}{9}$ cents.

23. If $\frac{3}{8}$ of a lot of land is worth 150 dollars, what is the value of $\frac{4}{5}$ of the same lot? Ans. 320 dollars.

24. If $\frac{7}{8}$ of a yard of cloth is worth $\frac{4}{5}$ of a dollar, what is the value of 12$\frac{3}{4}$ yards? Ans. \$11.65$\frac{5}{7}$.

25. If 5 men can mow 72$\frac{6}{7}$ acres of grass in 11$\frac{2}{3}$ days, how many acres can they mow in 8$\frac{2}{5}$ days?
Ans. 52$\frac{112}{245}$ acres.

26. If 7$\frac{5}{8}$ pounds of butter are worth 1$\frac{5}{11}$ dollars, what is the value of 27$\frac{4}{5}$ pounds? Ans. \$5.30$\frac{210}{671}$.

27. If a man perform a journey in 7$\frac{1}{2}$ days, travelling 12$\frac{1}{3}$ hours each day, in how many days will he perform it, if he travel but 9$\frac{1}{2}$ hours each day?

28. If 15 yards of cloth cost \$45.75, how many yards can be purchased with 366 dollars?

1st STATEMENT.

$\overset{\$\ \ \text{cts.}}{45.75} : \overset{\$\ \ \text{cts.}}{366.00} = \overset{\text{yds.}}{15} : \overset{\text{yds.}}{120}$. Ans.

2nd STATEMENT.

$\frac{366.00}{45.75} \times \overset{\text{yds.}}{15} = \overset{\text{yds.}}{120}$. Ans.

29. If 12.75 acres of land are worth \$255, what is the value of 102 acres of the same quality?

30. If .75 of a bushel of wheat is worth .90 of a dollar, what is the value of 24.5 bushels?

31. If 25 dollars will pay for the transportation of 5 tons 102.5 miles, what distance can 5 tons be carried for \$125.375?

32. Paid \$37.50 for 5 barrels of flour; how many barrels can be purchased with 450 dollars?

33. How many bushels of corn, at \$.625 a bushel, will amount to as much money as 75 bushels of wheat, at \$1.25 a bushel?

34. How many yards of cambric, .625 of a yard in width, will be required to line a lady's dress containing 10.75 yards, that is .875 of a yard in width?

35. If it take 4.75 yards of broadcloth that is 1.5 yards wide to make a gentleman's cloak, how many yards of silk that is only .875 of a yard wide will be required to line it?

COMPOUND RATIO AND COMPOUND PROPORTION.

Art. 132. A COMPOUND RATIO is the ratio of the products obtained by multiplying the corresponding terms of two or more simple ratios.

ILLUSTRATION. The simple ratio of 3 : 5 = $\frac{3}{5}$.
The simple ratio of 4 : 7 = $\frac{4}{7}$.

Multiplying the terms of the ratio 3 : 5 by the corresponding terms of the ratio 4 : 7, we obtain the composite or compound ratio 12 : 35.

Multiplying $\frac{3}{5}$ by $\frac{4}{7}$, we obtain the same compound ratio, $\frac{12}{35}$, in a fractional form.

Art. 133. Compound Proportion is the equality of two ratios, one of which is compound, and the other simple.

Every question in compound proportion always contains an odd number of terms, as five, seven, nine, &c. These terms are distinguished into terms of *supposition*, and terms of *demand;* the number of the former is always one more than that of the latter. One of the terms of supposition always has the same name, or is of a like kind, as the required term, or answer; which must always be made the *third* term in stating. Each of the remaining terms of supposition has its corresponding term of demand of a like kind, one of which must be made the first term of a simple ratio in stating, and the other the second.

ILLUSTRATION. 1. Suppose that 12 men can earn 600 dollars in 25 days; how many men will be required to earn 400 dollars in 20 days?

This question contains five numbers or terms; the first three are the terms of supposition, and the last two are the terms of demand.

1ST STATEMENT.

$$\left.\begin{array}{lcl} 600 \text{ dollars} & : & 400 \text{ dollars.} \\ 20 \text{ days} & : & 25 \text{ days.} \end{array}\right\} = 12 \text{ men.}$$

12000 dollars : 10000 dollars = 12 men : 10 men, the number required.

As the required term or answer is to be a number of men, we write 12, the supposed number of men, for the third term. 12 men are supposed to earn 600 dollars in 25 days, and it will require only $\frac{400}{600}$ of 12 men to earn 400 dollars in the same number of days; therefore, we write 400 dollars for the second term, and 600 dollars for the first term. As the 400 dollars is to be earned in $\frac{20}{25}$ of the number of days, it will require $\frac{25}{20}$ as many men to earn 400 dollars in $\frac{20}{25}$ of the time; therefore, we write 25 days for the second term, and 20 days for the first term.

Multiplying the terms of the first simple ratio by the corresponding terms of the second, we obtain the compound ratio 12000 dollars : 10000 dollars = 12 men : to the number of men required to earn 400 dollars in 20 days.

Multiplying the third term 12 by the second term 10000, the product is 120000; dividing this product by the first term 12000, the quotient is 10, the number of men required to earn 400 dollars in 20 days.

2ND STATEMENT.

$\overset{da.}{\frac{25}{20}} \times \overset{\$}{\frac{400}{600}} \times \overset{m.}{12} = \overset{m.}{10}$, the number required to earn 400 dollars in 20 days. Reducing each fraction to its lowest terms, we have $\frac{5}{4} \times \frac{2}{3} \times 12 = 10$ men, the number required.

From the above illustrations we derive the following

RULE. *Write that term of supposition which has the same name, or is of the same kind, as the required term or answer, for the third term.*

Then take one of the other terms of supposition and its corresponding term of demand of the same kind, and consider whether an answer depending on these terms alone must be gr[illegible] or less than the third term, and write one of them for a [illegible]nd term, or numerator of a fraction, and the other for a first term, or denominator, as directed in simple proportion. Arrange all the remaining corresponding terms of supposition and demand in a similar manner.

Then find the product of all the second terms or numerators; find also the product of all the first terms or denominators; multiply the third term by the product of the second terms or numerators; divide this product by the product of the first terms or denominators, the quotient will be the answer or term required.

The terms of each simple ratio, if of different denominations, must be reduced to the same, and the third term must be reduced to the lowest denomination in it.

Art. 134. It will be perceived that all the second terms, or numerators, and the third term also, are factors of a dividend, and that all the first terms, or denominators, are factors of a divisor.

Since diminishing the dividend and divisor proportionally does not alter the quotient, therefore all equal factors of the dividend and diviser may be crossed or cancelled, and not used in the operation.

When any two factors, one of them being a factor of the dividend, and the other a factor of the diviser, have a common measure, both may be crossed, and then divided by that common measure, and the quotients retained.

2. If 6 men can build a wall 20 feet long, 6 feet high, and 4 feet thick, in 16 days, in what number of days will 24 men build a wall 200 feet long, 8 feet high, and 6 feet thick?

1ST STATEMENT.

$$\left.\begin{array}{rrrrrr} 1 & \cancel{4} & \cancel{24}: & \cancel{6} & 1 & \\ & 1 & \cancel{20}: & \cancel{200} & \cancel{10} & 5 \\ 1 & \cancel{3} & \cancel{6}: & \cancel{8} & \cancel{4} & 1 \\ 1 & \cancel{2} & \cancel{4}: & \cancel{6} & \cancel{3} & 1 \end{array}\right\} = \overset{\text{da.}}{16} : \quad \overset{\text{da.}}{80}. \text{ Ans.}$$

Cancelling all equal factors, we have only to multiply the third term, 16 days, by 5, the product is the number of days required.

2ND STATEMENT.

$$\frac{6}{4} \times \frac{8}{6} \times \frac{200}{20} \times \frac{6}{24} \times \overset{\text{da.}}{16} = \overset{\text{da.}}{80}. \text{ Ans.}$$

Reducing each of the above fractions to its lowest terms, we have

$$\frac{\cancel{3}}{\cancel{2}} \times \frac{\cancel{4}}{\cancel{3}} \times \frac{\cancel{10}}{1} \times \frac{1}{\cancel{4}} \times \overset{\text{da.}}{16} = \overset{\text{da.}}{80}. \text{ Ans.}$$

Cancelling all equal factors, we have

$$\frac{1}{1} \times \frac{1}{1} \times \frac{5}{1} \times \frac{1}{1} \times \overset{\text{da.}}{16} = \overset{\text{da.}}{80}. \text{ Ans.}$$

In the above question, 6 men are supposed to build a wall 20 feet long, 6 feet high, and 4 feet thick, in 16 days; these five numbers or terms are the terms of supposition. It is required to find the number of days in which 24 men can build a wall 200 feet long, 8 feet high, and 6 feet thick; these four numbers or terms are the terms of demand.

As the answer is required to be a number of days, we write 16 days, the term of supposition of a like kind, for the third term.

Comparing 6 men, a term of supposition, with 24 men, its corresponding term of demand, we find that 24 men is $\frac{24}{6}$ of 6 men, hence it will take 24 men only $\frac{6}{24}$ of 16 days to perform the same work; therefore we write 6 for the second term and 24 for the first, in the first statement, and 6 for the numerator of a fraction and 24 for the denominator, in the second statement.

Comparing 20 feet in length, a term of supposition, with 200 feet in length, its corresponding term of demand, we find that 200 feet is $\frac{200}{20}$ of 20 feet; hence it will take $\frac{200}{20}$

many days to build a wall 200 feet in length as it will 20 feet in length; therefore we write 200 for the second term and 20 for the first; we also write 200 for the numerator of a fraction and 20 for the denominator.

Comparing 6 feet high, a term of supposition, with 8 feet high, its corresponding term of demand, we find that 8 feet is $\frac{8}{6}$ of 6 feet; hence it will take $\frac{8}{6}$ as many days to build a wall 8 feet high as it will 6 feet high; consequently we write 8 for the second term and 6 for the first; we also write 8 for the numerator of a fraction, and 6 for the denominator.

Comparing 4 feet thick, a term of supposition, with 6 feet thick, its corresponding term of demand, we find that 6 feet is $\frac{6}{4}$ of 4 feet; hence it will take $\frac{6}{4}$ as many days to build a wall 6 feet thick as it will 4 feet thick; consequently we write 6 for the second term, and 4 for the first; we also write 6 for the numerator of a fraction and 4 for the denominator, and the terms are all arranged and the statements completed.

3. If $100 will gain $6 interest in 12 months, in what time will $750 gain $30 dollars interest? Ans. 8 months.

4. If 7 men can reap 84 acres of wheat in 24 days, how many men will be required to reap 100 acres in 10 days? Ans. 20 men.

5. If $200 will gain $6 interest in 6 months, how many dollars will $400 gain in 9 months? Ans. 18 dollars.

6. If $1200 will support a family of 24 persons 8 months, how many months will $900 support a family of 16 persons? Ans. 9 months.

7. If I pay $24 for the transportation of 96 barrels of flour 200 miles, what must I pay for the transportation of 480 barrels 40 miles? Ans. 24 dollars.

8. If 8 men can build a wall 75 rods in length in 12 days, how many men will be required to build a wall 300 rods in length in 6 days? Ans. 64 men.

9. If 12 pounds of wool will make 8 yards of cloth 6 quarters wide, how many pounds will be required to make 144 yards 4 quarters wide? Ans. 144 pounds.

10. If 5 men can make 1200 pairs of shoes in 48 days, how many men will be required to make 4800 pairs of the same kind of shoes in 16 days? Ans. 60 men.

11. If 15 girls can perform 180 questions in arithmetic in 75 minutes, how many questions can 25 girls perform in 35 minutes? Ans. 140 questions.

12. If a garrison of 1200 men consume 500 barrels of flour in 9 months, how many barrels will a garrison of 2500 men consume in 15 months?

13. If 12 men can build a wall 100 feet long, 4 feet high, and 3 feet thick, in 40 days, in what time will 6 men build one 20 feet long, 6 feet high, and 4 feet thick?

14. If 180 men, in 6 days of 10 hours each, can dig a trench 1200 yards long, 4 feet wide, and 3 feet deep, in how many days, of 12 hours each, will 90 men dig a trench 400 yards long, 6 feet wide, and 4 feet deep?

15. A garrison of 1800 men has provision sufficient to allow each man 32 ounces a day 35 days; suppose the garrison to be reinforced with 600 men, what must be the daily allowance for each man 45 days?

16. If 6 men can reap 30 acres of wheat in 5 days, what number of acres can 8 men reap in 10 days?

17. If 3 horses eat 30 bushels of oats in 50 days, what number of bushels will 30 horses eat in 25 days?

18. If it requires $600 to support a family of 9 persons 8 months, what number of dollars will be required to support a family of 6 persons 12 months?

19. If 12 ounces of wool be sufficient to make 1½ yards of flannel 6 quarters wide, what number of pounds will be required to make 450 yards of flannel 4 quarters wide?

20. If 24 tailors can make 75 suits of clothes in 8 days, what number of tailors will be required to make 1200 suits of clothes in 6 days?

21. If 4 men can build a wall 30 rods in length in 6 days, what number of men will be required to build a wall 90 rods in length in 4 days?

22. If 14 men, working 9 hours each day, dig a ditch 420 feet long, 4 feet wide, and 3 feet deep, in 4 days, in what number of days will 35 men, working 12 hours each day, dig a ditch 840 feet long, 5 feet wide, and 4 feet deep?

23. A wall, to be built to the height of 27 feet, was raised to the height of 9 feet by 12 men in 6 days; how many men must be employed to finish the wall in 4 days at the same rate of working?

24. A wall 700 yards long was to be built in 29 days; at the end of 11 days 18 men had built 220 yards of it. What number of additional men was it then necessary to engage to work at the same rate, in order that the wall might be completed in the given time?

PROPORTION INVOLVING A CONSTANT QUANTITY.

Art. 135. We will explain what is to be understood by a constant quantity, by the following question.

1. A coal mine has the same quantity of water constantly running into it in the same time. At one time it was allowed to get full, and it required an engine of 12 horse-power 60 hours to empty it. At another time it was again allowed to get full, and it required an engine of 10 horse-power 80 hours to empty it. It is now full the third time; what must be the power of an engine that will empty it in 48 hours, and what must be the power of an engine that will discharge the water as fast as it runs into the mine after it is emptied?

It is to be understood that the horse-power and the flow of water into the mine are quite constant, that is, exactly the same in all cases. It is also to be understood that the quantity of water in the mine is the same each time, when full.

As it required an engine of 12 horse-power 60 hours to discharge the water that was in the mine when full the first time, including also the water that runs into the mine during those 60 hours, it is plain that it will require only $\frac{60}{80}$ or $\frac{3}{4}$ of 12 horse-power to discharge the same quantity of water in 80 hours, and $12 \times \frac{3}{4} = 9$, the number of horse-power required to discharge the quantity of water in the mine when full, including also the water that runs in during 60 hours.

The second time it was allowed to get full, it required an engine of 10 horse-power 80 hours to empty it, including also the water that runs into the mine during 80 hours.

As it requires only 9 horse-power 80 hours to empty the mine when full, including also the water that runs into the mine during 60 hours, it is plain that the other 1 horse-power, employed 80 hours the second time it was allowed to get full, is the power required to discharge the water that runs into the mine during the remaining 20 hours. If it requires 1 horse-power 80 hours to discharge the water that runs into the mine during the remaining 20 hours, it will require $\frac{80}{20}$ or 4 times 1 horse-power to discharge the same quantity of water in 20 hours, which is 4 horse-power. Hence it is plain that an engine of 4 horse-power will discharge the water as fast as it runs into the mine during any length of time; which is one of the answers required.

We will now find what power will be required to empty the mine in 48 hours. It is plain that, whatever power is required to empty the mine in any given number of hours, there is always 4 horse-power that does nothing towards emptying the mine, because it only discharges the water that runs into the mine during the given number of hours.

The first time that the mine was allowed to get full, it required an engine of 12 horse-power 60 hours to empty it. Deducting the 4 horse-power which is required to discharge the water as fast as it runs into the mine, there remains 8 horse-power, which was the number of horse-power required to empty the mine in 60 hours, not including the water that runs into the mine during those 60 hours.

To discharge the same quantity of water in 48 hours, it will require $\frac{60}{48}$ or $\frac{5}{4}$ of 8 horse-power, and $8 \times \frac{5}{4} = 10$, the number of horse-power that will be required to empty the mine in 48 hours, not including the water that will run into the mine during the 48 hours.

If to this we add 4, the number of horse-power which is required to discharge the water as fast as it runs into the mine, we have 14 for the power of an engine that will empty the mine in 48 hours, including also the water that will run into the mine during the 48 hours, which is the other answer required.

2. "If 12 horses eat $3\frac{1}{3}$ acres of grass in 4 weeks, and 21 horses eat 10 acres in 9 weeks, how many horses will be required to eat 24 acres in 18 weeks; the grass being at first equal on every acre, and growing constantly and uniformly?" Ans. 36 horses.

3. If 14 cows, in 3 weeks, can eat the grass on 2 acres of land, also all that grows during the 3 weeks, and 16 cows, in 4 weeks, can eat all the grass on 3 acres, also all that grows during the 4 weeks; how many cows will be required to eat the grass on 6 acres, in 5 weeks, also all that grows during the 5 weeks; there being the same quantity on each acre, and growing constantly and uniformly? Ans. 26 cows.

4. Suppose 8 sheep, in 7 weeks, to eat all the grass on 400 square rods of land, also all that grows in the same time; and suppose 9 sheep, in 8 weeks, to eat all the grass on 500 square rods of land, also all that grows in the same time; how many sheep will be required to eat all the grass on 600 square rods of land, in 12 weeks, also all that grows in the same time; there being the same quantity of grass upon each square rod, and growing constantly and uniformly? Ans. 8 sheep.

PER CENTAGE AND PER CENT.

Art. 136. The term *per centage* has but recently been introduced into text-books, and it is used to express a greater or less number of hundredths of any sum or quantity; thus, if a merchant invests his money and gains 25 hundredths of the sum invested, he gains a high per centage; if he gains only 5 hundredths of the sum invested, he gains a low per centage. Hence, any number of hundredths of any sum or quantity is called the per centage.

The term *per cent.* also signifies hundredths; thus, 1 per cent. of any sum or quantity is 1 hundredth of it, 2 per cent. is 2 hundredths of it, 6 per cent. is 6 hundredths of it; &c.

1. A man hired 20 dollars for one year, and agreed to pay 6 per cent. or 6 hundredths of the sum for its use; how much did he pay?

3. What is 4 per cent. of 5 dollars? Of 10 dollars? Of 15 dollars? Of 20 dollars?

5. A man purchased a watch for 25 dollars, and sold it so as to gain 10 per cent.; how much did he gain by trading?

7. What is 1 per cent. of 75 dollars? 2 per cent.? 3 per cent.? 4 per cent.?

2. A girl found a lady's purse containing 50 dollars; the owner gave her 5 per cent. of the money. How much did the girl receive?

4. What is 6 per cent of 12 dollars? Of 25 dollars? Of 30 dollars? Of 40 dollars?

6. A market-man sold 50 dollars' worth of butter for a farmer, who paid him 5 per cent. for selling it; what did the market-man receive?

8. What is 5 per cent. of 100 dollars? 6 per cent.? 7 per cent.? 8 per cent.?

Art. 137. *Since per centage and per cent. signify hundredths, we can express any per centage, or any number of per cent., by a decimal fraction; thus,*

1 per cent. = .01
2 per cent. = .02
3 per cent. = .03
4 per cent. = .04
5 per cent. = .05
6 per cent. = .06
7 per cent. = .07
8 per cent. = .08
9 per cent. = .09
10 per cent. = .10
15 per cent. = .15
20 per cent. = .20
25 per cent. = .25
30 per cent. = .30
50 per cent. = .50
75 per cent. = .75
100 per cent. = 1.00
106 per cent. = 1.06
125 per cent. = 1.25
$\frac{1}{2}$ per cent. = .005
$\frac{1}{4}$ per cent. = .0025
$\frac{3}{4}$ per cent. = .0075
$\frac{1}{5}$ per cent. = .002
$\frac{4}{5}$ per cent. = .008
$\frac{1}{8}$ per cent. = .00125
$\frac{7}{8}$ per cent. = .00875

Art. 138. Since, by multiplying any given sum or quantity by a decimal fraction, we obtain as great a part of the given sum or quantity for a product as the decimal is of a unit, we can find the amount of any per centage, or number of per cent., of any given sum or quantity by the following rule.

RULE. *Multiply the given sum or quantity by the decimal expressing the required per cent., the product will be the amount of per centage.*

1. What is 1 per cent. of 25 dollars ?	Ans. .25 cents.
2. What is 2 per cent. of 50 dollars ?	Ans. $1.00.
3. What is 3 per cent. of 75 dollars ?	Ans. $2.25.
4. What is 4 per cent. of 100 dollars ?	Ans. $4.00.
5. What is 5 per cent. of 125 dollars ?	Ans. $6.25.
6. What is 6 per cent. of 150 dollars ?	Ans. $9.00.
7. What is 7 per cent. of 175 dollars ?	Ans. $12.25.
8. What is 8 per cent. of 200 dollars ?	Ans. $16.00.
9. What is 9 per cent. of 225 dollars ?	Ans. $20.25.
10. What is 10 per cent. of 250 dollars ?	Ans. $25.00.
11. What is 12 per cent. of 275 dollars ?	Ans. $33.00.
12. What is 15 per cent. of 300 dollars ?	Ans. $45.00.
13. What is 20 per cent. of 325 dollars ?	Ans. $65.00.
14. What is 25 per cent. of 350 dollars ?	Ans. $87.50.
15. What is 50 per cent. of 375 dollars ?	Ans. $187.50.
16. What is 75 per cent. of 400 dollars ?	Ans. $300.00.
17. What is 100 per cent. of 425 dollars ?	Ans. $425.00.
18. What is 125 per cent. of 450 dollars ?	Ans. $562.50.
19. What is 2½ per cent. of 475 dollars ?	
20. What is 3¼ per cent. of 500 dollars ?	
21. What is 4¾ per cent. of 525 dollars ?	
22. What is 5⅛ per cent. of 550 dollars ?	
23. What is 6⅜ per cent. of 575 dollars ?	
24. What is 7⅝ per cent. of 600 dollars ?	
25. What is 12⅞ per cent. of 625 dollars ?	
26. What is ½ of 1 per cent. of $650.25 ?	
27. What is ¼ of 1 per cent. of $675.50 ?	
28. What is ¾ of 1 per cent. of $725.75 ?	
29. What is ⅛ of 1 per cent. of $750.875 ?	
30. What is ⅜ of 1 per cent. of $775.375 ?	
31. What is ⅝ of 1 per cent. of $800.625 ?	
32. What is ⅞ of 1 per cent. of $975.875 ?	
33. What is ⅖ of 1 per cent. of $1025.75 ?	
34. What is ⅘ of 1 per cent. of $1275.25 ?	

Art. 129. **35. What is $4\frac{1}{3}$ per cent. of 1728 dollars?**

OPERATION.

\$1728

.04$\frac{1}{3}$

69.12 = 4 per cent.

5.76 = $\frac{1}{3}$ per cent.

\$74.88 Ans.

Since $\frac{1}{3}$ per cent. cannot be exactly expressed by a decimal fraction, we first find 4 per cent. by multiplying by .04 hundredths, and we then multiply by $\frac{1}{3}$ of 1 hundredth; the sum of the products is the answer required.

36. What is $33\frac{1}{3}$ per cent. of 750 dollars? Ans. \$250.00.
37. What is $66\frac{2}{3}$ per cent. of 900 dollars? Ans. \$600.00.
38. What is $8\frac{1}{6}$ per cent. of 600 dollars? Ans. \$49.00.
39. What is $5\frac{5}{6}$ per cent. of 500 dollars? Ans. \$29.16$\frac{2}{3}$.
40. What is $15\frac{7}{10}$ per cent. of 1500 dollars? Ans. \$235.50.

The amount of per centage on any sum of English Money may be found by first changing the lower denominations to the decimal of a pound, and then multiplying by the decimal expressing the required per cent.

41. What is 5 per cent. of £125. 10s. 6d.?

OPERATION.

10s. 6d. = £.525.

£125.525

.05

£6.27625

20

s.5.52500

12

d.6.30000

4

qr.1.20000

We first change 10s. 6d. to the decimal of a pound, and annex it to the £125. We then multiply the whole sum £125.525 by .05, the product is £6.27625. We then reduce this decimal of a pound to its value in shillings, pence, and farthings, and find the answer to be £6. 5s. 6d. 1.2qr.

42. What is 6 per cent. of £750. 17s. 6d.?
Ans. £45, 1s. 0d. 2.4 qrs.
43. What is $7\frac{1}{2}$ per cent. of £75. 18s. 9d.?
Ans. £5. 13s. 10$\frac{3}{4}$d.
44. What is $8\frac{3}{4}$ per cent. of £45. 6s. 8d.?
Ans. £3. 19s. 8d. 3qrs.+.
45. What is 15 per cent. of £1275. 15s.?
Ans. £191. 7s. 3d.
46. What is 20 per cent. of £575. 15s.?

COMMISSION.

Art. 140. COMMISSION is the per centage charged by agents for transacting the business of others, and they usually charge a specified per cent. on the amount of money employed in the transaction.

An agent who is employed by merchants and manufacturers in purchasing and selling goods is called a Commission Merchant, or Factor.

To find the amount of commission, multiply the given sum by the decimal fraction expressing the given per cent., and the product will be the amount of commission required.

1. A commission merchant sold a quantity of cloth for a manufacturer amounting to $750.75, and charged a commission of 2½ per cent.; what was the amount of his commission? Ans. $18.76875.

2. An agent purchased goods for his employer to the amount of $675.25, and charged 1¾ per cent. commission; how much did he receive for his services?
Ans. $11.816875.

3. An auctioneer sold furniture to the amount of $1250; what did his commission amount to at 2⅝ per cent.?
Ans. $32.8125.

4. What amount of commission should my agent in Albany charge me for purchasing 500 barrels of flour on my account, at $5.25 a barrel, at 2 per cent. commission on the amount?

5. My agent, after selling merchandise for me to the amount of $1250.75, deducted his commission of 2¼ per cent., and remitted the balance; what was the amount of his commission? and how much did he remit?

6. A factor has in his possession $525, and being directed to invest it in purchasing cotton, what sum can he invest, after deducting his commission of 5 per cent. on the purchase?

As his commission of 5 per cent. on every $100 invested is $5, if we add $5 to $100, the amount is $105; hence the sum to be invested is $\frac{100}{105}$ of $525, which is $500.

7. My agent in England had in his hands $3895, which I directed him to lay out in the purchase of railroad iron at $95 a ton; how many tons did he purchase with the balance remaining to be invested, after deducting his commission of 2½ per cent. on the purchase?

INSURANCE.

Art. 141. INSURANCE is a contract or agreement, by which an individual or company engages, for a stipulated premium, to indemnify the person insured from loss or damage to the property at risk by certain perils enumerated in the contract.

The individual or company who takes the risk is called the *Insurer* or *Underwriter;* the person protected by the insurance is called the *Insured;* the sum paid is called the *Premium;* and the instrument containing the contract is called the *Policy.*

The premium is a specified per cent. or number of hundredths of the amount of property insured. Hence, to find the amount of premium, we multiply the sum insured by the decimal fraction expressing the per cent., the product will be the amount of premium.

1. What amount of premium must be paid for insuring a house against fire, which is worth $5000, at a premium of ⅝ of 1 per cent.? Ans. $31.25.

2. A gentleman obtained a policy of insurance on his furniture to the amount of $2500, at 1¼ per cent. a year; what amount of premium did he pay each year?
Ans. $31.25.

3. A merchant obtained a policy of insurance on his stock of goods, valued at $7500, at a premium of 2½ per cent. a year; what amount of premium did he pay annually?
Ans. $187.50

4. A manufacturer effected an insurance to the amount of $25000 on his buildings and stock, at a premium of 3⅜ per cent. a year; what amount of premium did he pay yearly?

5. A merchant shipped 1500 barrels of flour, valued at $6000, from New York to Liverpool, and effected an insurance at a premium of 2½ per cent.; what was the amount of premium?

6. A merchant in New Orleans shipped a cargo of cotton to Liverpool, valued at 25000 dollars, and obtained a policy of insurance on the amount at a premium of 2⅞ per cent.; what was the amount of premium?

7. The estimated value of my house is 5000 dollars. If I should obtain insurance upon ⅗ of its value, at a premium of ⅝ of 1 per cent., what would be the amount of premium? If it should be destroyed by fire while insured, what should *I save by having* my house insured?

STOCKS.

Art. 149. Stock is a general name for the capital of moneyed institutions; such as Banks, Manufactories, Railroads, Insurance Companies, United States Bonds, State Bonds, &c.

The capital stock of an institution is usually divided into equal shares, and the evidences of ownership are called certificates of stock.

The *par value* of a share is its original cost or sum named in the certificate or bond.

The *real value* of a share, at any time, is the sum for which it can be sold.

When a share can be sold for more than its original cost, it is said to be *above par*, or at a *premium*.

When the *market value* of a share is less than its original cost, it is said to be *below par*, or at a *discount*.

If $100 of stock can be sold for $115, the stock is 15 per cent. *above par;* if it can only be sold for $85, it is 15 per cent. *below par;* the per centage of premium or discount is always to be estimated on the *par value*.

The amount of profit arising from stocks is divided among the stockholders, at regular periods, each receiving a certain per cent. of the *par value* of his shares.

1. The semi-annual dividend of the Merchants' Bank, in Boston, has usually been 3½ per cent. on its capital stock; what sum did a stockholder receive semi-annually, who owned 25 shares, the *par value* of each share being $100?
Ans. $87.50.

2. A person purchased 5 shares of the Boston and Lowell railroad stock, at a premium of 17½ per cent., the par value of each share being $500; what did the 5 shares cost him?
Ans. $2937.50.

3. A capitalist deposited $5100 with a broker, to be invested in bank stock; the broker charged 2 per cent. commission on the amount paid for the stock; what was the value of the stock purchased? What was the amount of commission?

Ans. $5000 value of stock purchased. $100 amount of commission.

4. Suppose I wish to invest $5700 in bank stock which is 5 per cent. below par; what amount will it purchase at the par value?
Ans. $6000.

INTEREST.

Art. 143. Interest is an estimated number of hundredths of any sum of money, which is to be paid for its use during a specified time.

The rate of interest is established by the legislature of each state. In nearly all the states 6 per cent., or 6 hundredths of any sum of money, is paid for its use during one year.

The money hired, or the sum for which interest is paid, is called the *principal;* the sum paid for its use is the *interest;* and the *sum* of the principal and interest is called the *amount.*

When the rate of interest is 6 per cent. a year, the interest of one dollar for one year will be 6 hundredths of a dollar or 6 cents, and the interest of one dollar for any number of years must be as many times 6 cents as there are years.

Since the interest of one dollar for one year, or 12 months, is one half as many cents as there are months in a year, the interest of one dollar for any number of months must be one half as many cents as there are months.

Since the interest of one dollar for one year, or 360 days, is 6 cents or 60 mills, which is one sixth as many mills as there are days in a year, the interest of one dollar for any number of days must be one sixth as many mills as there are days.

The preceding definitions and illustrations furnish the following general rule for computing interest at 6 per cent.

Rule. *Find the interest of one dollar for the given time, then multiply this interest by any given number of dollars; the product will be the interest of that number of dollars for the same time.*

1. What is the interest of 1 dollar for 1 year at 6 per cent.? What is the interest of 1 dollar for 2 years? 3 years? 4 years? 5 years? 6 years? 7 years? 8 years? 9 years? 10 years? 11 years? 12 years?

3. What is the interest of 1 dollar for 1 day? 2 days? 3 days? 4 days? 5 days? 6 days?

2. What is the interest of 1 dollar for 1 month at 6 per cent.? What is the interest of 1 dollar for 2 months? 3 months? 4 months? 5 months? 6 months? 7 months? 8 months? 9 months? 10 months? 11 months? 12 months?

4. What is the interest of 1 dollar for 7 days? 8 days? 9 days? 10 days? 11 days? 12 days?

5. What is the interest of 1 dollar for 13 days? 14 days? 15 days? 16 days? 17 days? 18 days?

7. What is the interest of 1 dollar for 25 days? 26 days? 27 days? 28 days? 29 days?

9. What is the interest of 1 dollar for 2 years, 2 months, and 2 days?

11. What is the interest of 1 dollar for 4 years, 4 months, and 4 days?

13. What is the interest of 1 dollar for 6 years, 6 months, and 6 days?

15. What is the interest of 1 dollar for 8 years, 8 months, and 8 days?

17. What is the interest of 1 dollar for 10 years, 10 months, and 10 days?

19. What is the interest of 1 dollar for 12 years, 12 months, and 12 days?

21. What is the interest of 1 dollar for 3 years, 4 months, and 14 days?

23. What is the interest of 1 dollar for 5 years, 6 months, and 16 days?

25. What is the interest of 1 dollar for 7 years, 8 months, and 18 days?

27. What is the interest of 1 dollar for 9 years, 10 months, and 20 days?

29. What is the interest of 1 dollar for 11 years, 11 months, and 22 days?

31. What is the interest of 1 dollar for 4 years, 7 months, and 24 days?

33. What is the interest of 1 dollar for 3 years, 1 month, and 26 days?

35. What is the interest of 1 dollar for 6 years, 3 months, and 28 days?

37. What is the interest of 1 dollar for 2 years, 4 months, and *12 days?*

6. What is the interest of 1 dollar for 19 days? 20 days? 21 days? 22 days? 23 days? 24 days?

8. What is the interest of 1 dollar for 1 year, 1 month, and 1 day, at 6 per cent. a year?

10. What is the interest of 1 dollar for 3 years, 3 months, and 3 days?

12. What is the interest of 1 dollar for 5 years, 5 months, and 5 days?

14. What is the interest of 1 dollar for 7 years, 7 months, and 7 days?

16. What is the interest of 1 dollar for 9 years, 9 months, and 9 days?

18. What is the interest of 1 dollar for 11 years, 11 months, and 11 days?

20. What is the interest of 1 dollar for 2 years, 3 months, and 13 days?

22. What is the interest of 1 dollar for 4 years, 5 months, and 15 days?

24. What is the interest of 1 dollar for 6 years, 7 months, and 17 days?

26. What is the interest of 1 dollar for 8 years, 9 months, and 19 days?

28. What is the interest of 1 dollar for 10 years, 11 months, and 21 days?

30. What is the interest of 1 dollar for 12 years, 1 month, and 23 days?

32. What is the interest of 1 dollar for 5 years, 9 months, and 25 days?

34. What is the interest of 1 dollar for 2 years, 4 months, and 27 days?

36. What is the interest of 1 dollar for 8 years, 5 months, and 29 days?

38. What is the interest of 1 dollar for 3 years, 6 months, and 18 days?

39. What will be the interest of $1275 for 2 years, 9 months, and 27 days, at 6 per cent. a year.

cts.
.12 int. of $1 for 2 years.
.045 int. of $1 for 9 months.
.0045 int. of $1 for 27 days.

.1695 int. of $1 for 2 yrs. 9 mo. [and 27 days.
$1275

8475
11865
3390
1695

$216.1125 Ans.

In this question, we first find the interest of 1 dollar for 2 years, 9 months, and 27 days, which is $.1695; we then multiply this interest by $1275, the given number of dollars; the product is the interest required.

40. What is the interest of $750.75 for 3 years, 5 months, and 17 days, at 6 per cent. ?

cts.
.18 int. of $1 for 3 years.
.025 int. of $1 for 5 months.
.002$\frac{5}{6}$ int. of $1 for 17 days.

.207$\frac{5}{6}$
$750.75

62562$\frac{3}{6}$ product of $\frac{5}{6}$ of a mill [multiplied by $750.75.
1035
1449
10350
1449

156.03087\frac{3}{6}$ Ans.

Since the interest of 1 dollar for 6 days is .001 mill, the interest of 1 dollar for 1 day must be $\frac{1}{6}$ of a mill, for 2 days, $\frac{2}{6}$ or $\frac{1}{3}$ of a mill, for 3 days, $\frac{3}{6}$ or $\frac{1}{2}$ of a mill, for 4 days, $\frac{4}{6}$ or $\frac{2}{3}$ of a mill, for 5 days $\frac{5}{6}$, of a mill.

Since the vulgar fractions $\frac{1}{6}$, $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{5}{6}$, cannot be exactly expressed in decimals, if we change these fractions of a mill to hundredths of a mill, and reject the remainders, we shall obtain the interest sufficiently exact for most purposes. Thus, .000$\frac{1}{6}$ of a mill = .00016 +. .000$\frac{1}{3}$ of a mill = .00033 +. .000$\frac{2}{3}$ of a mill = .00066 +. .000$\frac{5}{6}$ of a mill = .00083 +.

Using .00083 + of a mill instead of .000$\frac{5}{6}$ of a mill in Question 40, we have .20783 × 750.75 = $156.0283725 for the answer.

41. What is the interest of $15.25 for 1 year, 2 months, and 15 days, at 6 per cent. ? Ans. $1.105+.

42. What is the interest of $25.75 for 2 years, 1 month, and 12 days, at 6 per cent. ? Ans. $3.27+.

43. What is the interest of $28.45 for 1 year, 7 months and 18 days, at 6 per cent. ? Ans. $2.788+.

44. What is the interest of $45.15 for 9 months and 15 days, at 6 per cent.? Ans. $2.144+.

45. What is the interest of $64.24 for 5 months and 12 days, at 6 per cent.? Ans. $1.734+.

46. What is the interest of $115.15 for 7 months and 24 days, at 6 per cent.? Ans. $4.49+.

47. What is the interest of $250 for 2 years, 3 months, and 10 days, at 6 per cent.? Ans. $34.165+.

48. What is the interest of $175.25 for 1 year, 7 months, and 17 days, at 6 per cent.? Ans. $17.144+.

49. What is the interest of $275 for 15 days, at 6 per cent.? Ans. $.687+.

50. What is the interest of $85.15 for 1 month and 27 days, at 6 per cent.? Ans. $.808+.

51. What is the interest of $325 for 2 months, at 6 per cent.? Ans. $3.25.

52. What is the interest of $95.45 for 4 months and 20 days, at 6 per cent.? Ans. $2.226+.

53. What is the interest of $145.50 for 5 months and 25 days, at 6 per cent.? Ans. $4.242+.

54. What is the interest of $350.75 for 8 months and 27 days, at 6 per cent.? Ans. $15.608+.

55. What is the interest of $425.40 for 10 months and 10 days, at 6 per cent.? Ans. $21.976+.

56. What is the interest of $440.80 for 11 months and 15 days, at 6 per cent.? Ans. $25.346.

57. What is the interest of $475.25 for 1 year and 4 months, at 6 per cent.?

58. What is the interest of $500 for 1 year and 6 months, at 6 per cent.?

59. What is the interest of $524.25 for 2 years and 8 months, at 6 per cent.?

60. What is the interest of $550 for 5 years and 10 months, at 6 per cent.?

61. What is the interest of $575 for 2 years, 6 months, and 15 days, at 6 per cent.? What is the amount?
Ans. $87.687+ int. $662.687+ amount.

62. What will be the interest of $330 from January 1st, 1847, to July 16th, 1847, at 6 per cent.? What will be the amount?

63. What will be the interest of $175 from January 15th, 1847, to April 20th, 1847, at 6 per cent.? What will be the amount?

Art. 144. *When the rate of interest is more or less than 6 per cent. a year, first find the interest at 6 per cent., and then make a proportional addition or subtraction. If it is 7 per cent. add one sixth. If 8 per cent. add one third. If 5 per cent. subtract one sixth. If 4½ per cent. subtract one fourth.*

64. What is the interest of $344.45 for 2 years, 2 months, and 3 days, at 7 per cent. a year? Ans. $52.442+.

65. What will be the interest of $75 for 8 months and 18 days, at 8 per cent.? What will be the amount?
Ans. $4.30 int. $79.30 amount.

66. What is the interest of $44.25 for 6 months and 6 days, at 5 per cent.?

67. What will be the interest of $175.50 for 10 months and 15 days, at 4½ per cent.? What will be the amount?

Art. 145. *Since the interest of one dollar for any given time expresses that decimal part of one dollar which the interest is of one dollar for that time, so it must express that decimal part of any number of dollars which the interest of that number of dollars will be for the same time. Hence, if we multiply any given sum by that decimal part of a dollar which the interest is of a dollar for any given time, the product must be the interest of that sum for the same time.*

Thus, the interest of 1 dollar for 2 years, 8 months, and 24 days, is $.164 thousandths of a dollar; hence, the interest of $475 for the same time is .164 thousandths of $475. And $475 × .164 = $77.90, the interest of $475 for the same time.

Art. 146. *In computing compound interest, and interest on English money, it will be more convenient to multiply the given sum or principal by that decimal part of* 1 *dollar, or of* 1 *pound, which expresses the interest of* 1 *dollar, or of* 1 *pound, for the given time.*

When the principal is given in English money, we must first reduce the shillings, pence, and farthings, to the decimal of a pound, and annex this decimal to the pounds.

68. What is the interest of £75. 15s. 6d. for 2 years, 8 months, and 12 days, at 6 per cent. a year?

15s. 6d. = £.775. Annexing this decimal to £75, we have £75.775 for a principal. The interest of £1 for 2 *years,* 8 months, and 12 days, at 6 per cent. a year, is .162

thousandths of a pound. And £75.775 × .162 = £12.27555 = £12. 5s. 6.132d., the interest required.

69. What is the interest of £25. 18s. 9d. for 10 months and 15 days, at 6 per cent. a year?

Ans. £1. 7s. 2d. 3qrs.+.

70. What will be the interest of £45. 12s. $6\frac{1}{2}$d. from January 1st, 1847, to July 16th, 1847, at 6 per cent. a year?

71. What will be the interest of £500 from January 20th, 1847, to September 20th, 1847, at 6 per cent. a year? What will be the amount?

Art. 147. Since the interest of 1 dollar for any given number of days is one sixth as many mills as there are days, it follows that the interest of any given number of dollars for any given number of days must be equal to one sixth of the product of the number of dollars multiplied by the number of days, in the denomination of mills. Hence the following rule for computing interest for any number of days.

RULE. *Multiply the given sum or number of dollars by the given number of days, divide this product by 6, the quotient will be the interest in mills, when the sum is dollars only; but when there are cents in the given sum, the quotient will be the interest in hundredths of mills.*

72. What is the interest of $25 for 25 days?

Ans. $.104+.

73. What is the interest of $10 for 10 days?

Ans. $.016.+

74. What is the interest of $15 for 15 days?

Ans. $.037+.

75. What is the interest of $75 for 33 days?

Ans. $.412+.

76. What is the interest of $100 for 63 days?

Ans. $1.05+.

77. What is the interest of $125 for 93 days?

78. What is the interest of $24.75 for 20 days?

79. What is the interest of $45.15 for 35 days?

80. What is the interest of $75.25 for 45 days?

81. What will be the interest of $175.75 for 33 days?

82. What will be the interest of $525.25 for 63 days?

In the above method of computing interest for days, the year is supposed to contain 360 instead of 365 days; hence, the interest found for any given number of days will be too large by its $\frac{1}{73}$ part, by which it must be diminished when perfect accuracy is required.

PARTIAL PAYMENTS.

Art. 148. When partial payments have been made and endorsed upon notes and bonds, the following rule has been adopted by the Supreme Court of the United States, also by the Courts in Massachusetts, New York, and by the Courts in most of the other States, for computing the interest. It is given in the language of Chancellor Kent, of New York.

"*The rule for casting interest, when partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due.*

"*If the payment exceeds the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due.*

"*If the payment be less than the interest, the surplus of interest must not be taken to augment the principal; but interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied towards discharging the principal; and interest is to be computed on the balance, as aforesaid.*"

83.

$1250.00. Boston, January 1st, 1846.

For value received, I promise to pay William Briggs, or order, twelve hundred and fifty dollars on demand, with interest at 6 per cent.
John Smith.

The following payments were endorsed on this note.

April 1st, 1846, received $175.75. W. Briggs.
August 1st, 1846, received $20.00. W. Briggs.
November 1st, 1846, received $360.00. W. Briggs.

What was the amount due on this note January 1st, 1847?

Principal, January 1st, 1846, ... $1250.00.
First payment, Jan. 1st, 1846, $175.75.
Interest to first payment (3 mo.) $18.75.
Excess of payment above the interest, ... $157.00.
Principal or sum due after first payment, ... $1093.00.
Second payment, Aug. 1st, 1846, $20.00, } $380.00.
Third payment, Nov. 1st, 1846, $360.00, }
Interest to Aug. 1st, 1846, (4 mo.) $21.86, } $38.255.
Interest to Nov. 1st, 1846, (3 mo.) $16.395, }
Excess of the payments above the interest, ... $341.745.
Principal or sum due after the third payment, . . . $751.255.
Interest to January 1st, 1847, ... 7.51255.
Amount due on note January 1st, 1847, ... $758.76755.

The day on which the note is dated, and the day on which it falls due, are not both reckoned in determining the time, but one of them is always excluded.

84.

$775.00. Lowell, March, 15th, 1846.

For value received, I promise to pay C. D. on demand, seven hundred and seventy-five dollars, with interest.

A. B.

This note is endorsed as follows:

June 1st, 1846, received $25.00. C. D.
Sept. 15th, 1846, received $45.75. C. D.
Dec. 1st, 1846, received $75.00. C. D.
May 15th, 1847, received $200.00. C. D.

What will be due Sept. 1st, 1847? Ans. $487.98+.

85.

$550.00. Worcester, January 25th, 1846

For value received, we, jointly and severally, promise to pay J. R., or order, five hundred and fifty dollars, in three months from date, with interest after three months.

W. P.
S. L.

The following payments are endorsed on this note:

July 20th, 1846, received $75.25. J. R.
Sept. 25th, 1846, received $50.75. J. R.
Dec. 20th, 1846, received $45.50. J. R.

What was due on this note January 25th, 1847?

Ans. $400.02+.

86.

$1200.00. Salem, June 10th, 1846.

For value received, I promise to pay A. B., or bearer, twelve hundred dollars on the 1st day of January, 1847, with interest from date. J. P.

Suppose the following payments to have been endorsed on this note, what was due January 1st, 1847?

July 1st, 1846, received $75.00. A. B.
Sept. 1st, 1846, received $5.25. A. B.
Nov. 1st, 1846, received $10.75. A. B.
Dec. 1st, 1846, received $375.00. A. B.

87.

$1500.00. Boston, July 15th, 1846.

For value received, I promise to pay T. P., or order, fifteen hundred dollars on demand, with interest.

G. B. C.

Suppose $200 to have been paid on this note on the first day of August, September, October, November, and December, what was due on the 1st of January, 1847?

Art. 149. The following rule is sometimes used for computing the interest on bonds and notes, upon which partial payments have been made.

Rule. *Compute the interest on the note from the time the interest commenced to the time of settlement, and find the amount.*

Then compute the interest on each payment from the time it was paid to the time of settlement, and find the amount of each. Lastly, deduct the total amount of the several payments from the amount of the note, the remainder is the balance due on the note.

88. A note dated January 1st, 1846, was given for $1000, payable on demand, with interest at 6 per cent., on which were the following endorsements:

March 1st, 1846, received $75.00. July 15th, 1846, received $125.75. September 25th, 1846, received $250.00. Nov. 10th, 1846, received $300.00. What was the balance due on this note, January 1st, 1847? Ans. $295.471.+.

89. A, by his note dated July 1st, 1846, promised to pay B $625.50 in six months from date, with interest after three months. On this note were the following endorsements:

October 16th, 1846, received $250.75. November 25th, 1846, received $125.50. December 15th, 1846, received $75.25.

What was the balance due on this note January 1st, 1847, interest being allowed at 6 per cent.? Ans. $179. 294+.

90. A note dated July 1st, 1846, was given for $750, payable in six months, with interest from the date at 6 per cent. a year. On this note were the following endorsements:

August 1st, 1846, received $75.75. September 16th, 1846, received $150.50. November 1st, 1846, received $250.25. What was the balance due on this note January 1st, 1847?

91. C, by his note dated April 1st, 1846, promised to pay D $1750 on demand, with interest at 6 per cent. a year. On this note were the following endorsements:

July 1st, 1846, received $125. October 1st, 1846, received $150. January 1st, 1847, received $500. What was the balance due on this note April 1st, 1847?

COMPOUND INTEREST.

Art. 150. COMPOUND INTEREST is that which is paid for the use of the principal, and also that which is paid for the use of the interest, after it becomes due; and it is found by adding the interest to the principal at the time it becomes payable, whether yearly, half-yearly, or quarter-yearly, and then making this amount a new principal.

RULE. *When the interest is payable yearly, find the interest of the given principal for one year, add it to the principal, the amount is the principal for the next year. Find the amount of this principal, and of each succeeding principal, in the same manner, for the given number of years.*

When the interest is payable half-yearly, or quarter-yearly, find the interest for a half-year, or a quarter-year, add the interest to the principal, the amount is the principal for the next half-year, or quarter-year. Find the amount of this principal and of each succeeding principal, in the same manner, for the given number of half-years or quarter-years.

Finally, subtract the given principal from the last amount; the remainder will be the compound interest.

1. What is the compound interest of $500 for 4 years, at 6 per cent., payable yearly?

OPERATION.

	$500	given principal.
$500 × .06 =	$30	interest for one year.
	$530	amount, principal for the second year.
$530 × .06 =	$31.80	interest for the second year.
	$561.80	amount, principal for the third year.
$561.80 × .06 =	$33.708	interest for the third year.
	$595.508	amount, principal for the fourth year.
$595.508 × .06 =	$35.73048	interest for the fourth year.
	$631.23848	amount for the fourth year.
	$500.00	given principal deducted.
	$131.23848	compound interest for four years.

2. What will be the compound interest of $1200 for 2 years, at 6 per cent., payable half-yearly? Ans. $150.61+.

3. What will be the compound interest of $675 for 2 years, at 6 per cent., interest payable quarter-yearly?

4. What will be the compound interest of $1500 for 5 years, at 6 per cent., payable yearly?

DISCOUNT.

Art. [illegible]. Discount is an allowance made for the payment of any sum of money or debt, which is not on interest, before it is due; or it is the difference between any given sum or debt payable at some future time, and its present worth.

The *present worth* of any given sum or debt, payable at a future time without interest, is such a sum of money as would amount to the given sum or debt, if put on interest, at the time it becomes payable.

The interest of $1.00 for one year at 6 per cent. is .06 cents, and the amount is $1.06; hence, $1.00 is the present worth of $1.06 payable in one year without interest, and .06 cents is the discount.

Therefore, the present worth of any sum or debt, payable in one year and not on interest, is $\frac{100}{106}$ of the sum or debt, and the discount is $\frac{6}{106}$ of the sum. From the above illustrations we obtain the following rules.

Rule I. *Make* $1.00 *the numerator of a fraction, and the amount of* $1.00 *at the given rate and time the denominator; multiply the given sum or debt by this fraction, the product will be the present worth.*

Rule II. *Make the interest of* $1.00 *at the given rate and time the numerator of a fraction, and the amount of* $1.00 *at the same rate and time the denominator; multiply the given sum or debt by this fraction, the product will be the discount.*

1. When money is worth 6 per cent. a year, what part of any sum or debt, payable in 1 year, and not on interest, is the present worth? What part of the sum is the discount?

2. When money is worth 6 per cent. a year, what part of any sum or debt, payable in 6 months, and not on interest, is the present worth? What part of the sum is the discount?

3. When money is worth 6 per cent. a year, what part of any sum or debt, due at the end of 9 months, and not on interest, is its present worth? What part of the sum is the discount?

4. When money is worth 6 per cent. a year, what part of any sum or debt, due at the end of 3 months, and not on interest, is its present worth? What part of the sum is the discount?

5. When money is worth 7 per cent. a year, what part of any sum or debt, payable in 1 year, and not on interest, is the present worth? What part of the sum is the discount?

6. When money is worth 7 per cent. a year, what part of any given sum or debt, payable in 6 months, and not on interest, is its present worth? What part of the sum is the discount?

7. Purchased goods to the amount of $1224, on a credit of 4 months; what discount must be allowed for present payment, when money is worth 6 per cent. a year? Ans. 24 dollars.

8. A merchant purchased a quantity of goods for $250, cash, and sold them for $300, payable in 9 months, without interest; what did he gain, supposing the use of money to be worth 6 per cent. a year? Ans. $37.081+.

9. What is the present worth of three notes, one for $960, payable in three months; one of $640, payable in six months; and the other for $375, payable in nine months; supposing money to be worth 6 per cent. a year? Ans. $1926.023+.

10. Purchased goods of a merchant in New York, to the amount of $1275, and gave him my note for the amount, payable in six months, without interest; what discount could the merchant in New York make for present payment, where money is worth 7 per cent. a year? Ans. $43.115+.

11. A merchant in Boston holds a note against a trader in Worcester, for $575.25, dated January 16th, 1847, payable in six months, without interest; what was the present worth of this note at the time it was dated?

12. Sold goods to the amount of $750; one half of the amount to be paid in three months, and the other half to be paid in six months; what discount can I make for present payment, when money is worth 6 per cent. a year?

13. What is the difference between the interest of $775 for 18 months, at 6 per cent. a year, and the discount of the same sum for the same time?

14. A mechanic, having a carriage for sale, offered it for $750, cash, or $775, payable in 4 months, without interest; the purchaser chose the latter. Did the purchaser gain or lose by his choice, and how much?

15. A owes B $575, to be paid September 1st, 1848, without interest; but B wishes A to pay him April 1st, 1848, to which A agrees. What sum must A pay B, supposing the use of money to be worth 6 per cent. a year?

16. A manufacturer sold 10 bales of sheeting for $1260, on a credit of 6 months; what discount can he afford to make for present payment when the use of money is worth 6 per cent. a year?

The only correct principle of calculating discount is by the preceding rule; yet banks and merchants, in discounting notes, deduct the interest for the time, including 3 days of grace, for the discount.

BANK DISCOUNT.

Art. 159. The bank discount of a note is the interest of the sum of money for which the note is given, from the time it is dated to the time when it will become due, including 3 days of grace.

It is customary in banks to compute the interest on every note for 3 days more than the time specified in the note. These 3 days are called days of grace, and it is not necessary that the note should be paid until the last day of grace, except when the last day happens to be a Sunday, or a holiday, as the 4th of July; then the note must be paid on the second day of grace.

The sum named in the note is called the *face* of the note. The *avails*, or present value, of a note discounted at a bank, is the sum of money received from the bank for the note; or it is the difference between the face of the note and the bank discount.

There are two kinds of notes discounted at banks. 1st. Notes given by one individual to another for property actually purchased; notes of this kind are called *business notes*, or *business paper*. 2d. Notes made for the purpose of hiring money of the bank. Notes of this kind are called *accommodation notes*, or *accommodation paper*.

Business notes are not usually on interest. The *legal* rate of discount is the *legal* rate of interest in the state in which the note is discounted.

1. What is the bank discount on a note of $600, for 30 days and grace, at 6 per cent. a year? Ans. $3.30.

2. What is the bank discount on a note of $750, for 60 days and grace, at 6 per cent. a year? Ans. $7.875.

3. A merchant of Boston purchased goods of a merchant in New York to the amount of $1275, and gave him his note, payable in ninety days and grace, for the amount; the New York merchant had the note discounted at a bank in New York, where the *legal* rate of interest is 7 per cent. What was the avails of the note, or what sum of money did he receive from the bank for the note? Ans. $1251.943+.

4. A merchant of Philadelphia sold a cargo of coal, containing 325 tons, for $1950, for which he received a note payable in six months, without interest; he immediately went to a bank and had this note discounted. What was the bank discount, and how much ready money did he receive for the cargo of coal?

5. A trader from the country purchased goods to the amount of $489.75 of a merchant in Boston, at his lowest cash prices. After purchasing the goods, the trader proposed to the merchant to give him his note for the amount, payable in 4 months, without interest. Yes, says the merchant, provided you will write the note for such an amount that the avails of the note shall amount to just $489.75, after I shall get it discounted at a bank. For what amount must this note be written?

It is plain that the note must be given for such a sum as will leave $489.75 after the bank discount of it for 4 months and 3 days has been deducted from that sum. The interest of $1 for 4 months and 3 days, at 6 per cent. a year, is $.0205, and $1 — $.0205 = $.9795; and $489.75 ÷ $.9795 = $500, the required amount of the note. Hence, to find the amount for which a note must be given, payable at any future time, which shall leave any required sum after deducting the bank discount for the time specified, we have the following

RULE. *Find the bank discount of* $1 *for the specified time, including* 3 *days of grace; deduct this bank discount from* $1; *the remainder will be the present value of* $1 *payable at the future specified time.*

Then divide the sum of money which you wish to obtain from the bank, by this present value of $1; *the quotient will be the amount of the required note.*

6. Suppose I wish to obtain from a bank $994.50 for 30 days and grace, when the rate of discount is 6 per cent. a year; for what amount must I give my note? Ans. $1000.

7. Suppose I want a loan from a bank in the city of New York, where the rate of discount is 7 per cent. a year, of $578.65 for 6 months; for what amount must a note be given to obtain this sum of money? Ans. $600.

8. A merchant sold 500 barrels of flour, and received a note at 60 days and grace in payment; he went immediately to a bank and had the note discounted, and received $2968.50. What was the amount of the note?

9. A merchant sold a cargo of coal, and received a note at 90 days and grace in payment; he had this note discounted at a bank, and received $2953.50. What was the amount of the note?

PROFIT AND LOSS.

Art. 153. PROFIT and loss are mercantile terms used to express what is gained or lost by business transactions.

The profit or loss on mercantile transactions is usually estimated by the per centage of the first cost of the articles which is gained or lost in trade.

ILLUSTRATION. 1. Samuel purchased a sled for which he paid $1.50, and sold it at a profit of 10 per cent.; how much did he get for his sled?

In order to make a profit of 10 per cent., he must sell it for the first cost, plus 10 per cent. of the first cost. 10 per cent. of $1.50 is .15 cents, and $1.50 + .15 is $1.65, the sum for which he sold the sled.

2. Purchased a sleigh for $20, and was under the necessity of selling it at a loss of 5 per cent.; how much did I get for the sleigh?

It is plain that I must have sold it for the first cost, minus 5 per cent. of the first cost. 5 per cent. of $20 is $1, and $20 — $1, is $19, the sum for which I sold the sleigh.

3. A farmer purchased a sheep for $4, and sold it at 12½ per cent. profit; what did he get for the sheep?

5. A boy purchased a pair of skates for $1.25, and sold them to another boy at 10 per cent. profit; what did he get for his skates?

7. A man bought a lot of land for $500; he afterwards sold it at a profit of 15 per cent.; how many dollars did he gain?

9. A man purchased a house for $5000, and sold it at a profit of 5 per cent.; what number of dollars did he gain?

4. A trader paid $50 for a gold watch, and sold it at a loss of 4 per cent.; what did he get for the watch?

6. William purchased a knife for 50 cents, and sold it at a loss of 20 per cent.; how much did he get for his knife?

8. A man purchased a horse for $125, and sold him at a loss of 8 per cent.; how many dollars did he lose?

10. A broker purchased 10 shares in the Fitchburg Railroad for $1000, and sold them at 8 per cent. advance; what was his gain?

11. Bought a piece of cloth containing 36 yards, at $2.50 a yard, and sold it at $3.25 a yard; what was the whole gain? Ans. $27.

12. A merchant purchased 750 barrels of flour, at $5.75 a barrel, and the next day sold the same at $6.50 a barrel. how much did he gain? Ans. $562.50.

13. A man purchased a farm containing 175 acres, for $3500; he afterwards sold the same farm at $25 an acre; what number of dollars did he gain? Ans. $875.

Art. 154. 14. Purchased a watch for $40, and sold it for $45; what per cent. did I gain?

$45 — $40 = $5, the gain, which is $\frac{5}{40}$ of the cost of the watch. If we change $\frac{5}{40}$ to hundredths, by annexing two decimal ciphers to the numerator and dividing this product by the denominator, we shall have 5.00 ÷ 40 = .12½ hundredths, the per cent. gained.

Hence, when goods are purchased at one price and sold at another, we have the following rule for finding the per cent. which is gained or lost.

RULE. *Make the gain or loss the numerator of a vulgar fraction, and the first cost the denominator; this fraction will express what part the gain or loss is of the first cost; then change this fraction to hundredths; the number of hundredths will be the per cent. gained or lost. If there is a remainder, it will express the fraction of another hundredth, and must be written at the right of the number of hundredths. If the gain or loss is less than* 1 *per cent., it may be expressed by a fraction.*

15. If $300 be paid for insuring a cargo of flour from Boston to Liverpool, valued at $15000, what per cent. is the insurance? Ans. .02 per cent.

16. If the price of flour be raised from $5.75 to $6.50 a barrel, on account of the scarcity in Europe, what per cent. is the rise? Ans. .13$\frac{1}{23}$ per cent.

17. A bankrupt's debts amount to $25000; his property amounts to only $16250; what per cent. of his debts can he pay? Ans. .65 per cent.

18. A grocer bought a chest of tea at 35 cents a pound, and sold it at retail at 50 cents a pound; what per cent. did he gain? Ans. .42$\frac{6}{7}$ per cent.

19. A speculator purchased a tract of new land, for which he paid $625; he afterwards sold it for $500; what per cent. did he lose? Ans. .20 per cent.

20. If I pay $18.75 for insuring $3000 on my house against fire annually, what per cent. do I pay? Ans. .00⅝ of 1 per cent.

21. A merchant imported 5 pieces of broadcloth, which cost $4.62½ a yard, and sold it at $5.25 a yard; what per cent. profit did he make?

22. A grocer purchased 1500 pounds of coffee, for which he paid $120, and sold it at 9 cents a pound; what per cent. did he gain?

Art. 155. 23. A merchant sold a quantity of goods for $247.50, by which he gained 10 per cent. of the first cost; how much did the goods cost?

10 per cent. is $10 on $100, and $100 + 10 is $110; then $247.50 must be $\frac{110}{100}$ of the first cost; hence, the first cost must be $\frac{100}{110}$ of $247.50, and $247.50 × $\frac{100}{110}$ = $225, the first cost of the goods.

24. A mechanic sold a new house for $8550, and gained 12½ per cent. of what it cost him to build it; what did it cost to build it? Ans. $7600.

25. A merchant sold a piece of broadcloth containing 48 yards for $147.84, and lost 12 per cent.; what did the cloth cost him?

12 per cent. is $12 on $100, and $100 — $12 is $88; then $147.84 must be $\frac{88}{100}$ of the cost; hence, the cost must be $\frac{100}{88}$ of $147.84, and $147.84 × $\frac{100}{88}$ = $168, the cost of the cloth.

26. An auctioneer sold 15 shares in the Western Railroad for $1275, by which the owner lost 15 per cent.; what was the cost of the 15 shares? Ans. $1500.

27. A merchant sold a quantity of goods for $22.50 more than he gave for them, and gained 7½ per cent. of the first cost; what did the goods cost him, and for how much did he sell them?

7½ per cent. is $7½ on $100; hence, $22.50 must be $\frac{750}{10000}$ of the first cost, and the first cost must be $\frac{10000}{750}$ of $22.50, and $22.50 × $\frac{10000}{750}$ = $300, the cost of the goods. $300 + $22.50 = $322.50, the sum for which he sold them.

28. A speculator sold a building-lot for $300 more than it cost him, and gained 20 per cent.; what did he pay for the lot, and for how much did he sell it?

Ans. It cost $1500, and he sold it for $1800.

29. A trader sold a chaise for $15.50 less than it cost him, and lost 6 per cent.; what did the chaise cost, and for how much did he sell it?

30. When flour is sold at $4.62 a barrel, there is a loss of 12½ per cent.; what will be the gain or loss per cent. when the same kind of flour is sold at $5.50 a barrel?

31. A merchant sold sugar at 8 dollars a cwt., and lost 10 per cent.; what will be the gain or loss per cent. when he sells the same kind of sugar at 9 dollars a cwt.?

32. A watchmaker sold a watch for $40, and lost 12½ per cent.; for what number of dollars should he have sold the same watch to have made a profit of 12½ per cent.?

EQUALIZING OR AVERAGING THE TIME OF PAYMENTS.

Art. 156. Equalizing or averaging the time of payments is finding a *mean* time for the payment of several debts due at different times, so that no loss shall be sustained by either debtor or creditor.

Suppose A owes B $80 to be paid in 2 months, and $120 to be paid in 7 months; at what time may A pay B the whole sum of $200, and no loss of interest be sustained by either of them?

$80 × 2 = $160; hence, the int. of $80 for 2 mo. = the int. of $160 for 1 month.
$120 × 7 = $840; hence, the int. of $120 for 7 mo. = the int. of $840 for 1 month.
$200 $1000

The interest on the two debts to the times of payment is equal to the interest of $1000 for 1 month, which is found by multiplying each debt by its time of payment, and adding the products.

If the interest of $1 for 1000 months is $5, the interest of the sum of the two debts, which is $200, for $\frac{1}{200}$ of 1000 months, which is 5 months, is also $5. Therefore, 5 months is the *average* or *mean* time for the payment of the sum of the two debts.

Proof. The interest of $ 80 for 2 months is $ .80.
The interest of $120 for 7 months is $4.20.
The interest of $200 for 5 months is $5.00.

From the above illustration we obtain the following rule.

Rule. *Multiply each debt by the number of days, months, or years, between its date and the time it becomes due, and divide the sum of the products by the sum of the debts; the quotient will be the average or mean time for the payment of the sum of all the debts.*

1. William owes 20 cents to be paid in 20 days, 30 cents to be paid in 10 days, and 40 cents to be paid in 5 days; what is the average or mean time for the payment of the whole sum?

2. If I owe a man $200 to be paid in 2 months, $300 to be paid in 6 months, and $500 to be paid in 8 months, what is the average or mean time for the payment of the whole sum?

3. C owes D $1200, of which $200 is to be paid in 3 months, $400 in 6 months, and $600 in 9 months; what is the average or mean time for the payment of the whole sum? Ans. 7 months.

4. A merchant has due him $1200 to be paid in 30 days, $800 to be paid in 60 days, and $1000 to be paid in 90 days; what is the mean time for the payment of the whole sum? Ans. 58 days.

If one of the payments is due on the day from which we begin to count the time, this payment must be added in finding the sum of the payments.

5. A owes B $1800, of which sum, $600 is due at the present time, $600 will be due in 4 months, and $600 in 8 months; what will be the average or mean time of payment? Ans. 4 months.

6. A merchant purchased goods to the amount of $2000, for which he agreed to pay $500 at the time he made the purchase, $500 in 30 days, $500 in 60 days, and $500 in 90 days; he afterwards proposed to pay the whole sum at one time. What will be the average or mean time for the payment of the whole amount? Ans. 45 days.

This rule, which is universally adopted, is founded upon the supposition that the interest of the several debts which are due before the average or mean time of payment is equal to the interest of the several debts which are due after the average or mean time.

The debtor, by keeping a sum of money after it is due, gains the interest of the sum for the time he keeps it; but the loss he sustains by paying a sum of money before it is due is equal to the discount of the sum for the time he pays it before it is due, which is less than the interest; therefore, the rule is not accurately true.

The following rule will be found to be accurately true.

Suppose A owes me $103, to be paid in 6 months, and $106 to be paid in 12 months; what will be the exact mean time for the payment of the whole sum?

The present worth of $103 payable in 6 months is $100; the present worth of $106 payable in 12 months is $100. The sum of the present worth of the two debts is $200. The sum of the two debts is $209. In what time will $200 amount to $209, at 6 per cent. a year?

$209 — $200 = $9. The interest of $200 for 1 year is $12, and $9 ÷ $12 = .75 of a year or 9 months, the *exact* mean time. Hence, the following rule.

RULE. *Find the present worth of each of the debts; then find the time at which the sum of these present worths will amount to the sum of the debts. The time thus found will be the true average or mean time.*

The time at which any given principal will amount to any required sum may be found by the following rule.

RULE. *Subtract the given principal from the required amount or sum, the remainder will be the interest of the given principal for the required time; then divide this interest by the interest of the given principal for one year, the quotient will be the time required.*

7. A owes B $1976, of which $510 is to be paid in 4 months, $618 in 6 months, and the remainder in 12 months; what will be the true average or mean time for the payment of the whole amount at a single payment?

Ans. 8 months.

8. A merchant purchased goods to the amount of $2033, of which $404 is to be paid in 60 days, $609 in 90 days, and the remainder in 120 days; what will be the true average or mean time for the payment of the whole amount?

Art. 157. In finding the average or mean time for the payment of several sums due at different times, any convenient date may be taken from which to count the time.

9. Suppose a trader purchased goods as follows, on a credit of 60 days:

May 1st, 1847, purchased to the amount of $ 50.
June 1st, " " " " $150.
Aug. 1st, " " " " $200.

What was the average or mean time of payment, counting from the 1st of May?

From 1st of May to 1st payt., 60 d's, $ 50 × 60 = 3000.
" " " to 2d payt., 91 d's, $150 × 91 = 13650.
" " " to 3d payt., 152 d's, $200 × 152 = 30400.
400 47050.

47050 ÷ 400 = 117⅝ days. Hence, the average or mean time for the payment of the whole amount was 117⅝ days, counting from the 1st of May, which was August 26th, 1847.

10. Purchased goods as follows, on a credit of 4 months:

May 1st, 1846, purchased to the amount of $ 50.
" 12th, " " " " $150.
" 19th, " " " " $120.
" 27th, " " " " $100.

What was the average or mean time of payment, counting from 1st of May?

The day *from* which you compute is not to be counted, but the day *to* which you compute is always to be counted.

Art. 158. *When goods are purchased at different times, and on different terms of credit, find the time when each amount will become due, then find what will be the average or mean time for the payment of the whole amount.*

11. A merchant purchased the following bills of goods:

Feb. 1, a bill of $100 on a credit of 120 days, due June 1.
Feb. 15, a bill of $150 on a credit of 90 days, due May 16.
May 7, a bill of $ 70 on a credit of 60 days, due July 6.
Aug. 12, a bill of $ 90 on a credit of 80 days, due Oct. 31.

What will be the average or mean time of payment for the whole amount?

May 16, ...	$150.			
June 1, ...	$100	× 16	=	1600.
July 6, ...	$ 70	× 51	=	3570.
Oct. 31, ...	$ 90	× 168	=	15120.
	410			20290.

20290 ÷ 410 = 49 days, from May 16, which will carry the average or mean time of payment to July 4.

12. A merchant sold the following bills of goods:

Jan. 16th, sold a bill amounting to $350, on a credit of 4 months.

March 11th, sold a bill amounting to $475, on a credit of 6 months.

June 21st, sold a bill amounting to $500, on a credit of 8 months.

What was the average or mean time of payment?

Art. 159. When partial payments are made before the expiration of the term of credit, to find the time when the balance should be paid, so that no loss of interest shall be sustained by either debtor or creditor.

Rule. *Multiply the whole debt by the term of credit. Then multiply each payment by the time it was paid after the date of purchase, and subtract the sum of the products from the product of the whole debt multiplied by the whole term of credit. Divide the remainder by the balance due at the expiration of the term of credit; the quotient will be the time when this balance should be paid.*

13. A merchant purchased goods to the amount of $1200 on a credit of 8 months; but he offers to pay $400 in 4 months, on condition that the balance shall remain unpaid *until it* shall cancel the interest of the $400. When will the balance of $800 become due? Ans. 10 months.

PARTNERSHIP.

Art. 160. A PARTNERSHIP is an association of two or more persons for the transaction of business. Such an association is called a *firm* or *house;* and each individual of the association is called a *partner*.

The whole amount of money contributed by the firm is its capital or joint stock; and the sum contributed by each partner is his part of the joint stock.

A and B formed a partnership with a capital of $25000, of which A furnished $15000, and B $10000; at the expiration of three years they had gained $7500. What was each partner's share of the gain?

Since A furnished $\frac{15000}{25000} = \frac{3}{5}$ of the whole capital or joint stock, he must have $\frac{3}{5}$ of the gain, and $\$7500 \times \frac{3}{5} = \$4500$, A's gain. B furnished $\frac{10000}{25000} = \frac{2}{5}$ of the whole capital or joint stock, and he must have $\frac{2}{5}$ of the whole gain, and $\$7500 \times \frac{2}{5} = \$3000$, B's gain.

When the stock of each of the several partners is employed the same length of time, it is plain that each partner's share of the gain or loss must be in proportion to his share of the joint stock; hence, we have the following rule.

RULE. *Make each partner's stock the numerator of a fraction, and the whole capital or joint stock the denominator; the several fractions thus formed will express each partner's part of the joint stock; then multiply the whole gain or loss by each partner's fractional part of the joint stock; the product will be his share of the gain or loss.*

1. William and James bought a sheep. William paid $3, and James paid $2; they afterwards sold the sheep, and gained $1. What was the share of each?

2. A and B entered into partnership. A contributed $500, and B $300; at the end of one year, they had gained $160. What was the share of each?

3. A, B, and C, formed a partnership. A furnished $1200, B $1000, and C $800; they gained $360. What was each partner's share of the gain?

Ans. A's share $144. B's share $120. C's share $96.

4. A, B, C, and D, traded in partnership. A furnished $800, B $500, C $300, and D $150; by misfortune they lost $350. What must each partner sustain of the loss?

Ans. A's loss $160. B's loss $100. C's loss $60. D's loss $30.

5. Three men trade in company. The first furnishes $750, the second $600, and the third $525; they gain $325. What is each partner's share?

Ans. The first $130. The second $104. The third $91.

6. A, B, and C, purchased a ship, for which they paid $7500. A paid $2750, B paid $2500, and C the rest; in her first voyage she cleared $3500. What was each partner's share?

Art. 161. When the stock of each of the several partners is employed for different periods of time, it is plain that each partner's share of the gain or loss must be in proportion to his share of the joint stock multiplied by the time it is employed.

Illustration. A and B traded in company. A furnished $500 for 8 months; B furnished $600 for 5 months; they gained $280. What was each partner's share of the gain?

The use of $500 for 8 months is equal to the use of 8 times $500 for 1 month, or $4000 for 1 month. The use of $600 for 5 months is equal to the use of 5 times $600 for 1 month, or $3000 for 1 month. $4000 + $3000 = $7000, the whole amount of capital employed for 1 month. A furnished $\frac{4000}{7000} = \frac{4}{7}$ of the joint stock, and he must have $\frac{4}{7}$ of the gain. B furnished $\frac{3000}{7000} = \frac{3}{7}$ of the joint stock, and he must have $\frac{3}{7}$ of the gain. $280 × $\frac{4}{7}$ = $160, A's gain. $280 × $\frac{3}{7}$ = $120, B's gain.

Hence, we have the following rule.

Rule. *Multiply each partner's stock by the time it was employed; make each product the numerator of a fraction, and the sum of the several products the denominator; the several fractions thus formed will express each partner's part of the joint stock. Then multiply the whole gain or loss by each partner's fractional part of the joint stock; the product will be his share of the gain or loss.*

7. A and B hired a pasture for 6 months, for which they paid $24. A had 5 cows pastured 6 months, B had 6 cows pastured 3 months. How much should each pay?

8. C and D traded in company. C furnished $500 for 6 months, D furnished $1000 for 2 months; they gained $300. What was each partner's share of the gain?

9. A commenced business January 1, with a capital of $4500. April 1, he took B into partnership, with a capital of $5000; at the expiration of one year they had gained $660. What was each partner's share of the gain?

Ans. A's share $360. B's share $300.

10. Charles Adams, John Brown, and William Eaton formed a partnership, under the firm of Adams, Brown & Co., with a capital of $15000; of which Adams furnished $6000, Brown $5000, and Eaton $4000. At the expiration of 6 months, Adams furnished $1500 more; at the expiration of 9 months, Brown furnished $2000 more; and at the expiration of 12 months, Eaton withdrew $2000. At the end of 2 years, they found their profits amounted to $6500. What was each partner's share?

Art. 162. METHOD OF DIVIDING A BANKRUPT'S EFFECTS AMONG HIS CREDITORS.

A bankrupt is a trader, or other person, who is unable to pay his just debts.

1. A bankrupt owes A $500, B $650, and C $850; the whole amount of his property is $1500. How much can he pay on a dollar, and how much will each creditor receive?

$\$500 + \$650 + \$850 = \$2000$, the amount of his debts. And $\$1500 \div \$2000 = \$.75$ the number of cents he can pay on a dollar. $\$.75 \times \$500 = \$375.00$, the number of dollars A will receive. $\$.75 \times \$650 = \$487.50$, the number of dollars B will receive. $\$.75 \times \$850 = \$637.50$, the number of dollars C will receive. Hence, the following rule.

RULE. *Divide the amount of the bankrupt's property by the amount of his debts; the quotient will be the per cent., or what he can pay on a dollar. Then multiply the per cent., or what he pays on a dollar, by the amount due each creditor; the several products will express the several shares.*

2. A merchant died insolvent, and it was found that his debts amounted to $25000; his property was sold for $6250. How much can he pay on a dollar, and how much will D receive, to whom he owed $4000?

Ans. 25 cents on a dollar. D will receive $1000.

3. A trader failing in business, owes E $1250, F $940, G $1575, and H $1885; his effects amount to $875. How much will each of his creditors receive?

4. If the money and other property of a bankrupt amount to $3750.75, and he owes A $2650.25, B $1975.50, and C $1250.75, what per cent. can he pay, and what will each creditor receive?

ASSESSMENT OF TAXES.

Art. 163. Taxes are sums of money assessed upon the persons and property of citizens, to defray the expenses of the state, county, city, or town. Each individual, liable to be taxed, is usually assessed a specific sum, called a *poll tax*, also a specified per centage on his property.

When any required amount of taxes is to be assessed by any city or town, it will be necessary to ascertain the name of each individual liable to be taxed, and the amount of his property; also the number of individuals who pay a poll tax, and the total amount of taxable property in the city or town.

1. Suppose the city of Boston should assess a tax to the amount of $790000, and the whole amount of taxable property in the city to be valued at $130000000, the number of individuals who pay a poll tax to be 8000, and each of them to be assessed a poll tax of $1.25; what is the amount of poll tax, what per centage is the tax upon the whole amount of property in the city, and what is the amount of A's tax, who pays a poll tax, and whose property is valued at $125000?

$1.25 × 8000 = $10000, the whole amount of poll tax. $790000 — $10000 = $780000, to be assessed on the whole amount of property. $780000 ÷ $130000000 = .006 tenths of 1 per cent., or 6 mills on a dollar.

$.006 × $125000, the value of A's property = $750.00, A's tax on his property, and $750.00 + $1.25, his poll tax, = $751.25 the whole amount of A's tax. Hence, the following rule.

Rule. *Find the whole amount of poll tax by multiplying the poll tax that each individual is assessed by the number of individuals; the product will be the whole amount of poll tax.*

Subtract this amount of poll tax from the whole amount of tax to be assessed; the remainder will be the amount to be assessed on the property.

Divide this amount by the whole amount of taxable property; the quotient will be the per cent. or tax on 1 dollar. Multiply the tax on 1 dollar by the amount of each individual's taxable property; the product will be the tax on his property. Add each individual's poll tax to the tax on his property; the sum will be his whole tax.

2. B, an inhabitant of Boston, pays a poll tax for himself and son; his taxable property is valued at $25675. What is the whole amount of his tax? Ans. $156.55.

AMERICAN DUTIES.

Art. 164. Duties are taxes imposed by the government of a country upon most articles of merchandise imported from foreign countries. They are of two kinds, *ad valorem* and *specific*.

An *ad valorem* duty is a specified per cent. on the actual cost of the goods in the country from which they were imported.

A *specific* duty is a specified sum on a square yard, pound, gallon, &c.

Duties are to be paid only on the articles of merchandise, and not on the boxes, casks, bags, &c., which contain them; hence, certain deductions are to be made from their gross weight or measure, called *allowances*. These allowances are draft, tare, leakage, and breakage.

Gross weight is the whole weight of the goods, including the box, bag, &c., which contains them.

Neat weight is the weight of the goods after all allowances have been deducted.

Draft is an allowance made for waste, which is to be deducted from the gross weight, and is as follows:

On	112 lbs.				1 lb.
Above	112 lbs.	and not	exceeding	224 lbs.	2 lbs.
"	224 lbs.	"	"	336 lbs.	3 lbs.
"	336 lbs.	"	"	1120 lbs.	4 lbs.
"	1120 lbs.	"	"	2016 lbs.	7 lbs.
"	2016 lbs.				9 lbs.

Tare is an allowance for the *actual* or *supposed* weight of the cask, box, chest or bag, which contains the article of merchandise, and is to be deducted after the draft has been deducted.

Leakage is an allowance of 2 per cent., which is to be deducted from the gauge or measure of all liquids imported in casks.

Breakage is an allowance of 10 per cent. on porter, ale, and beer; and of 5 per cent. on all other liquors, when imported in bottles. A dozen bottles of the common size are estimated to contain 2½ gallons.

In deducting allowances, a fraction is disregarded, unless it exceeds one half, when it is considered a unit.

1. What is the amount of duty on an invoice of sheetings, imported from Liverpool, which cost £125. 10s. sterling, at 20 per cent. ad valorem? Ans. $121.484.

ALLIGATION.

Art. 165. Alligation is the method of finding the mean value of a mixture which is composed of several ingredients, when the price and quantity of each of the ingredients are given; also, of finding the quantity of each of several ingredients, whose prices are given, that will be required to compose a mixture of a given mean value.

Art. 166. To find the mean value of a mixture composed of several ingredients, when the price and quantity of each of the ingredients are given.

Suppose a farmer should mix 20 bushels of corn, worth 60 cents a bushel, with 20 bushels of oats, worth 30 cents a bushel; what would be the mean value of the mixture?

$.60 × 20 = $12.00, the value of 20 bushels of corn.
$.30 × 20 = $ 6.00, the value of 20 bushels of oats.

Bu. 40 = $18.00, the total value of the mixture.
$18.00 ÷ 40 = 45 cents, the mean value required.

Hence, the following rule.

Rule. *Find the value of the several ingredients by multiplying the price of each by its quantity; then divide the sum of the several products by the sum of the several quantities, the quotient will be the mean value of the mixture.*

1. A grocer mixed 2 pounds of tea, worth 50 cents a pound, with 3 pounds, worth 40 cents a pound; what was the mean value of the mixture?

2. If 4 pounds of sugar, worth 8 cents a pound, be mixed with 8 pounds, worth 10 cents a pound, what will be the value of each pound of the mixture?

3. A grocer mixed 125 pounds of sugar, worth 9 cents a pound, and 75 pounds, worth 8 cents a pound, with 50 pounds, worth 7 cents a pound; what was the mean value of each pound of the mixture? Ans. $8\frac{3}{10}$ cents.

4. A goldsmith melted together 4 ounces of gold of 15 carats fine, 5 ounces of 18 carats fine, and 6 ounces of 20 carats fine; what was the fineness of the mixture?
Ans. 18 carats fine.

Note. A carat is the twenty-fourth part of any quantity of gold, and the fineness of gold depends upon the number of carats of pure gold it contains. Thus, gold that is 18 carats fine, is composed of 18 parts of pure gold, and 6 parts of some other metal. The fineness of silver depends upon the number of ounces of pure silver in a pound.

Art. 167. To find the quantity of each of several ingredients, whose prices are given, that will be required to compose a mixture of a given mean value.

Suppose we wish to mix tea that is worth 60 cents a pound, with tea that is worth only 40 cents a pound, in such proportions that the mixture shall be worth 50 cents a pound; it is plain that we must take equal quantities of each, because the price of one kind is as much greater than the mean price, as the price of the other is less.

Again; suppose we wish to mix sugar that is worth 7 cents a pound, with sugar that is worth 10 cents a pound, in such proportions that the mixture shall be worth 9 cents a pound.

The sugar that is worth 10 cents a pound is worth 1 cent more than the mean price, and the sugar that is worth 7 cents a pound is worth 2 cents less than the mean price; hence, it will be necessary to use twice as much sugar at 10 cents a pound as we use at 7 cents a pound.

A goldsmith has gold of 12, 15, 20, and 22 carats fine, and he wishes to mix the four kinds in such proportions that the mixture shall be 18 carats fine.

The proportion of each kind of gold may be found by writing the fineness of the several kinds under one another, and then connecting one that is less than the mean fineness with one that is greater, and writing the difference between the fineness of each kind and the required mean fineness of the mixture, opposite to that with which it is connected; these differences will express the proportion or quantity of each kind.

Thus,	12	4 parts of 12 carats fine.
Mean fineness 18	15	2 parts of 15 carats fine.
	20	3 parts of 20 carats fine.
	22	6 parts of 22 carats fine.

By connecting a less rate of fineness with a greater, and placing the differences between them and the mean rate alternately, there is precisely as much gained by one quantity as is lost by the other; therefore the gain and the loss upon the whole must be equal.

From the preceding illustrations we obtain the following

Rule. *Write the rates or prices of the ingredients in a column under one another, and place the mean rate or price at the left of the column. Connect with a continued line, the price of each ingredient which is less than the mean rate* *or price, with one or more that is greater; then write the difference between the price of each ingredient and the mean rate, opposite the price of the ingredient with which it is connected, and the difference, or the sum of the differences, if there is more than one opposite the price of each ingredient, will be the required quantity of that ingredient.*

5. A grocer would mix tea at 35 cents, 40 cents, 50 cents, and 55 cents a pound, in such proportions that the mixture shall be worth 45 cents a pound; how many pounds of each kind must he use?

Ans. 10 lbs. at 35 cents, 5 lbs. at 40 cents, 5 lbs. at 50 cents, and 10 lbs. at 55 cents.

Questions of this kind admit of as many different answers as there are various ways of connecting the rates or prices of the ingredients, and all of them equally correct, which may be proved by Art. 165.

Art. 168. WHEN ONE OF THE INGREDIENTS IS LIMITED TO A GIVEN QUANTITY.

RULE. *Find the proportion of each ingredient, as in Art.* 167; *then make the number opposite that ingredient whose quantity is given, a common denominator, and the number opposite each of the other ingredients, the numerator of a fraction; multiply the given quantity by each of these fractions, the several products will express the required quantities of the other ingredients.*

6. A trader has 90 pounds of sugar worth 7 cents a pound, which he would mix with some at 8 cents a pound, some at 10 cents, and some at 12 cents a pound. How much of each kind must he mix with the 90 pounds to make a mixture worth 9 cents a pound?

Ans. 30 lbs. at 8 cts., 30 lbs. at 10 cts., and 60 lbs. at 12 cents.

Art. 169. WHEN THE WHOLE COMPOUND IS LIMITED TO A GIVEN QUANTITY.

RULE. *Find the proportional part of each ingredient as in Art.* 167; *then make the sum of these proportional parts a common denominator, and the proportional part of each ingredient the numerator of a fraction; multiply the given quantity by each of these fractions; the several products will express the required quantity of each ingredient.*

7. A grocer has three kinds of sugar, at 7, 8, and 10 cents a pound; what quantity of each kind must he take to fill a cask holding 230 pounds, so that the mixture shall be worth 9 cents a pound?

INVOLUTION.

Art. 170. Involution is multiplying a number by itself.

The number itself is called the first power, or root, and the product obtained by multiplying any number by itself is called the second power, or square of that number. If the second power or square be multiplied by the first power, the product is called the third power, or cube of that number.

The power of a number is sometimes indicated by a small figure placed at the right and a little above the number, called the exponent or index, and it shows how many times the number is to be used as a factor to produce the required power.

Thus, $5^2 = 5 \times 5 = 25$, the second power of 5.
$5^3 = 5 \times 5 \times 5 = 125$, the third power of 5.
$5^4 = 5 \times 5 \times 5 \times 5 = 625$, the fourth power of 5.

Any required power of a number may be found by continually multiplying each succeeding higher power by the given number until the required power is obtained.

Any required power of a fraction may be found in a similar manner; thus, the second power of $\frac{3}{4}$ is $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$; the third power is $\frac{9}{16} \times \frac{3}{4} = \frac{27}{64}$. The second power of the decimal fraction .3 is $.3 \times .3 = .09$; the third power is $.09 \times .3 = .027$.

If the power of a mixed number be required, reduce it to an improper fraction, or reduce the fractional part to a decimal, and then involve it to the required power.

1. What is the second power of 2?
2. What is the third power of 2?
3. What is the second power of 3?
4. What is the third power of 3?
5. What is the second power of 4?
6. What is the third power of 4?
7. What is the second power of 5?
8. What is the third power of 5?
9. What is the second power of 6?
10. What is the third power of 6?
11. What is the second power of 7?
12. What is the third power of 7?
13. What is the second power of 8?
14. What is the third power of 8?
15. What is the second power of 9?
16. What is the third power of 9?
17. What is the second power of 10?
18. What is the third power of 10?

EVOLUTION.

Art. 171. Evolution is the reverse of involution. It explains the method of resolving a number into equal factors, which factors are called *roots.*

When a number is resolved into two equal factors, each of these factors is called the *square root* of the number.

When a number is resolved into three equal factors, each of these factors is called the *cube root* of the number.

Any required power of a number may be found, but there are numbers whose exact root cannot be obtained; yet by the use of decimals, an approximate root may be obtained sufficiently exact for all practical purposes.

A number whose exact root cannot be found is called a *surd number*, and its approximate root is called a *surd root.*

The square root of a number is indicated by this character $\sqrt{}$ placed before the number; the other roots are indicated by the same character with the index of the root placed over it, or by the fractional index placed at the right.

The square root of 64 is expressed thus, $\sqrt{64}$, or $64^{\frac{1}{2}}$; and the cube root of 64, thus, $\sqrt[3]{64}$, or $64^{\frac{1}{3}}$.

When the power is expressed by several numbers, with the sign $+$ or $-$ between them, a line, called a vinculum, is drawn from the top of the sign over all the numbers; thus the square root of $\sqrt{25+11}$ is 6; the cube root of $\sqrt[3]{64-27+88}$ is 5.

EXTRACTION OF THE SQUARE ROOT.

Art. 172. The square root of a number is one of its two equal factors.

The extraction of the square root of a number is the method of finding one of its two equal factors.

The second powers or squares of the first twelve integral numbers are exhibited in the following table.

1st powers or roots.	1	2	3	4	5	6	7	8	9	10	11	12
2d powers or square numbers.	1	4	9	16	25	36	49	64	81	100	121	144

1. What is the square root of 1?

2. What is the square root of 4?

3. What is the square root of 9?

4. What is the square root of 16?

5. What is the square root of 25?

6. What is the square root of 36?

7. What is the square root of 49?

8. What is the square root of 64?

9. What is the square root of 81?

10. What is the square root of 100?

11. What is the square root of 121?

12. What is the square root of 144?

The second power of each of the numbers 1, 2, and 3, is expressed by a single figure. The second power of each of the numbers 4, 5, 6, 7, 8, and 9, is expressed by two figures. The second power of each of the numbers 10, 11, and 12, is expressed by three figures. Hence, the second power of any given number can never contain more than twice its number of figures, and never but one less than twice its number.

Therefore, we can determine the number of figures that the square root of any number will contain, by dividing the given number into periods of two figures each, by points, counting from the right; the period on the left may contain one or two figures. The root will contain as many figures as there are periods.

We will now find the second power of 48, and then extract its root. 48 × 48 = 2304. The number 48 is composed of 4 tens or 40, plus 8 units.

We will find the second power of 40 + 8, the several products will exhibit its component parts.

	40 + 8.
	40 + 8.
Product of 40 + 8 × 8	320 + 64.
Product of 40 + 8 × 40	1600 + 320
Second power of 40 + 8	1600 + 640 + 64.

By examining the several component parts of the second power of 40 + 8, we perceive that the first part, 1600, is the square of the tens, or the product of the tens multiplied by the tens; hence, each of its equal factors must be the tens, or 40, and 1600 ÷ 40 = 4 tens or 40, which is the first part of the root.

We see that the second part, 640, is twice the product of the tens multiplied by the units; hence, its greater factor must be twice the tens, or 80, and 640 ÷ 80 = 8, the less factor or the units.

We see, also, that the third part, 64, is the square of the units, or the product of the units multiplied by the units; hence, each of its equal factors must be the units, or 8; and 64 ÷ 8 = 8, the units.

Moreover, we see that 80 + 8, or 88, must be the greater factor of 640 + 64, or 704, and 704 ÷ 88 = 8, the less factor, or the units, which is the second part of the root. Therefore, the whole root of 1600 + 640 + 64, is 40 + 8, or 48.

We may consider every number greater than 9, whose right hand figure is not a cipher, to be composed of tens and units, because all the figures which compose any number, except its right hand figure, express tens; hence, we infer that the second power of every number composed of tens and units must contain the square of the tens, plus twice the product of the tens multiplied by the units, plus the square of the units.

We will now reverse the process of finding the second power of 40 + 8, and we shall obtain one of its two equal factors or root.

Divisor 40)1600 + 640 + 64(40 + 8 = 48, the root.
1600

Divisor completed 80 + 8)640 + 64
640 + 64

As 1600 is the square of the tens, we find by trial that one of its two equal factors is 40, which we write at its left for a divisor; we also place it at the right for the first part of the root. We then multiply the divisor by the first part of the root, and subtract the product from 1600 + 640 + 64; the remainder is 640 + 64, which is a new dividend.

As 640 contains twice the product of the tens multiplied by the units, twice 40, or 80, must be the greater factor of 640, which we write at its left for a trial divisor; this trial divisor is contained 8 times in 640, which is its less factor, and it is also one of the two equal factors of 64. We write this factor 8 for the second part of the root, and also at the right of the trial divisor; the sum is the divisor completed. We then multiply this completed divisor by 8, the product is 640 + 64, which we write under the dividend; this product being equal to the dividend, the operation is completed, and the root is 40 + 8 = 48.

We will now find one of the two equal factors, or the square root, of 2304, by a shorter process.

Divisor 4)2304(48, the square root.
16

Trial divisor 80)704, new dividend.
Less factor 8)
Divisor completed 88)704

We first divide 2304 into periods of two figures each, by placing a point over the units, and another over the hundreds; the number of periods always indicating the number of figures that there will be in the root.

The first period on the left is 23, (hundred) which must contain the square of the tens, because tens multiplied by tens produce hundreds. The greatest square number contained in 23 (hundred) is 16, (hundred) and each of its two equal factors is 4, (tens) one of which we write at the left for a divisor, and the other on the right, for the first figure of the root. We then multiply the divisor by the figure placed in the root, and subtract the product from the first period, the remainder is 7, (hundred) we then annex the next period of figures to this remainder, and we have 704 for a new dividend.

This dividend contains twice the product of the tens multiplied by the units, plus the square of the units. Twice 4, (tens) the root already obtained, is 8, (tens) which is the greater factor of twice the product of the tens multiplied by the units; we then write the 8 (tens) with a cipher annexed at the left of the new dividend, for a trial divisor; this trial divisor is contained 8 times in the dividend; then 8 must be the less factor of twice the product of the tens multiplied by the units, and it must also be one of the two equal factors of the square of the units; we then write the 8 at the right of the 4 (tens) in the root, and also under the trial divisor; the sum of the trial divisor and this less or equal factor is the divisor completed; we then multiply this completed divisor, 88, by the figure last placed in the root, the product is 704, which we write under the dividend; this product being equal to the dividend, the operation is completed, and we have found one of the two equal factors or square root of 2304 to be 48.

From the preceding illustrations, we obtain the following rule for extracting the square root of numbers.

Rule. *Divide the given number into periods of two figures each, by placing a point over the units, and another over every second figure at the left; also, over every second figure at the right, when the number contains decimals.*

Find, by trial, the greatest square number in the first period on the left, and write one of its two equal factors at the left for a divisor, and the other on the right for the first figure of the root; then multiply this divisor by the figure in the root, subtract the product from the first period, and annex the next period of figures to the remainder for a new dividend.

Write twice the number of the root already found with a cipher annexed at its left for a trial divisor; then find the number of times this trial divisor is contained in the new dividend, and write a figure expressing the number for the second figure of the root, and write it also under the new trial divisor; add this figure to the trial divisor, their sum is the divisor completed.

Multiply this completed divisor by the figure last placed in the root, subtract the product from the dividend, and annex the next period of figures to the remainder for a new dividend.

Write twice the number of the root already found, with a cipher annexed at the left of the new dividend, for a new trial divisor; then repeat each of the successive steps of the previous operation, and thus continue until the required root is obtained.

If there should be a remainder after the root of all the periods has been extracted, the operation may be continued by annexing periods of ciphers.

1. What is the square root of 729? Ans. 27.
2. Find the square root of 4096. Ans. 64.
3. Extract the square root of 15625. Ans. 125.
4. What is the square root of 46656? Ans. 216.
5. Find the square root of 117649. Ans. 343.
6. Extract the square root of 262144. Ans. 512.
7. What is the square root of 531441? Ans. 729.
8. Find the square root of 1048576. Ans. 1024.
9. Extract the square root of 9765625. Ans. 3125.
10. What is the square root of 60466176? Ans. 7776.
11. Find the square root of 282475249. Ans. 16807.
12. Extract the square root of 3486784401. Ans. 59049.
13. What is the square root of 17.3056? Ans. 4.16.
14. Extract the square root of 373. Ans. 19.3132079+.
15. What is the square root of 3.15? Ans. 1.7748239+.
16. What is the square root of 8.93? Ans. 2.9883105+.
17. What is the square root of 3648764025?
18. What is the square root of 5047534116?
19. What is the square root of 7404086209?
20. What is the square root of 8245911249?
21. What is the square root of 651353914225?
22. What is the square root of 884867574276?
23. What is the square root of 97546105558146624?
24. What is the square root of 204397348660.5625?
25. What is the square root of 162425233245.6196?
26. What is the square root of 415067140056.4175?
27. What is the square root of 976514574641.1416?

Art. 173. To EXTRACT THE SQUARE ROOT OF A VULGAR FRACTION.

Reduce the fraction to its lowest terms, then extract the square root of the numerator and denominator, if each of them is a square number; if not, reduce the vulgar fraction to a decimal and then extract its root.

28. What is the square root of $\frac{144}{15129}$? Ans. $\frac{4}{41}$.
29. What is the square root of $\frac{343}{1008}$? Ans. $\frac{7}{12}$.
30. What is the square root of $42\frac{1}{4}$? Ans. $6\frac{1}{2}$.
31. What is the square root of $\frac{3}{5}$? Ans. .774+.

EXTRACTION OF THE CUBE ROOT.

Art. 174. We have before shown, Art. 170, that the third power of any number is the product of its three equal factors; hence, extracting the cube root of the third power of any number is the method of finding *one* of its three equal factors: consequently, the second power or square of one of its equal factors must be its greater factor or divisor.

The second and third powers of the first twelve integral numbers are exhibited in the following table.

1st powers or roots.	1	2	3	4	5	6	7	8	9	10	11	12
2d powers or squares.	1	4	9	16	25	36	49	64	81	100	121	144
3d powers or cubes.	1	8	27	64	125	216	343	512	729	1000	1331	1728

1. What is the cube root of 1?
2. What is the cube root of 8?
3. What is the cube root of 27?
4. What is the cube root of 64?
5. What is the cube root of 125?
6. What is the cube root of 216?
7. What is the cube root of 343?
8. What is the cube root of 512?
9. What is the cube root of 729?
10. What is the cube root of 1000?
11. What is the cube root of 1331?
12. What is the cube root of 1728?

The third power of each of the numbers 1 and 2 is expressed by a single figure; the third power of each of the numbers 3 and 4 is expressed by two figures; and the third power of each of the numbers 5, 6, 7, 8, and 9, is expressed by three figures. The third power of each of the numbers 10, 11, and 12, is expressed by four figures. *Consequently*, the third power of any given number can never contain more than three times its number of figures, and never but two less than three times its number.

Therefore, we can determine the number of figures that the cube root of any number will contain, by separating the given number into periods of three figures each, by points, counting from the right; the period on the left may contain one, two, or three figures. The root will contain as many figures as there are periods.

We will find the third power of 25, and then extract its root. $25 \times 25 \times 25 = 15625$. The number 25 is composed of 2 tens, or 20, plus 5 units.

We will find the third power of $20+5$, and the several products will exhibit its component parts.

			20 +	5
			20 +	5
Product of 20 + 5 × 5,			100 +	25
Product of 20 + 5 × 20,		400 +	100	
The second power or square of 20 + 5,		400 +	200 +	25
			20 +	5
Product of 400 + 200 + 25 × 5,		2000 +	1000 +	125
Product of 400 + 200 + 25 × 20,	8000 +	4000 +	500	
The third power or cube of 20 + 5,	8000 +	6000 +	1500 +	125

By examining the several component parts of the third power of 20 + 5, we perceive that the first part, 8000, is the cube of the tens, or the product of the square of the tens multiplied by the tens; consequently, its greater factor or divisor must be 20 × 20, or 400, and 8000 ÷ 400 = 20, the less factor, or the tens, which is the first part of the root.

The second part, 6000, is the product of three times the square of the tens multiplied by the units; therefore, its greater factor or divisor must be 20 × 20 × 3, or 1200, and 6000 ÷ 1200 = 5, the less factor, or the units.

The third part, 1500, is the product of three times the tens multiplied by the square of the units; hence, its greater factor or divisor must be 20 × 3 × 5, or 300, and 1500 ÷ 300 = 5, the less factor, or the units.

The fourth part, 125, is the cube of the units, or the product of the square of the units multiplied by the units; then its greater factor or divisor must be 5 × 5, or 25, and 125 ÷ 25 = 5, the less factor, or the units.

Moreover, 1200 + 300 + 25, or 1525, must be the greater factor, or divisor completed, of 6000 + 1500 + 125, or 7625; and 7625 ÷ 1525 = 5, the less factor, or the units, which is the second part of the root.

The whole root of 8000 + 6000 + 1500 + 125 is 20 + 5, or 25.

By examining the component parts of the third power of 20 + 5, we perceive that it is composed of the cube of the tens, plus three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.

We will now reverse the process of finding the third power of 20 + 5, and we shall obtain one of its three equal factors, or root.

$$\begin{array}{l} 20 \times 20 = 400\,)8000 + 6000 + 1500 + 125(\,20 + 5 = 25, \text{ the root} \\ \qquad\qquad\quad\;\; 8000 \end{array}$$

$$\left.\begin{array}{r} 20 \times 20 \times 3 = 1200 \\ 20 \times \;\; 3 \times 5 = \;\; 300 \\ 5 \times 5 = \quad 25 \end{array}\right\} \begin{array}{l} 6000 + 1500 + 125 \\ 6000 + 1500 + 125 \end{array}$$

Divisor completed, = 1525

As 8000 is the product of the second power of the tens multiplied by the tens, we find, by trial, that the greatest second power or square number in 8000 is 400, which we write at the left of 8000, for a divisor, and its first power, 20, on the right, for the first part of the root. We then multiply 400 by 20, and subtract the product, 8000, from 8000 + 6000 + 1500 + 125, the remainder is 6000 + 1500 + 125, which is the product of three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the square of the units multiplied by the units; this remainder we call a dividend.

We next find a divisor for each part of the dividend, and write it at the left, and then find the number of times each divisor is contained in its appropriate part of the dividend, and write a figure expressing the number on the right, for the second part of the root. Lastly, we multiply each divisor by this figure, and write each product under its appropriate part of the dividend, these several products being equal to the several parts of the dividend, the operation is completed, and the whole root is 20 + 5 = 25.

We will now find one of the three equal factors, or cube root, of 15625, by a shorter process.

$$\begin{array}{rrl}
2 \times 2 = & 4\,) & 1\dot{5}62\dot{5}\,(\,25\text{, the root.} \\
 & & \;\;8 \\
\cline{3-3}
2 \times 2 \times 3 = & 1200\,) & 7625\text{, new dividend.} \\
\cline{3-3}
 & \left.\begin{array}{r} \\ 300 \\ 25 \end{array}\right\} & \begin{array}{r} 6000 \\ 1500 \\ 125 \end{array} \\
\cline{2-3}
\text{Completed Divisor, } & 1525 & 7625
\end{array}$$

with the left-hand products: $2 \times 3 \times 5 = 300$, $5 \times 5 = 25$.

We first place a point over the units, and another over the thousands; the two periods indicate the number of figures there will be in the root.

The period on the left is 15, (thousand) which must contain the cube of the tens, or the square of the tens multiplied by the tens, because the square of tens is hundreds, and hundreds multipled by tens produce thousands.

We find, by trial, the greatest third power or cube in 15, (thousand) and write its first power, 2, (tens) on the right, for the first figure of the root, and its second power or square, 4, (hundred) on the left, for a divisor; we multiply this divisor by the 2 (tens) in the root, and subtract the product from the 15, (thousand) the remainder is 7; (thousand) we then annex the next period of figures to the remainder for a new dividend.

As the first part of this dividend is the product of three times the square of the tens of the root multiplied by the units of the root, we find three times the square of the 2 tens of the root, which we write with two ciphers annexed, at the left of the dividend. for a trial divisor. This trial divisor is contained 6 times in the dividend, but this trial divisor being only a part of the whole divisor when completed, we write 5 for the second or unit figure of the root, and then multiply the trial divisor by this figure, and write the product under the dividend.

As the second part of the dividend is the product of three times the tens of the root multiplied by the square of the units of the root, we find three times the 2 tens of the root, which are 6 tens, and five times 6 tens are 30 tens, which we write, with a cipher annexed, under the trial divisor, and then multiply it by the 5 units of the root, and write the product under the preceding product.

As the third part of the dividend is the cube of the units of the root, or the product of the square of the units multiplied by the units, we find the square of the 5 units to be 25 units, which we write under the second part of the divisor, and then multiply it by the units of the root, and write the product under the two preceding products; the sum of these three products being equal to the dividend, the operation is completed, and the whole root is obtained.

If we multiply the completed divisor by the 5 units of the root, the product will be equal to the dividend.

From the preceding illustrations, we derive the following

RULE. *Divide the given number into periods of three figures each, by placing a point over the units, and another over every third figure at the left; also, over every third figure at the right, when the number contains decimals.*

Find, by trial, the greatest third power, or cube number, in the first period on the left; write its first power or root at the right, and the second power of this root at the left, for a divisor; multiply this divisor by the figure in the root, subtract the product from the first period, and annex the figures of the second period to the remainder, for a new dividend.

Find three times the square of the root, and write it with two ciphers annexed at the left of the dividend, for a trial divisor. Find the number of times this trial divisor is contained in the dividend, and write the number for the second figure of the root.

Find three times the tens of the root, multiply it by the units of the root, and write the product, with one cipher annexed, under the trial divisor. Find the square of the units of the root, and write the square under the preceding product. Find the sum of these three parts of the divisor, their sum is the divisor completed.

Multiply the divisor thus completed, by the figure last placed in the root; subtract the product from the dividend, and annex the figures of the third period to the remainder, for a new dividend.

Find three times the square of the whole root, and write it with two ciphers annexed at the left of the new dividend, for a new trial divisor, and repeat the successive steps of the previous operation, and thus continue, until the required root is obtained.

13. What is the cube root of 19683? Ans. 27.
14. Find the cube root of 262144. Ans. 64.
15. Extract the cube root of 1953125. Ans. 125.
16. What is the cube root of 10077696? Ans. 216.
17. Find the cube root of 40353607. Ans. 343.
18. What is the cube root of 52734375? Ans. 375.
19. What is the cube root of 64964808? Ans. 402.
20. What is the cube root of 68921000? Ans. 410.
21. What is the cube root of 76765625? Ans. 425.
22. What is the cube root of 97336000? Ans. 460.
23. What is the cube root of 121287375? Ans. 495.
24. What is the cube root of 125000000? Ans. 500.
25. What is the cube root of 136590875? Ans. 515.
26. What is the cube root of 140608000? Ans. 520.
27. What is the cube root of 153990656? Ans. 536.
28. What is the cube root of 205379000? Ans. 590.
29. What is the cube root of 216000000? Ans. 600.
30. What is the cube root of 257259456? Ans. 636.
31. What is the cube root of 343000000? Ans. 700.
32. What is the cube root of 411830784? Ans. 744.
33. What is the cube root of 460099648? Ans. 772.
34. What is the cube root of 512000000? Ans. 800.
35. What is the cube root of 561515625? Ans. 825.
36. What is the cube root of 667627624?
37. What is the cube root of 716917375?
38. What is the cube root of 729000000?
39. What is the cube root of 870983875?
40. What is the cube root of 8552241125?
41. What is the cube root of 17073859375?
42. What is the cube root of 43169672512?
43. What is the cube root of 65890311319?
44. What is the cube root of 253395799552?
45. What is the cube root of 178263.433152?
46. What is the cube root of 35655654571?

Art. 175. To EXTRACT THE CUBE ROOT OF A VULGAR FRACTION.

RULE. *Reduce the fraction to its lowest terms, then extract the cube root of the numerator and denominator, if each of them is a perfect third power; if not, reduce the vulgar fraction to a decimal, and then extract its root.*

47. What is the cube root of $\frac{125}{343}$? Ans. $\frac{5}{7}$.
48. What is the cube root of $\frac{1520}{5130}$? Ans. $\frac{2}{3}$.
49. What is the cube root of $\frac{2}{3}$? Ans. .873+.

ARITHMETICAL PROGRESSION.

Art. 176. An Arithmetical Progression is a series of numbers which *increase* or *decrease*, continually, by a common difference.

When the numbers increase by the continual addition of the common difference, they form an ascending series; thus, 2, 4, 6, 8, 10, 12, is an ascending series.

When the numbers decrease by the continual subtraction of the common difference, they form a descending series; thus, 12, 10, 8, 6, 4, 2, is a descending series.

The numbers which compose the series are called the *terms* of the series. The first and last terms are called the *extremes*, and the other terms, the *means*.

It is plain that any required term of a series may be found by continued addition or subtraction, but this process would be very tedious when a very remote term is required.

There is a method much more expeditious.

Suppose a parent has 9 children whose ages form a series of numbers in arithmetical progression. The age of the youngest is 3 years, and the common difference of their ages is 2 years; what is the age of the eldest? In other words, what is the last term of an arithmetical series, whose first term is 3, the common difference 2, and the number of terms 9?

$$3+2=5+2=7+2=9+2=11+2=13+2=15+2=17+2=19.$$

By examining this series, we perceive that the common difference, 2, is added 8 times, which is one less than the number of terms, and that the last term, 19, is 8 times the common difference greater than the first term; therefore, when the first term, the common difference, and the number of terms, are given to find the last term, we have the following rule.

Rule. *Multiply the common difference by the number of terms less one, add the first term to the product, the sum will be the required term.*

1. The first term of an arithmetical series is 5, the common difference 5, and the number of terms 5; what is the fifth term?

2. Suppose the first term of an arithmetical series to be 12, and the common difference 12, what is the twelfth term of the series?

3. A owes B a sum of money, which he agrees to pay in 52 weekly payments; the first payment is to be 5 dollars; the second, 10 dollars; the third, 15 dollars; increasing each succeeding payment 5 dollars. What will be the amount of the last payment?

Ans. $260.

4. Suppose 1 dollar to be let on simple interest at 6 per cent.; what would be the amount in 20 years? Ans. $2.20.

The yearly amounts of any sum at simple interest form an arithmetical series, of which the principal is the *first term*, the yearly *interest* is the *common difference*, and the number of years is *one less than the number of terms*.

Art. 177. Since the last term of an ascending arithmetical series is equal to the common difference multiplied by one less than the number of terms, *plus* the first term, it follows, that if the first term be subtracted from the last term, the remainder will be equal to the product of the common difference multiplied by the number of terms *less* 1. Hence, if the extremes and number of terms of an arithmetical series are given, we have the following rule for finding the common difference.

RULE. *Subtract the less extreme from the greater; divide the remainder by one less than the number of terms; the quotient will be the common difference.*

1. Suppose the first term of an arithmetical series to be 5, the last term 60, and the number of terms 12; what is the common difference?

2. Suppose the first term of an arithmetical series to be 12, the last term 144, and the number of terms 12; what is the common difference?

3. A man purchased a house, and agreed to pay for it in 52 weekly payments. The first payment was to be 5 dollars, and each succeeding payment was to be uniformly increased, so that the last payment should be 260 dollars. What was the difference between any two succeeding payments? Ans. $5.

4. Suppose that 100 dollars, which has been on simple interest 20 years, amounts to $220; what was the rate per cent. or yearly interest? Ans. 6 per cent. or $6.

5. A merchant purchased 100 yards of broadcloth; for the first yard he gave 5 cents, and for the last yard 500 cents; what was the common difference, or increase of price, on each succeeding yard? Ans. 5 cents.

Art. 178. The sum of the two extremes of any series of numbers in arithmetical progression, is equal to the sum of any two means which are equally distant from the extremes, and to twice the middle term, when the series is composed of an odd number of terms. We will take the series 2, 4, 6, 8, 10, 12, 14. The extremes $2 + 14 = 16$; $4 + 12 = 16$; $6 + 10 = 16$; and $8 \times 2 = 16$.

Suppose we wish to find the sum of all the terms in a series, whose first term is 4, the common difference 2, and the number of terms 12.

$$\begin{array}{cccccccccccc}
4, & 6, & 8, & 10, & 12, & 14, & 16, & 18, & 20, & 22, & 24, & 26. \\
26, & 24, & 22, & 20, & 18, & 16, & 14, & 12, & 10, & 8, & 6, & 4. \\
\text{—} & \text{—} & \text{—} & \text{—} & \text{—} & \text{—} & \text{—} & \text{—} & \text{—} & \text{—} & \text{—} & \text{—} \\
30, & 30, & 30, & 30, & 30, & 30, & 30, & 30, & 30, & 30, & 30, & 30.
\end{array}$$

The first row of numbers above represents the given series; the second, the same series in an inverted order; and the third, the sums of the corresponding terms of the two series.

By examining these series, we find that the sums of the corresponding terms are the same; and that each of them is equal to the sum of the extremes, viz., 30. Now, as there are 12 of these pairs of terms in the two series, the sum of all the terms in both must be 30 × 12 = 360. But it is evident, that the sum of all the terms in one series can be only one half the sum of both; therefore, if we divide 360 by 2, the quotient will be the sum of all the terms in one series, which is the thing required. And 360 ÷ 2 = 180, the sum of the given series.

From the above illustration, we derive the following rule for finding the sum of all the terms of a series of numbers in arithmetical progression, when we have the extremes and number of terms given.

Rule. *Multiply the sum of the extremes by the number of terms, and divide the product by 2, the quotient will be the sum of all the terms.*

1. The extremes of an arithmetical series are 5 and 25, the number of terms 5; what is the sum of the series?

2. The extremes of an arithmetical series are 12 and 144, the number of terms 12; what is the sum of the series?

3. A man purchased a farm, and agreed to pay for it in 52 successive monthly payments; the first payment was 5 dollars, and each succeeding payment was uniformly increased, until the last payment amounted to 260 dollars. What did he pay for his farm?

Ans. $6890.

4. Suppose a man rents a farm for $100 a year, payable annually, what sum will the rent amount to in 20 years, allowing simple interest at 6 per cent. for the use of the money?

The last year's rent will evidently be $100 without interest; the last but one will be the amount of $100 for 1 year = $106; the last but two, the amount of $100 for 2 years; and so on in an arithmetical series to the first, which will be the amount of $100 for 19 years = $214. Ans. $3140.

5. A man purchased 100 acres of land, paying 10 cents for the first acre, 20 cents for the second, and so on, — the price of each succeeding acre exceeding the price of the preceding one 10 cents; what was the price of the 100th acre? What did he pay for the 100 acres?

6. A man travelled 20 days in succession, increasing each succeeding day's travel the same number of miles. The first day he travelled 15 miles, and the 20th day he travelled 72 miles. What was the daily increase, and what distance did he travel in the 20 days?

7. Suppose a number of dollars were placed on a railroad at a distance of 10 rods from each other, over the space of 5 miles, and the first 10 rods from a depot; what distance must a boy travel, who, starting from the depot, should go and pick them up one at a time, returning to the depot with each single dollar and depositing it in a bag?

Every series of numbers in arithmetical progression consists of the following component parts, viz., the first term, the last term, the common difference, the number of terms, and the sum of all the terms; any three of which being given, the other two may be found.

GEOMETRICAL PROGRESSION.

Art. 179. GEOMETRICAL PROGRESSION is a series of numbers which increase continually by a common multiplier, or decrease continually by a common divisor.

Thus, the series 2, 4, 8, 16, 32, 64, 128, is composed of numbers each of which is *twice* the preceding; this is an *increasing* or *ascending* geometrical series.

The series 128, 64, 32, 16, 8, 4, 2, is composed of numbers each of which is one half of the preceding; this is a *decreasing* or *descending* geometrical series.

The *common* multiplier or divisor is called the *ratio*, and the numbers which compose the series are called the *terms* of the series.

The component parts of a series of numbers in geometrical progression are the *first* term, the *last* term, the *ratio*, the *number* of terms, and the *sum* of all the terms; any three of which being given, the other two can be found.

In a geometrical series, the product of the extremes is equal to the product of any two means which are equally distant from the extremes, and to the second power of the middle term, when the number of terms is odd. In the series 2, 4, 8, 16, 32; $2 \times 32 = 64$; $4 \times 16 = 64$; and $8 \times 8 = 64$.

1. A lady wishing to purchase a dress, thought 12 dollars too great a price to pay for 10 yards of silk; but she agreed to give what the 10th yard would cost, calling the price of the first yard 5 cents, the second 10 cents, the third 20 cents, and so on, doubling the price of each succeeding yard to the last. What did the lady pay for the 10th yard?

$$\overset{c.}{5} \times 2 = \overset{c.}{10} \times 2 = \overset{c.}{20} \times 2 = \overset{c.}{40} \times 2 = \overset{c.}{80} \times 2 = \overset{c.}{160} \times 2 = \overset{c.}{320} \times 2 = \overset{c.}{640} \times 2 = \overset{c.}{1280} \times 2 = \overset{c.}{2560}$$ cents, or \$25.60. Ans.

From the above operation we obtain the following rule for finding any required term of an increasing geometrical series, when the first term and ratio are given.

RULE. *Multiply the first term by the ratio, then multiply the product by the ratio, and continue to multiply each succeeding product by the ratio until the required term is obtained.*

2. What is the 8th term of an ascending geometrical series, whose first term is 4, and ratio 3?

$4 \times 3 = 12 \times 3 = 36 \times 3 = 108 \times 3 = 324 \times 3 = 972 \times 3 = 2916 \times 3 = 8748$, the 8th term.

By examining the above process, it will be seen that the ratio, 3, is used as a factor 7 times, or once less than the number of terms. Hence, it is plain that the 8th term may be found by raising the ratio, 3, to the 7th power, and then multiplying the first term, 4, by this 7th power. Hence, the following rule.

RULE. *Raise the ratio to a power whose index is* 1 *less than the required term; then multiply the first term by this power of the ratio; the product will be the required term.*

In involving the ratio, the operation may be shortened, by multiplying one power by another. Thus, the 4th power of 2 is 16, and $16 \times 16 = 256$, the 8th power of 2.

3. Suppose a man had put out one cent at compound interest in 1620, what will be the amount in 1848, allowing it to double once in 12 years? Ans. $5242.88.

4. A merchant bought a piece of silk measuring 20 yards, and by agreement was to give what the last yard would come to, reckoning a cent for the first yard, 2 cents for the second, and so on, doubling the price of each succeeding yard, to the last; what did the piece of silk cost him?

Ans. $5242.88.

Art. 180. We at times wish to find the sum of all the terms of a geometrical series, and it is obvious that their sum may be found by addition, but this is not the most expeditious method.

Suppose a man should purchase 5 acres of land, and should pay 5 dollars for the first acre, 20 dollars for the second, and so on in a fourfold ratio; what would the 5 acres cost him?

5. 20. 80. 320. 1280.
    20. 80. 320. 1280 5120.

The first series of numbers above expresses the price of each of the 5 acres. The second, that series multiplied by the ratio, 4, the product of each term being removed one place to the right.

By comparing these two series, it will be seen that their terms are all alike, except the first term of the first series, and the last term of the second. If the first series be subtracted from the second, the remainder will be the last term in the second, less the first term in the first, or $5120 - 5 = 5115$. This remainder is three times the sum of the first series; therefore, if we divide this remainder by 3, the quotient will be the sum of the first series; and $5115 \div 3 = 1705$, the sum of the first series.

In the above process, it will be seen that all the terms disappear in subtracting the first series from the second, except the first term of the first series and the last term of the second, and it is plain the result would have been the same, if the last term only had been multiplied, and the first subtracted from the product. Hence, we have the following rule for finding the sum of all the terms, when we have the first term and ratio given.

RULE. *Find the last term by the rule in Art.* 179, *then multiply the last term by the ratio, subtract the first term from the product, and divide the remainder by the ratio, less* 1; *the quotient will be the sum of all the terms.*

1. If the first term of a geometrical series be 2, and the ratio 3, what is the 4th term? What is the sum of all the terms?

2. If the first term of a geometrical series be 10, and the ratio 4, what is the 5th term? What is the sum of all the terms?

3. If the first term of a geometrical series be 1, the ratio 3, and the number of terms 12, what is the sum of all the terms? Ans. 265720.

4. A merchant sold 20 yards of broadcloth, at 3 cents for the first yard, 9 cents for the second, 27 cents for the third, and so on, tripling the price of each succeeding yard to the last. What did he get for the 20 yards of cloth?
Ans. $52301766.00.

5. A rich miser thought 1000 dollars a price too much for 12 fine horses, but agreed to give 4 cents for the first, 16 cents for the second, 64 cents for the third, and so on, in quadruple or fourfold proportion, to the last; what did the 12 horses cost him at that rate? Ans. $223696.20.

6. An ignorant fop wanting to purchase an elegant house, a facetious gentleman told him he had one which he would sell him on these moderate terms, viz., that he should give him a cent for the first door, 2 cents for the second, 4 cents for the third, and so on, doubling the price for every door, which were 36 in all. It is a bargain, cried the simpleton, and here is a guinea to bind it. What did the house cost him?

COMPOUND INTEREST BY GEOMETRICAL PROGRESSION.

Art. 181. We have shown the usual method of computing compound interest in Art. 150. If we multiply any given principal by the amount of $1 for one year, at any given rate per cent., the product will be the amount of that principal for one year. Again, if we multiply this amount by the amount of $1 for one year, the product will be the amount for the second year, and so on for any given number of years. These successive amounts form the terms of a geometrical series, whose first term is the original principal, the ratio is the amount of $1 for one year, and the number of terms is the number of years, *plus* 1.

From the above illustration, we perceive that finding the amount of any given principal for a specified number of years, at any given rate per cent., consists in finding the last term of a geometrical series, whose first term is the principal, whose ratio is the amount of $1 for one year, and whose number of terms is the number of years, plus 1. The last term will be the required amount. Art. 179.

1. What is the amount of $250, for 5 years, at 6 per cent. compound interest? What is the interest?

Ans. Amt. $334.555. Int. $84.555.

2. What will be the amount of $1250 for 4 years, at 5 per cent. compound interest? What will be the interest?

Ans. Amt. $1519.3825. Int. $269.3825.

On the next page we have placed a table exhibiting the amount of $1 or £1, from 1 year to 40, at 5 and 6 per cent. compound interest, the interest being added yearly. This table will be found very convenient in finding the amount of any given sum of money at compound interest.

3. What will $500 amount to in 12 years, at 6 per cent. compound interest?

In the table we find the amount of $1 for 12 years at 6 per cent. to be $2.012196, which multiplied by the given principal $500, the product is $1006.098, which is the required amount.

4. What will be the amount and compound interest of $745.75 for 15 years, at 6 per cent.?

5. What will be the amount and compound interest of $975.50 for 20 years, at 5 per cent.?

6. What will be the amount and compound interest of *$1500 for 25* years, at 6 per cent.?

TABLE I.

Showing the amount of $1 or £1, from 1 year to 40, at 5 and 6 per cent. compound interest.

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	1.050000	1.060000	21	2.785963	3.399564
2	1.102500	1.123600	22	2.925261	3.603537
3	1.157625	1.191016	23	3.071524	3.819750
4	1.215506	1.262477	24	3.225100	4.048935
5	1.276282	1.338226	25	3.386355	4.291871
6	1.340096	1.418519	26	3.555673	4.549383
7	1.407100	1.503630	27	3.733456	4.822346
8	1.477455	1.593848	28	3.920129	5.111687
9	1.551328	1.689479	29	4.116136	5.418388
10	1.628895	1.790848	30	4.321942	5.743491
11	1.710339	1.898299	31	4.538039	6.088101
12	1.795856	2.012196	32	4.764941	6.453387
13	1.885649	2.132928	33	5.003189	6.840590
14	1.979932	2.260904	34	5.253348	7.251025
15	2.078928	2.396558	35	5.516015	7.686087
16	2.182875	2.540352	36	5.791816	8.147252
17	2.292018	2.692773	37	6.081407	8.636087
18	2.406619	2.854339	38	6.385477	9.154252
19	2.526950	3.025600	39	6.704751	9.703507
20	2.653298	3.207135	40	7.039989	10.285718

The interest of $1 for one year, at 6 per cent., is $.06, and the amount is $1.06. And $1.06 × $1.06 = $1.1236, the amount for the second year, and $1.1236 × $1.06. = $1.191016, the amount for the third year, and so on for any number of years; the amount of $1 for any number of years being multiplied by the amount of $1 for one year, will give the amount for the next succeeding year, and in this manner Table I. has been constructed. Hence, to find the amount of any given sum for any given number of years, we have the following rule.

Rule. *Find the amount of $1 in the Table against the given number of years, and under the given rate per cent.; multiply this amount by the given sum; the product will be the amount required.*

If the interest is payable half yearly, the amount of $1 must be found for the required number of half years, by making the amount of $1 for a half year the ratio or common multiplier.

If the interest is payable quarterly, the amount of $1 must be found for the required number of quarter years, by making the amount of $1 for a quarter year the ratio or common multiplier.

What will $750 amount to in 19 years, at 6 per cent. compound interest? Ans. $2269.20.

COMPOUND DISCOUNT.

Art. 182. COMPOUND DISCOUNT is an allowance made for the payment of money before it is due, on the supposition that money can be let at compound interest.

The present worth of a debt payable at some future time without interest, is that sum which, being on compound interest, will amount to the given sum or debt at the time it becomes payable.

Hence, to find the present worth of any sum of money or debt payable at some future specified time without interest, is to find what sum of money or principal will amount to the given sum or debt at the time it becomes payable.

Suppose we wish to find the present worth of $400, payable in 5 years without interest, on the supposition that money can be let at 6 per cent. a year, compound interest.

In questions of this kind, we have given the amount, which is the last term of a geometrical series, the rate per cent., which is the ratio less 1, and the time, which is the number of terms less 1, to find the principal or present worth, which is the first term of a geometrical series.

Under the head of compound interest by geometrical progression, it was shown that the amount was equal to the principal multiplied by the amount of $1 for one year, raised to a power whose index or exponent was the number of years.

Hence, we have the following rule for finding the principal or present worth, and compound discount.

RULE. *Divide the given sum or debt by the amount of $1 for one year, raised to a power whose exponent is equal to the number of years; the quotient will be the principal or present worth. Subtract the present worth from the given sum or debt; the remainder will be the discount.*

1. What is the present worth of $334.555, due 5 years hence, discounting at 6 per cent. compound interest?

The amount of $1 for one year at 6 per cent., raised to the 5th power, is $1.33822, and $334.555 ÷ $1.33822 = $250, the principal or present worth.

2. What is the present worth of $1519.3825, due in 4 years, discounting at 5 per cent. compound interest? what is the discount?

Ans. $1250, present worth. $269.3825, discount.

3. What is the present worth of $1790.848, due in 10 years, discounting at 6 per cent. compound interest? what is the discount?

Ans. $1000, present worth. $790.848, discount.

TABLE II.

Showing the present worth of $1 or £1, from 1 year to 40, allowing compound discount, at 5 and 6 per cent.

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	.952381	.943396	21	.358942	.294155
2	.907029	.889996	22	.341850	.277505
3	.863838	.839619	23	.325571	.261797
4	.822702	.792094	24	.310068	.246979
5	.783526	.747258	25	.295303	.232999
6	.746215	.704961	26	.281241	.219810
7	.710681	.665057	27	.267848	.207368
8	.676839	.627412	28	.255094	.195630
9	.644609	.591898	29	.242946	.184557
10	.613913	.558395	30	.231377	.174110
11	.584679	.526788	31	.220359	.164255
12	.556837	.496969	32	.209866	.154957
13	.530321	.468839	33	.199873	.146186
14	.505068	.442301	34	.190355	.137912
15	.481017	.417265	35	.181290	.130105
16	.458112	.393646	36	.172657	.122741
17	.436297	.371364	37	.164436	.115793
18	.415521	.350344	38	.156605	.109239
19	.395734	.330513	39	.149148	.103056
20	.376889	.311805	40	.142046	.097222

The present worth of a sum of money to be received at the end of any number of years, is that principal which will, at compound interest, amount to the sum to be received at the expiration of the given period.

In Table I., against 19 years and under 6 per cent., we find $3.025600; multiplying this amount of $1 for 19 years by $500, the product is $1512.80 which is the amount of $500 for 19 years.

In Table II., against 19 years and under 6 per cent., we find $.330513; multiplying this present worth of $1 by $1512.80, the product is $500, the present worth of $1512.80 payable at the expiration of 19 years. The same result is obtained by dividing $1512.80 by the amount of $1 for 19 years. $1512.80 ÷ $3.025600 = $500. In this manner Table II. has been formed, — *unity* being divided by the amount against each number of years in Table I. Hence, the following rule for finding the present worth of any sum at 5 or 6 per cent.

Rule. *Multiply the present value of $1 for the given number of years, at the given rate per cent., by any given amount; the product will be its present worth.*

What is the present worth of $2269.20, due in 19 years, allowing compound discount at 6 per cent.? Ans. $750.

ANNUITIES.

Art. 183. An Annuity is a sum of money payable at regular periods for a specified number of years, or during the life of a person, or forever.

Although the term *annuity*, in the proper sense of the word, applies only to annual payments, yet payments which are made semi-annually or quarterly are also called annuities.

When annuities are not paid at the time they become due, they are said to be in *arrears*. The sum of all the annuities in arrears, with the interest on each for the time it has remained due, is called the *amount*.

The *present worth* of an annuity which is to continue forever, is that sum of money whose yearly interest is equal to the annuity.

Now, as multiplying the principal by that decimal which expresses the rate per cent. gives the interest, so dividing the interest by that decimal will give the principal or present worth. Hence, to find the present worth of an annuity which is to continue forever, we have the following rule.

Rule. *Divide the annuity by that decimal which expresses the rate per cent. for one year; the quotient will be the present worth.*

1. What is the present worth of an annuity of $600, to continue forever, allowing the use of money to be worth 6 per cent. a year to the purchaser? Ans. $10000.

2. What is the value of an estate which rents for $300 a year, allowing the use of money to be worth 5 per cent. a year? Ans. $6000.

ANNUITIES AT COMPOUND INTEREST.

Art. 184. It has been shown, (Art. 150,) that compound interest is that which arises from adding the interest to the principal at the end of each year, and making the amount a new principal for the next succeeding year. The amount of $1 for one year at 6 per cent. is $1.06. If any given principal be multiplied by 1.06, the product will be the amount for one year; if this amount be multiplied by 1.06, the product will be the amount for the second year, and so on for any given number of years. Hence, the successive yearly amounts of any sum of money at compound interest, form a geometrical series, whose first term is the given sum, whose ratio is the amount of $1 for one year, and whose number of terms is the number of years.

Suppose an annuity of $100 to have remained unpaid during 5 years, what would be its amount, allowing compound interest at 6 per cent.?

It is evident that for the 5th year the annuity only would be due; for the 4th, the amount of the annuity for 1 year; for the 3d, the amount of the annuity for 2 years; for the 2d, the amount of the annuity for 3 years; for the 1st, the amount of the annuity for 4 years; and that the sum of the annuity and these several amounts would be the whole amount of the annuity at the end of 5 years.

The annuity due at the end of 5 years is $100.
The amount of the annuity for 1 year is $106.
The amount " " for 2 years is $112.36.
The amount " " for 3 years is $119.1016.
The amount " " for 4 years is $126.247696.

Whole amount due at the end of 5 years, $563.709296.

From the above illustrations, we derive the following rule for finding the amount of an annuity in arrears, at compound interest.

Rule. *Find the amount of an increasing geometrical series, whose first term is the annuity, whose ratio is the amount of* $1 *for one year, and whose number of terms is the number of years. Art.* 179, *and Art.* 180. *Or multiply the amount of* $1 *annuity for the given number of years, by the given annuity; the product will be the required amount. Table III.*

1. What is the amount of an annuity of $100, in arrears for 4 years, allowing compound interest at 6 per cent. a year?

$1.06 \times 1.06 \times 1.06 = 1.191016 \times 100 = 119.1016$, the last term. And $119.1016 \times 1.06 = 126.247696 - 100 = 26.247696 \div .06 =$ $437.4616, the amount.

2. What is the amount of an annuity of $500, it being in arrears 12 years, allowing 5 per cent. compound interest?
Ans. $7958.563+.

3. If the annual rent of a farm, which is $400, be in arrears for 6 years, what is the amount, allowing money to be worth 6 per cent. a year, compound interest?
Ans. $2790.127+.

TABLE III.

Showing the amount of $1 or £1 *annuity*, from 1 year to 40, at 5 and 6 per cent. compound interest.

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	1.000000	1.000000	21	35.719252	39.992727
2	2.050000	2.060000	22	38.505214	43.392290
3	3.152500	3.183600	23	41.430475	46.995828
4	4.310125	4.374616	24	44.501999	50.815577
5	5.525631	5.637093	25	47.727099	54.864512
6	6.801913	6.975319	26	51.113454	59.156383
7	8.142008	8.393838	27	54.669126	63.705766
8	9.549109	9.897468	28	58.402583	68.528112
9	11.026564	11.491316	29	62.322712	73.639798
10	12.577893	13.180795	30	66.438848	79.058186
11	14.206787	14.971643	31	70.760790	84.801677
12	15.917127	16.869941	32	75.298829	90.889778
13	17.712983	18.882138	33	80.063771	97.343165
14	19.598632	21.015066	34	85.066959	104.18376
15	21.578564	23.275970	35	90.320307	111.43478
16	23.657492	25.672528	36	95.836323	119.12087
17	25.840366	28.212880	37	101.62814	127.26812
18	28.132385	30.905653	38	107.70955	135.90421
19	30.539004	33.759992	39	114.09502	145.05846
20	33.065954	36.785591	40	120.79977	154.76197

The above Table was constructed in the following manner. To $1.000000 was added the amount of $1 at the expiration of 1 year, at the given rates of interest, obtained from Table I., which gives the amount of an annuity of $1 at those rates, forborne 2 years; to this sum was added the amount of $1 in 2 years, which gives the amount of the annuity for 3 years, and so on to the end of the period of 40 years.

The amount of any other sum may be found by the following

Rule. *Find the amount of $1 annuity in the table against the given number of years and under the given rate per cent.; multiply this amount by any given sum; the product will be the amount of that sum for the same time at the same rate per cent.*

What will an annuity of $200 a year amount to in 5 years, at 6 per cent. compound interest?

In Table III., opposite 5 years and under 6 per cent., we find $5.637093, the amount of $1 annuity for 5 years at 6 per cent., which multiplied by $200, gives $1127.4186, the amount required.

What will an annuity of $600 a year amount to in 40 years, at 6 per cent. compound interest? Ans. $92857.182.

Art. 185. We have given a rule, (Art. 183) for finding the present worth of an annuity which is to continue forever; we will now illustrate the method of finding the present worth of an annuity which is to continue for a specified number of years.

Suppose we wish to ascertain the present worth of an annual pension of $200, to continue 5 years, at 6 per cent. compound interest.

The present worth is, evidently, that sum of money, which, at 6 per cent. compound interest, would in 5 years produce an amount equal to the amount of the annuity for the same time.

We can find the amount of a given sum of money at compound interest, for any specified number of years, by multiplying the given sum by the amount of $1 at compound interest for the given time.

Hence, to find a sum which will produce a given amount in a specified time, we must *reverse* the process, and divide the amount by the amount of $1 for the specified time; the quotient will be the principal or present worth.

Applying this principle to the question proposed, we find the amount of $200 annuity for 5 years to be $1127.4186. Dividing this by the amount of $1 for 5 years, we find the principal or present worth. And $1127.4186 ÷ 1.3382256 = $842.472+.

Hence, we have the following rule.

Rule. *Find the amount of the annuity for the specified number of years; divide this amount by the amount of* $1 *at compound interest for the same time; the quotient will be the present worth. Or, multiply the present worth of* $1 *annuity by the given annuity; the product will be its present worth. Table IV.*

1. What is the present worth of an annuity of $300, to continue 10 years, at 6 per cent. compound interest?
Ans. $2208.0261.

2. What is the present worth of an annuity of $100, to continue 20 years, at 5 per cent. compound interest?
Ans. $1246.222.

3. What is the present worth of an annuity of $500, to continue 10 years, allowing compound interest at 6 per cent.?
Ans. $3680.043+.

4. What amount of ready money will purchase an annuity of $300, to continue 25 years, at 6 per cent. compound interest?

TABLE IV.

Showing the present worth of $1 or £1 *annuity*, from 1 year to 40, at 5 and 6 per cent. compound interest.

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	.952381	.943396	21	12.821153	11.764077
2	1.859410	1.833393	22	13.163003	12.041582
3	2.723248	2.673012	23	13.488574	12.303379
4	3.545951	3.465106	24	13.798642	12.550358
5	4.329477	4.212364	25	14.093945	12.783356
6	5.075692	4.917324	26	14.375185	13.003166
7	5.786373	5.582381	27	14.643034	13.210534
8	6.463213	6.209794	28	14.898127	13.406164
9	7.107822	6.801692	29	15.141074	13.590721
10	7.721735	7.360087	30	15.372451	13.764831
11	8.306414	7.886875	31	15.592811	13.929086
12	8.863252	8.383844	32	15.802677	14.084043
13	9.393573	8.852683	33	16.002549	14.230230
14	9.898641	9.294984	34	16.192904	14.368141
15	10.379658	9.712249	35	16.374194	14.498246
16	10.837770	10.105895	36	16.546852	14.620987
17	11.274066	10.477260	37	16.711287	14.736780
18	11.689587	10.827603	38	16.867893	14.846019
19	12.085321	11.158116	39	17.017041	14.949075
20	12.462210	11.469921	40	17.159086	15.046297

The total present value of an annuity for a term of years is manifestly equal to the sum of the present values of each year's payment; and by the continued addition of these at the two rates of interest, Table IV. has been formed. For example,—in Table II.,

$ .943396 is the present value of $1, to be received at the end of 1 year, at 6 per cent. interest.
$ .889996 is the present value of $1, to be received at the end of 2 years.
$1.833392 sum of the above, or present value of an annuity of $1 for 2 years.
$ .839619 present value of $1, to be received at the end of 3 years.
$2.673011 sum of the above, or present value to be received at the end of 3 years, and so on to the end of 40 years.

The present value of any other sum may be found by the following

RULE. *Find the present value of $1 annuity in the table against the given number of years, and under the given rate per cent.; multiply this present value by any given sum; the product will be the present value of that sum for the same time at the same rate per cent.*

What is the present value of an annuity of $200 a year, to continue 5 years, at 6 per cent. compound interest? Ans. $842.4728.

Art. 186. An annuity in reversion is an annuity which does not commence until a specified time has elapsed, or some particular event has taken place.

Suppose we wish to find the present worth of an annuity of $100, which is to commence at the end of 2 years, and then to continue 4 years, allowing 6 per cent. compound interest.

The present worth is evidently such a sum as would, in 2 years, at 6 per cent. compound interest, produce an amount equal to the present worth of the annuity, were it to commence immediately.

We must therefore find the present worth of an annuity of $100 to commence immediately and to continue 4 years, which is $346.5106. We wish now to obtain a sum whose amount in 2 years will equal this present worth. This may be found by dividing $346.5106 by the amount of $1 for 2 years; thus, $346.5106 ÷ 1.1236 = $308.394+.

Hence, to find the present worth of an annuity in reversion, at compound interest, we have the following rule.

Rule. *Find the present worth of the given annuity for the time of its continuance, supposing it to commence immediately; then divide this present worth by the amount of* $1 *for the time in reversion; the quotient will be the present worth of the annuity in reversion.*

Or, find the present worth of the annuity from the present time to the end of the time of its continuance, by Table IV.; find also, by Table IV., its present worth before it commences; the difference of these two results will be the present worth.

1. What is the present worth of an annuity of $200, which is to continue 5 years, but not to commence until the end of 2 years, allowing compound interest at 6 per cent.? Ans. $749.7976.

2. What amount of ready money will purchase the reversion of an estate which rents for $60 a year, to continue 6 years, but not to commence until the end of 3 years, allowing compound interest at 6 per cent.? Ans. $247.72+.

3. Suppose a parent leaves to his son an annual rent of $310 for 8 years, and the reversion of the same rent to his daughter for the next succeeding 14 years. What is the present worth of the legacy of each, allowing compound interest at 6 per cent.? Son's, $1925.036+.
Daughter's, $1807.854+.

LIFE INSURANCE.

Art. 187. Life Insurance is a contract whereby one or more, or a company of persons, in consideration of the payment of an amount called the *premium*, promise, either on the decease of a person named, whenever it may happen, or on his decease within a given time, to pay a certain sum to his heirs, or others for whose benefit the contract is made. This contract is now more frequently made by Mutual Life Insurance Companies, all the members of which contribute, in certain agreed proportions, to a common fund, out of which the sums due on the decease of a member, that is, the *losses*, are to be paid; the surplus, if any, being distributable, from time to time, among the members.

In the early stages of life insurance in England, in the seventeenth century, a premium of five per cent. on the amount insured was charged upon each life, without discrimination on account of difference of age. It, however, early became obvious that, all other things being equal, the younger of two persons has a better chance, or probability, or, as it is usually termed, *expectation* of life, than the elder; and consequently, if the two, or any greater number, by the annual payment of equal sums during their whole lives, make up a common fund for the payment of a given sum to the heirs of each, it is plain that the younger associates will pay a greater amount into this joint stock than the older, so that the partnership will be a very unequal one. The question then arises, what different yearly amounts the contributors of different ages ought to pay into the common fund, out of which the respective heirs of each are to be entitled to a given sum on his decease. To answer this question, it is necessary to determine upon some *law of mortality*, that is, how many out of a given number will die annually at each age. Registers of deaths have been kept at various places in Europe at different periods, for the purpose of ascertaining this law in those places, and the experience of life insurance companies has also been resorted to for a similar purpose. Life insurance has hitherto been little practised in the United States, and the records of population and registers of deaths have been very imperfect. Some such registers have been kept, which, with the censuses of the population of the United States taken every ten years, afford some materials for making an estimate of the rate of mortality. At the time of the New England Mutual Life Insurance Company going into operation, in 1843, in Boston, very considerable investigation was made, in order to ascertain a rate of mortality which could be safely adopted as the basis of the premiums to be charged by the company. A very material inquiry was, whether the European registers and experience would be a safe guide, and it appeared from the registers kept and the censuses of population, that in the older and long-settled parts of New England and the Middle States, at least, the rate of mortality approximated very nearly to those assumed in Europe as the basis of calculations of premiums of insurance of lives, and the value of life annuities. From the various materials thus offered, the following table of mortality was formed for the ages of ten and upwards, as the basis of the computation of the premiums to be paid by the members of that company. This table starts with 10000 lives at the age of ten, and notes the annual *decrements* by deaths, until the whole number is exhausted.

Age.	Lives.	Decrement.	Expectation.	Age.	Lives.	Decrement.	Expectation.
10	10000	65	47.43	54	6262	134	16.97
11	9935	61	46.73	55	6128	138	16.33
12	9874	57	46.02	56	5990	145	15.69
13	9817	54	45.28	57	5845	152	15.07
14	9763	54	44.53	58	5693	162	14.44
15	9709	58	43.78	59	5531	173	13.86
16	9651	62	43.03	60	5358	186	13.11
17	9589	65	42.31	61	5172	194	12.57
18	9524	67	41.60	62	4978	198	12.04
19	9457	69	40.89	63	4780	200	11.51
20	9388	71	40.17	64	4580	205	10.99
21	9317	72	39.49	65	4375	209	10.49
22	9245	73	38.79	66	4166	214	9.99
23	9172	75	38.10	67	3952	220	9.50
24	9097	76	37.41	68	3732	225	9.03
25	9021	77	36.72	69	3507	232	8.58
26	8944	78	36.03	70	3275	236	8.15
27	8866	79	35.34	71	3039	237	7.75
28	8787	80	34.65	72	2802	234	7.36
29	8707	84	33.97	73	2568	230	6.99
30	8623	84	33.29	74	2338	228	6.63
31	8539	84	32.62	75	2110	217	6.29
32	8455	84	31.94	76	1893	208	5.95
33	8371	84	31.25	77	1685	197	5.63
34	8287	84	30.56	78	1488	187	5.31
35	8203	85	29.87	79	1301	174	5.00
36	8118	85	29.18	80	1127	165	4.69
37	8033	86	28.48	81	962	153	4.41
38	7947	87	27.78	82	809	138	4.15
39	7860	88	27.09	83	671	126	3.91
40	7772	93	26.39	84	545	110	3.70
41	7679	96	25.70	85	435	93	3.50
42	7583	99	25.02	86	342	77	3.31
43	7484	100	24.35	87	265	64	3.14
44	7384	100	23.67	88	201	51	2.99
45	7284	101	22.99	89	150	37	2.84
46	7183	102	22.30	90	113	31	2.60
47	7081	105	21.62	91	82	24	2.40
48	6976	107	20.96	92	58	18	2.80
49	6869	111	20.25	93	40	14	1.95
50	6758	117	19.58	94	26	10	1.73
51	6641	123	18.91	95	16	7	1.50
52	6518	126	18.26	96	9	5	1.27
53	6392	130	17.61	97	4	2	1.25

This table gives at the age of eleven, that is, after one year, 9935 lives surviving, 65 out of this society of 10000 having died during the year. Let it be proposed that the heirs of each one of the society of 10000, of the age of ten, who shall die within the year, shall receive 100 dollars out of a fund to be raised by a contribution from the whole 10000. This fund must be 6500 dollars, and the assessment or *premium* for each to pay will be 65 cents. Looking to the age of twenty in the above table, we find the number of surviving lives to be 9388, of whom, according to the chance or probability presented by this table, the number of deaths or decrement during a year will be 71, leaving 9317 survivors at the age of twenty-one. The number of deaths or decrement during a year is, as we see, greater than at the age of ten to eleven; and this greater number is out of a less number of lives; the ratio at ten to eleven being $\frac{65}{10000}$, and that at twenty to twenty-one being $\frac{71}{9388}$. Therefore, the fund requisite for the payment of 100 dollars to the heirs of each member of the society who shall die during a year at this age, is 7100 dollars; which assessed upon 9388 members of the society at the beginning of their twenty-first year, gives a premium of $75\frac{5900}{9388}$ cents to be paid by each.

So if the society is for more than one year, — either for a term of years or for life, — it is to be ascertained what sum must be paid out each year, which is readily ascertained by the above table, and a sufficient assessment, payable annually, or at other fixed periods, or all at once, or payable in less or diminishing amounts, or greater or increasing amounts, at the beginning of each successive year or other period, must be made upon the members to meet the payments.

But when the association is for more than a year, and the fund is to be raised by the same assessment or premium paid by a member every year as long as he continues to be a member, it is plain that a part of the payment made the first year, and so on to the expiration of about one half of the period of his membership, is paid to meet the increasing amounts of the losses during the latter part of the period; the consequence is that the company must accumulate a fund to be held over into the subsequent part of the period.

This brings a new element into the calculation, namely, the interest to be gained on this accumulated fund, which the company is supposed to invest in mortgages, stocks or otherwise, so that it may yield an income. Accordingly, in the computation of the premium to be paid by each member whose insurance is for more than one year, the interest to be derived upon a part of his early payments must be taken into the account. If the company is formed for a period of a few years only, it may be presumed that the existing rate of interest will continue without any material change; but such companies are usually formed for an indefinite period, and must be so if they insure for the whole life of a member, for the chance is, as we see by the table, and know very well otherwise, that some of the members may survive to a very old age; and it is absurd for a company, which is by its constitution to subsist but twenty years, to agree to pay a sum on the decease of a member who may live ninety or a hundred; a member of such a company, who is insured for his whole life, ought to get insurance in another that he shall die within twenty years. The *probable rate* of interest for a long series of years to come must be assumed in calculating the premium. That assumed by the New England Mutual Life Company was four per cent. per annum.

Other expenses and contingencies besides the payment of losses resulting from the ordinary rate of mortality, must be taken into the account in calculating the premium, such as the chance of a greater mortality than that assumed, incidental expenses, loss on investments, which may take place notwithstanding the utmost prudence. To provide against those contingencies and expenses, that company added twenty per cent. to the rate of premium requisite for merely meeting the losses at the assumed rate of mortality; and the calculation thus made results in the following rates of annual premiums to be paid for insurance on the lives of persons of different ages, from ten to sixty In determining the rate for a mutual company, the premium should be put at a point which will be certainly high enough; and if it should prove higher than is necessary, the excess is distributed back, whereas if it be too low, the company will sooner or later become insolvent.

TABLE OF PREMIUMS.

Annual payment for each hundred dollars insured.

Age.	For Life.	For seven years.	For one year.	Age.	For Life.	For seven years.	For one year.
10	1.34	0.73	0.78	36	2.74	1.38	1.26
11	1.37	0.72	0.74	37	2.84	1.43	1.30
12	1.41	0.74	0.70	38	2.93	1.48	1.32
13	1.44	0.76	0.67	39	3.04	1.53	1.35
14	1.48	0.80	0.67	40	3.16	1.58	1.44
15	1.53	0.84	0.72	41	3.27	1.63	1.51
16	1.56	0.87	0.78	42	3.39	1.68	1.57
17	1.61	0.90	0.82	43	3.52	1.73	1.61
18	1.65	0.92	0.85	44	3.65	1.79	1.63
19	1.70	0.95	0.88	45	3.81	1.87	1.67
20	1.74	0.98	0.91	46	3.96	1.96	1.71
21	1.78	1.00	0.93	47	4.13	2.07	1.78
22	1.83	1.02	0.95	48	4.31	2.18	1.85
23	1.88	1.05	0.99	49	4.50	2.30	1.94
24	1.94	1.08	1.01	50	4.71	2.44	2.08
25	1.98	1.11	1.03	51	4.92	2.58	2.23
26	2.03	1.13	1.05	52	5.15	2.74	2.32
27	2.09	1.15	1.07	53	5.39	2.94	2.45
28	2.15	1.17	1.10	54	5.66	3.17	2.57
29	2.21	1.20	1.16	55	5.94	3.43	2.71
30	2.28	1.21	1.17	56	6.24	3.72	2.91
31	2.34	1.23	1.19	57	6.57	4.02	3.13
32	2.42	1.25	1.20	58	6.92	4.34	3.42
33	2.49	1.28	1.21	59	7.28	4.67	3.76
34	2.57	1.30	1.22	60	7.65	5.01	4.17
35	2.66	1.34	1.25				

It is apparent that if the business of a mutual company is prudently managed, there is a probability amounting to almost a certainty of a handsome return to the members on the premiums paid in by them according to the above table, or, instead of it, a considerable addition to the amount insured, without the payment of any additional annual premium.

PERMUTATIONS.

Art. 188. PERMUTATIONS are the changes which are made in arranging any given number of things in every possible order or relative position.

ILLUSTRATION. One thing, as *a*, admits of but one position. Two things, as *a* and *b*, admit of two positions or changes in the order of arranging them, viz., *ab* and *ba*; and this number of changes is expressed by 1 × 2 = 2. Three things, as *a*, *b*, and *c*, admit of six changes in the order of arranging them, viz., *abc*, *acb*, *bac*, *bca*, *cab*, *cba*; and this number of permutations, or changes, is expressed by 1 × 2 × 3 = 6. In the same manner, we may show that four things, as *a*, *b*, *c*, and *d*, will admit of 24 changes; because every three of them will admit of 6 changes, and taking the fourth with the other three, there will be 4 times 6 or 24 changes; and so on, *ad infinitum*, according to the same law. Hence, to find the number of permutations, or changes, that can be made in the arrangement of any given number of different things, we have the following

RULE. *Find the product of the natural series of numbers, from one up to the given number of things; this product will be the number of permutations, or changes, required.*

1. Christ's Church, in Boston, has 8 bells; what number of changes may be rung on them? Ans. 40320 changes.

2. In how many different relative positions may a family of 10 persons be seated at dinner? Ans. 3628800 different positions.

3. What number of different arrangements can there be made in seating a class of 24 scholars?

When several of the individual things are alike, it is evident that the number of changes that can be made upon the whole number of things is not so great, as when all the things are different.

ILLUSTRATION. We have shown that the letters *abc* admit of six changes or variations; but if two of the letters be alike, as *aab*, the six variations are reduced to three, viz., *aab*, *aba*, *baa*, which may be expressed by $\frac{1\times2\times3}{1\times2}=3$.

We have also shown that the letters *abcd* admit of 24 variations; but, if we have *aabb*, the 24 variations are reduced to six, viz., *aabb*, *abab*, *abba*, *bbaa*, *baba*, *baab*; and this number of variations may be expressed by $\frac{1\times2\times3\times4}{2\times1\times2}=6$. Hence, we have the following rule for finding the number of changes that can be made in the arrangement of any given number of things, when several of them are alike.

RULE. *Find the number of changes that could be made in the arrangement of the whole number of things, as if they were all different things.*

Then find the number of changes that could be made in the arrangement of the several things of each kind, as if they were different things.

Lastly, divide the number of changes that could be made in the arrangement of the whole number of things, by the joint product of the number of changes that could be made in the arrangement of the several things of each kind; the quotient will be the number of changes required.

1. How many different numbers can be formed of the following figures, 1223334444? Ans. 12600.

2. How many variations may take place in the succession of the following musical notes, *fa*, *fa*, *fa*, *sol*, *sol*, *la*, *mi*, *fa*?

Ans. 3360 variations.

3. What number of changes can be made in the arrangement of the letters in the word Cincinnati?

When the given number of things are all different, and it is required to find the number of permutations, or changes, that can be made in arranging them, by taking any given number of them at a time.

ILLUSTRATION. We will take the four letters *abcd*, and find the number of changes that can be made in arranging them, by taking two of them at a time. We will write the letter *a* at the left of each of the other letters, and the changes will be three, viz., *ab*, *ac*, *ad*; we will do the same with each of the other letters, thus, *ba*, *bc*, *bd*; *ca*, *cb*, *cd*; *da*, *db*, *dc*. Now, we have all the changes that can be made in arranging the four letters, taking two of them at a time, and they are $4 \times 3 = 12$.

We will also find the number of changes that can be made in arranging the four letters, by taking three of them at a time. Writing *a* at the left, thus, *abc*, *abd*; *acb*, *acd*; *adb*, *adc*; we have $3 \times 2 = 6$ variations or changes. Now, since each of the other letters is to be written at the left in the same manner, we shall have four such classes of variations or changes, and the whole number will be $4 \times 3 \times 2 = 24$ variations or changes. From the above illustration, we deduce this

RULE. *Take a series of numbers, beginning with the number of things given, and decreasing by* 1 *until the number of terms is equal to the number of things to be taken at a time; find the product of all the terms of this series; this product will be the number of changes required.*

1. What number of changes can be made in arranging the letters *a e i o u y*, by taking three of them at a time? Ans. 120 changes.

2. How many different numbers can be expressed with the nine digits, by using four of them at a time?

Ans. 3024 different numbers.

3. How many different numbers can you express with the nine digits and a cipher, by using five of them at a time?

COMBINATIONS.

COMBINATION, in arithmetic, is the forming of any given number of different things into several groups, each group differing from all the others in respect to some of the several things of which it is composed.

ILLUSTRATION. When the whole of any given number of things is taken at a time, it can form but one combination; but any given number of things may be parcelled out into groups or combinations, by taking two, three, four, or any number of them, at a time. Two letters, *a* and *b*, admit of but one combination, but if we add a third letter, *c*, we can form three combinations, by taking only two of them at a time, thus *ab*, *ac*, *bc*; and this number of combinations may be expressed thus, $\frac{3\times2}{1\times2}=3$. If we add a fourth letter, *d*, we can form six combinations, by taking only two of them at a time; thus, *ab*, *ac*, *bc*, *ad*, *bd*, *cd*; and this number of combinations may be expressed thus, $\frac{4\times3}{1\times2}=6$.

From the above illustration, we derive the following rule for finding the number of combinations that can be made out of any given number of different things, by taking any given number of them at a time.

RULE. *Take a series of numbers, the first term of which is equal to the number of things out of which the combinations are to be made, and decreasing by* 1, *until the number of terms is equal to the number of things to be taken at a time, and find the product of all the terms.*

Then take the natural series, 1, 2, 3, *&c., up to the given number of things to be taken at a time, and find the product of all the terms of this series.*

Divide the former product by the latter; the quotient will be the number of combinations required.

1. How many different numbers of 4 figures each may be expressed by the figures from 1 to 8, inclusive; no two of the numbers having all their figures alike? Ans. 70 different numbers.

2. What is the number of combinations which can be formed out of 10 individual things, taking 5 of them at a time? Ans. 252.

3. What number of combinations can be formed out of 100 individual things, by taking 10 of them at a time?

4. A drover purchased a dozen sheep of a farmer, which were to be selected out of a flock of three dozen; while the drover was selecting them, the farmer told him that he might have the whole flock, if he would give him a cent for every different dozen that could be selected from it; to which he readily agreed. What number of dollars did the whole flock come to, at that rate?

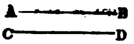
GEOMETRICAL DEFINITIONS.

Art. 189. 1. A POINT is position only, and mathematically considered, has no dimensions.

2. A LINE is extension in length only. Points are the extremities of lines.

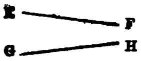
3. A STRAIGHT LINE is the shortest distance between two points, or it is extension in one direction only. The line A B in the margin is a straight line.

4. A CURVE LINE is extension which is continually changing its direction. The line C D is a curve line.



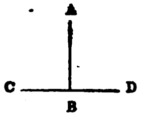
5. PARALLEL LINES are straight lines which are equally distant in every part, and if extended ever so far, would never meet. The lines A B and C D are parallel lines.

6. HORIZONTAL LINES are those which are parallel to the plane of the horizon, or to the spirit-level.

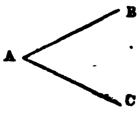


7. INCLINED *or* OBLIQUE LINES are straight lines which continually approach each other, and if sufficiently extended, would meet at some point. The lines E F and G H are inclined lines.

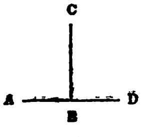
8. VERTICAL LINES are those which are perpendicular to the plane of the horizon.



9. A PERPENDICULAR LINE is a line which meets another line so as to make the angle on each side a right-angle. The line A B is a perpendicular line, and meets the line C D at right angles. Lines are the boundaries of surfaces.

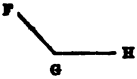


10. AN ANGLE is the opening between two inclined lines which meet in a point; the point of meeting is called the *vertex* of the angle. The opening between the two lines A B and A C is called an angle, and the angular point at A is called the vertex. The magnitude or size of an angle depends on the width of the opening, and not on the length of the lines. An angle is usually denoted by three letters.

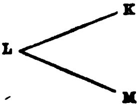


11. A RIGHT ANGLE is an angle that contains 90 degrees, and its measure is the fourth part of the circumference of a circle. A B C and C B D are right angles.

12. AN OBLIQUE ANGLE is an angle that contains either more or *less than 90 degrees.*



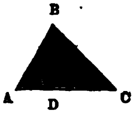
13. AN OBTUSE ANGLE is greater than a right angle, and contains more than 90 degrees. F G H is an obtuse angle.



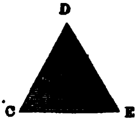
14. AN ACUTE ANGLE is less than a right angle, and contains less than 90 degrees. K L M is an acute angle.

15. A SURFACE is the exterior or outside of anything that has length and breadth, or it is extension in two directions.

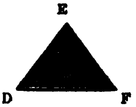
16. A PLANE SURFACE is one that is perfectly flat or level. A plane surface which has definite boundaries is called a geometrical figure. Lines are the boundaries of surfaces. The figure A B C D is a plane surface.



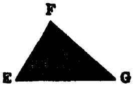
17. A TRIANGLE is a plane surface bounded by three straight lines. The figure A B C is a triangle. Its base is the line A C. Its *altitude* or height is the dotted line B D, drawn from the vertex at B perpendicularly to the base.



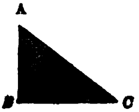
18. AN EQUILATERAL TRIANGLE is a plane surface bounded by three straight lines of *equal* length. The triangle C D E is an equilateral triangle.



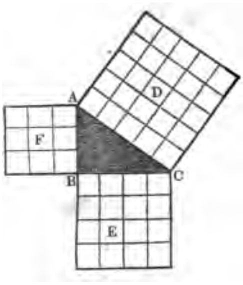
19. AN ISOSCELES TRIANGLE is a plane surface bounded by three straight lines, *two* of which are of equal length. The triangle D E F is an isosceles triangle.



20. A SCALENE TRIANGLE is a plane surface bounded by three straight lines of *unequal* length. The triangle E F G is a scalene triangle.



21. A RIGHT-ANGLED TRIANGLE is a plane surface bounded by three straight lines, the perpendicular making an angle with the base of 90 degrees. A B C is a right-angled triangle, the perpendicular or line A B making an angle of 90 degrees with the base B C.

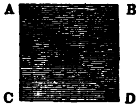


22. In every right-angled triangle, the square described on the hypothenuse or side opposite the right angle, is equal to the sum of the squares described upon the base and perpendicular. If A B C be a right-angled triangle, and right-angled at B, then will the square D, described on the hypothenuse A C, be equal to the sum of the squares E and F, described on the base B C and perpendicular A B. By counting the small squares, we find the number of small squares in the square D to be equal to the number of small squares in the squares E and F.

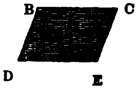
23. A QUADRILATERAL is a figure bounded by four straight lines, having four sides and four angles. A DIAGONAL is a line joining any two opposite angles of a quadrilateral.

24. A PARALLELOGRAM is a quadrilateral which has its opposite sides equal and parallel. It takes the following particular names, viz.: *Rectangle*, *Square*, *Rhombus*, and *Rhomboid*.

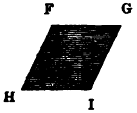
25. A RECTANGLE is a right-angled parallelogram, having its opposite sides equal and parallel. The plane surface bounded by the lines E F, F H, H G, and G E, is a rectangle.



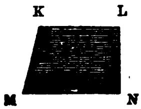
26. A SQUARE is an equilateral rectangle, having its length and breadth equal, and all its angles right angles. The plane surface bounded by the lines A B, B D, D C, and C A, is a square. The line A D is a diagonal.



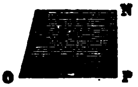
27. A RHOMBOID is an oblique-angled parallelogram, having its opposite sides equal and parallel, and its opposite angles equal. The plane surface bounded by the lines B C, C E, E D, and D B, is a rhomboid.



28. A RHOMBUS is an equilateral rhomboid; having all its sides equal, its opposite sides parallel, and its opposite angles equal. The plane surface bounded by the lines F G, G I, I H, and H F, is a rhombus.



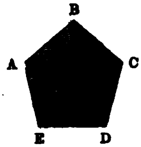
29. A TRAPEZOID is a quadrilateral, which has only one pair of its opposite sides parallel. The plane surface bounded by the lines K L, L N, N M, and M K, is a trapezoid.



30. A TRAPEZIUM is a quadrilateral whose opposite sides are not parallel. The plane surface bounded by the lines M N, N P, P O, and O M, is a trapezium.

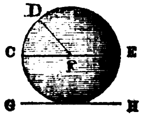
31. PLANE FIGURES, bounded by more than four straight lines, are, in general, called *Polygons;* and they have other particular names, according to the number of their sides.

32. A PENTAGON is a polygon of five sides; a *Hexagon*, of six sides; a *Heptagon*, seven; an *Octagon*, eight; a *Nonagon*, nine; a *Decagon*, ten; an *Undecagon*, eleven; a *Dodecagon*, twelve sides.



33. A REGULAR POLYGON has all its sides and angles equal. If they are not equal, the polygon is irregular. The plane figure A B C D E is a regular pentagon.

34. The *perimeter* of a figure is the sum of all its sides.

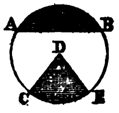


35. A CIRCLE is a plane surface bounded by a curve line, called the circumference; every part of which is equally distant from a point within the figure, called the centre of the circle. The figure C D E is a circle.

36. THE RADIUS of a circle is a straight line extending from the centre of the circle to the circumference. The line F D is the radius, and the extremity of the line at F is the centre of the circle.

37. THE DIAMETER of a circle is a straight line passing through its centre, and extending each way to the circumference. The line C E is the diameter of the circle C D E.

38. A TANGENT is a straight line which touches the circumference only in one point, called the point of contact. The line G H is a tangent.

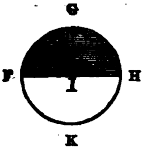


39. AN ARC of a circle is any part of the circumference. The part of the circumference A B is an arc.

40. A CHORD is a straight line joining the extremities of an arc. The line A B is a chord.

41. A SEGMENT is any part of a circle bounded by an arc and its chord. The part of the circle bounded by the arc A B and its chord is a segment.

42. A SECTOR is any part of a circle which is bounded by an arc and two radii, drawn to its extremities. That part of the circle which is bounded by the arc C E, and the two radii C D and D E, is a sector.

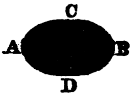


43. A QUADRANT is a quarter of a circle, having a quarter of the circumference for its arc, and its two radii perpendicular to each other. The quarter circles F I G and G I H are quadrants.

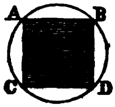
44. A SEMICIRCLE is half of a circle, having half the circumference for its arc, and the diameter for its chord. The half circle F G H is a semicircle.

45. The *circumference* of every circle is supposed to be divided into 360 equal parts, called degrees; and each degree into 60 equal parts, called minutes; and each minute into 60 equal parts, called seconds. Hence, a semicircle measures 180 degrees, and a quadrant 90 degrees.

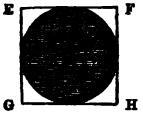
46. The *measure* of an angle is an arc of a circle contained between the two lines which form the angle; the angular point being at the centre of the circle; and its measure is estimated by the number of degrees in that arc.



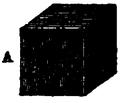
47. AN ELLIPSIS is an oval figure bounded by a curve line, which returns into itself like the circumference of a circle, having two diameters of different length; the longer, A B, is called the *transverse*, and the shorter, C D, the *conjugate diameter*.



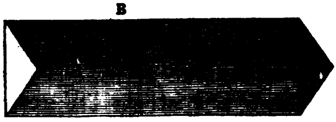
48. A right-lined figure is inscribed in a circle, or the circle circumscribes it, when all the angular points of the figure are in the circumference of the circle. The square A B C D is inscribed in the circle, and the circle circumscribes the square.



49. A right-lined figure circumscribes a circle, or the circle is inscribed in it, when all the sides of the figure touch the circumference of the circle. The square E F G H circumscribes the circle, and the circle is inscribed in the square.

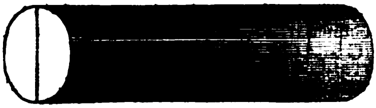


Art. 190. 1. A SOLID is extension in three directions, length, breadth, and thickness. Surfaces are the boundaries of solids. The figure A is the representation of a cube. Arithmetically, a *line* is expressed by a single number; a *surface* by the product of two factors; and a *solid* by the product of three factors. Length and breadth are the factors of a surface; and length, breadth and thickness are the factors of a solid.



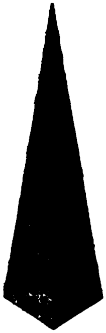
2. A PRISM is a solid whose sides are parallelograms, and whose ends are equal and parallel plane figures. A prism is called a *cube* when its sides and ends are equal and parallel squares; it is called a triangular prism when its ends are equal and parallel triangles; the figure of the ends giving name to the prism. The figure B is the representation of a triangular prism.

B

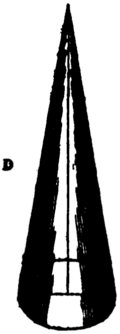


3. A **cylinder** is a round solid, uniform in diameter, whose ends are equal and parallel circles. The figure B is the representation of a cylinder.

C



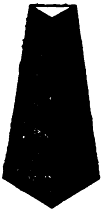
D



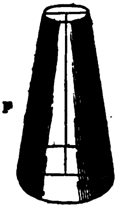
4. A **pyramid** is a solid, having a plane rectilinear figure for its base, its sides being equal triangles whose vertices meet in a point at the top, called the *vertex* of the pyramid. The figure C is the representation of a triangular pyramid.

5. A **cone** is a solid, which has a circle for its base, and tapers uniformly to a point at the top. The figure D is the representation of a cone.

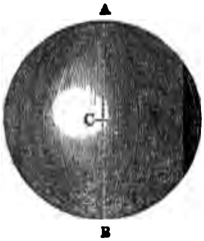
E



F



6. A **frustum** of a pyramid or cone is the part which remains after the top has been cut off by a section parallel with its base. The figure E represents the frustum of a pyramid, and the figure F, the frustum of a cone.



7. A **Sphere**, or **Globe**, is a solid, bounded by a convex surface, every point of which is equally distant from a point within, called the *centre*. The sphere may be conceived to be formed by the revolution of a semicircle about its diameter. The figure A C B represents a sphere.

8. A *hemisphere* is half a sphere. A *segment* of a sphere, or of any other solid, is a part cut off the top by a plane parallel to its base.

9. The *axis* or *diameter* of a sphere is a straight line passing through its centre, and terminating at the surface.

10. The *height* or *altitude* of a solid is a line drawn from its vertex, or top, perpendicular to its base.

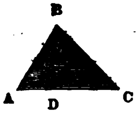
MENSURATION OF SURFACES.

Art. 191. The area or surface of a figure is the number of square inches, feet, yards, &c., which it contains.

A square constructed upon a straight line, of which the length is an inch, is called a *square inch;* and the same is to be understood of a square foot, &c. This is called the *measuring unit*, and the area of any figure is the number of times it contains the measuring unit.

Art. 192. To find the area of a triangle.

Rule. *Multiply the base by the altitude or perpendicular height; divide the product by 2; the quotient will be the area. Or, multiply the base by half the altitude; the product will be the area.*



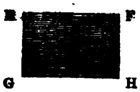
1. What is the area of the triangle A B C, supposing the base A B to be 24 feet, and the perpendicular B D, 18 feet?
Ans. 216 square feet.

2. What is the area of a triangular piece of land, the base of which is 30 rods, and its altitude 20 rods?
Ans. 300 square rods, or 1 a. 3 r. 20 rods.

3. What number of feet of boards will be required to cover the gable end of a barn, 36 feet in width, the perpendicular height of the roof being 12 feet? Ans. 216 square feet.

Art. 193. To find the area of a rectangle, or right-angled parallelogram.

Rule. *Multiply the base or longer side by the altitude or shorter side; the product will be the area.*



1. What is the area of the rectangle E F, G H, supposing the base or longer side, G H, to be 36 feet, and the altitude, E G, 24 feet?
Ans. 864 square feet.

2. What number of feet of boards would be required for the upper floor of a rectangular school-room, 72 feet in length, and 42 feet in width? Ans. 3024 square feet.

3. What number of acres are there in a rectangular lot of land, each of the longer sides measuring 275 rods, and each of the shorter sides, 225 rods?

Art. 194. To find the area of a square, or equilateral rectangle.

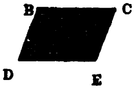
Rule. *Multiply the base by the altitude; or multiply the length of its side by itself; the product will be the area.*

1. What is the area of the square A B, C D, supposing the length of its side to be 36 feet? Ans. 1296 square feet.

2. What number of acres are there in a square township of land, each of its sides being 1920 rods in length?

Art. 195. To find the area of a rhomboid, or oblique-angled parallelogram.

Rule. *Multiply the base by the altitude; or multiply one of the longer sides by the perpendicular let fall from its opposite angle upon the base; the product will be the area.*

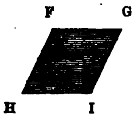


1. What is the area of the rhomboid B C, D E, supposing the base, D E, to be 60 feet, and the perpendicular let fall from the angle B upon the base D E, 40 feet? Ans. 2400 square feet.

2. What is the area of a lot of land in the form of a rhomboid, its base being 72 feet and its altitude 24 feet?

Art. 196. To find the area of a rhombus, or equilateral rhomboid.

Rule. *Multiply the base by the altitude; the product will be the area.*

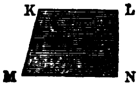


1. What is the area of the rhombus F G, H I, supposing the base H I to be 75 feet, and its altitude, or length of a perpendicular let fall from the angle F upon the base H I, to be 68 feet? Ans. 5100 square feet.

2. What is the area of a rhombus, its base being 25.5 feet, and its altitude 20.75 feet?

Art. 197. To find the area of a trapezoid.

Rule. *Multiply one half of the sum of the two parallel sides by the altitude; the product will be the area.*



1. What is the area of the trapezoid K L, M N, supposing the base M N to be 50 feet, its parallel side K L 44 feet, and its altitude 30 feet? Ans. 1410 square feet.

2. What number of square yards of carpeting will be required to cover a floor in the form of a trapezoid, its base or longest side being 18 feet, its parallel side 16 feet, and its altitude or width 15 feet?

Art. 198. To find the area of a trapezium.

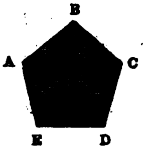
Rule. *Divide the trapezium into two triangles by a diagonal line; find the area of each of these triangles; the sum of their areas will be the area of the trapezium.*

1. What is the area of the trapezium M N, O P, supposing a diagonal line drawn from M to P, to be 40 feet, the altitude of the triangle M N P, 24 feet, and the altitude of the triangle M O P, 30 feet?
Ans. 1080 square feet.

The area of any right-lined figure having more than four unequal sides may be found by dividing it into triangles by diagonal lines, and then finding the area of each; the sum of these areas will be the area of the whole figure.

Art. 199. To find the area of a regular polygon.

Rule. *Multiply the perimeter of the polygon by half the perpendicular let fall from the centre upon one of its sides; the product will be the area.*



1. What is the area of the pentagon A B C D E, supposing each side to be 50 feet, and the perpendicular let fall from the centre to one of its sides 34.41 feet?
Ans. 4301.25 square feet.

2. What is the area of a regular hexagon whose side is 29.20 feet, and perpendicular from the centre 25.28 feet?

Art. 200. The following table shows the areas of the ten regular polygons when the side of each is equal to 1; it also shews the length of the perpendiculars or radii of the inscribed circles.

Number of sides.	Names of Polygons.	Areas or Multipliers.	Perpendiculars or Radii of inscribed circles.
3	Triangle,	0.4330127	0.2886751
4	Square,	1.0000000	0.5000000
5	Pentagon,	1.7204774	0.6881910
6	Hexagon,	2.5980762	0.8660254
7	Heptagon,	3.6339124	1.0382607
8	Octagon,	4.8284272	1.2071068
9	Nonagon,	6.1818242	1.3737387
10	Decagon,	7.6942088	1.5388418
11	Undecagon,	9.3656411	1.7028439
12	Dodecagon,	11.1961524	1.8660254

Since the areas of similar figures are to each other as the squares of their corresponding dimensions, we have the following rule for finding the area of any regular polygon.

Rule. *Find the square of the side of the given polygon; then multiply the area in the table opposite the polygon of the same number of sides by this square number; the product will be the required area.*

3. What is the area of a regular hexagon whose side is 12 feet? 12 × 12 = 144, and 2.5980762 × 144 = 374.1229728 square feet, the area.

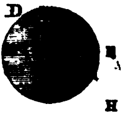
4. What is the area of a heptagon whose side is 10 yards?

5. What is the area of an octagon whose side is 20 rods?

6. What is the area of a dodecagon whose side is 30 feet?

Art. 201. TO FIND THE CIRCUMFERENCE OF A CIRCLE WHEN THE DIAMETER IS GIVEN.

Rule. *Multiply the diameter by 3.1416; the product will be the circumference.*



1. Suppose the diameter of the circle C D E to be 12 feet; what is the circumference?
12 × 3.1416 = 37.6992 feet, the circumference.

2. What is the circumference of a circle whose diameter is 20 feet? Ans. 62.832 feet.

3. What is the circumference of a circle whose diameter is 30 rods?

4. What is the circumference of the earth; the mean diameter being about 7912 miles?

Art. 202. TO FIND THE DIAMETER OF A CIRCLE WHEN THE CIRCUMFERENCE IS GIVEN.

Rule. *Divide the circumference by 3.1416; the quotient will be the diameter.*

1. Suppose the circumference of a circle to be 37.6992 feet, what is the diameter? 37.6992 ÷ 3.1416 = 12 feet, the diameter.

2. The circumference of a circular pond is 62.832 rods; what is the diameter?

Art. 203. TO FIND THE AREA OF A CIRCLE WHEN THE DIAMETER AND CIRCUMFERENCE ARE GIVEN.

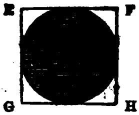
Rule. *Multiply half the circumference by half the diameter; the product will be the area.*

1. What is the area of a circle whose circumference is 37.6992 feet, and diameter 12 feet?
37.6992 ÷ 2 = 18.8496, and 12 ÷ 2 = 6; then 18.8496 × 6 = 113.0976 square feet, the area.

2. What is the area of a circular pond, whose circumference is 62.832 rods, and diameter 20 rods?

Art. 204. TO FIND THE AREA OF A CIRCLE WHEN THE DIAMETER ONLY IS GIVEN.

RULE. *Find the square of the diameter, and multiply it by the decimal* .7854; *the product will be the area.*



1. Suppose the circle inscribed in the square E F, G H, to be 100 inches in diameter, what is the area?

100 × 100 = 10000, the number of square inches in the square which circumscribes the circle, and 10000 × .7854 = 7854 square inches, the area of the circle.

2. There is a circular grass plat 60 feet in diameter; what is the area?

Art. 205. TO FIND THE AREA OF A CIRCLE WHEN THE CIRCUMFERENCE ONLY IS GIVEN.

RULE. *Divide the square of half the circumference by* 3.1416; *the quotient will be the area.*

1. The circumference of a circle is 37.6992 rods; what is the area?

37.6992 ÷ 2 = 18.8496 × 18.8496 = 355.30742016, and 355.30742016 ÷ 3.1416 = 113.0976 square feet, the area.

2. How many square rods are there in a bowling green, whose circumference is 188.496 rods?

Art. 206. TO FIND THE SIDE OF A SQUARE EQUAL IN AREA TO A CIRCLE WHOSE DIAMETER OR CIRCUMFERENCE IS GIVEN.

RULE. *Multiply the diameter by* .8862; *the product will be the side of the equal square.*

Or, multiply the circumference by .2821; *the product will be the side of a square equal in area to the given circle.*

1. The diameter of a circle is 60 feet; what is the side of a square of equal area?

60 × .8862 = 53.172 feet, the side of the square.

2. The circumference of a circle is 188.496 feet; what is the side of a square equal in area?

188.496 × .2821 = 53.174 + feet, the side of the required square.

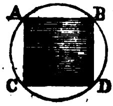
3. There is a circular grass plat 90 feet in diameter; what is the side of a square of equal area?

4. A man has a circular meadow, of which the circumference is 282.744 rods; what must be the side of a square of equal area?

Art. 207. HAVING GIVEN THE DIAMETER OR CIRCUMFERENCE OF A CIRCLE, TO FIND THE SIDE OF THE INSCRIBED SQUARE.

RULE. *Multiply the diameter by* .7071; *the product will be the side of the inscribed square.*

Or, multiply the circumference by .2251; and the product will be the side of the inscribed square.



1. Suppose the diameter of the circle at A B C D, to be 12 feet, what is the side of the inscribed square?

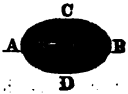
12 × .7071 = 8.4852 feet, the side of the inscribed square.

2. The circumference of a circle is 94.248 rods; what is the side of the inscribed square?

NOTE. The side of the circumscribed square is equal to the diameter of the circle.

Art. 208. TO FIND THE AREA OF AN ELLIPSIS OR OVAL.

RULE. *Multiply the longer diameter by the shorter, and their product by .7854; this product will be the area.*



1. Suppose the longer diameter A B, of the ellipsis A C B D, to be 12 inches, and the shorter diameter C D 9 inches, what is the area?

12 × 9 = 108 × .7854 = 84.8232 square inches, the area.

2. A lady has an elliptical flower-garden; its longer diameter is 60 feet, and its shorter diameter 40 feet; what is the area?

PRACTICAL QUESTIONS.

Art. 209. 1. How many square feet in a lot of land 72 feet 6 inches in length and 18 feet 9 inches in breadth?

2. Bought a house-lot in the form of a trapezoid; its base or longest side measures 75 feet, its parallel side 69 feet, its altitude or width 21 feet 6 inches. What is the number of square feet in the lot?

3. There is a marble pavement in the form of a regular decagon; the length of each side is 12 feet. What number of square feet does it contain?

4. There is a circular floor 60 feet in diameter; how many square yards of carpeting will be required to cover the floor?

5. There is an elliptical flower-garden; its transverse diameter is 90 feet, its conjugate diameter 60 feet. What number of square yards does it contain?

6. What must be the diameter of a circle that shall contain 1600 square feet? What is the side of its inscribed square?

7. There is an elliptical pavement; its transverse diameter is 24 feet, its conjugate diameter 18 feet. What is its superficial content in square yards?

Art. 210. TABLE OF PROPORTIONAL MULTIPLIERS AND DIVISORS.

1. *If the diameter of any circle*

$$\text{be}\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}\text{by}\left\{\begin{array}{l}3.14159\text{, the product}\\ .31831\text{, the quotient}\end{array}\right\}\text{is the circumference.}$$

2. *If the diameter of any circle*

$$\text{be}\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}\text{by}\left\{\begin{array}{l}.886227\text{, the product}\\ 1.128379\text{, the quotient}\end{array}\right\}\begin{array}{l}\text{is the side of an equal}\\ \text{square.}\end{array}$$

3. *If the diameter of any circle*

$$\text{be}\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}\text{by}\left\{\begin{array}{l}.866024\text{, the product}\\ 1.1547\text{, the quotient}\end{array}\right\}\begin{array}{l}\text{is the side of the equi-}\\ \text{lateral triangle inscr.}\end{array}$$

4. *If the diameter of any circle*

$$\text{be}\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}\text{by}\left\{\begin{array}{l}.707106\text{, the product}\\ 1.414213\text{, the quotient}\end{array}\right\}\begin{array}{l}\text{is the side of the square}\\ \text{inscribed.}\end{array}$$

5. *If the square of the diameter of any circle*

$$\text{be}\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}\text{by}\left\{\begin{array}{l}.785398\text{, the product}\\ 1.273241\text{, the quotient}\end{array}\right\}\text{is the area.}$$

6. *If the circumference of any circle*

$$\text{be}\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}\text{by}\left\{\begin{array}{l}.31831\text{, the product}\\ 3.14159\text{, the quotient}\end{array}\right\}\text{is the diameter.}$$

7. *If the circumference of any circle*

$$\text{be}\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}\text{by}\left\{\begin{array}{l}.282094\text{, the product}\\ 3.544907\text{, the quotient}\end{array}\right\}\begin{array}{l}\text{is the side of the}\\ \text{equal square.}\end{array}$$

8. *If the circumference of any circle*

$$\text{be}\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}\text{by}\left\{\begin{array}{l}.2756646\text{, the product}\\ 3.6275939\text{, the quotient}\end{array}\right\}\begin{array}{l}\text{is the side of the equi-}\\ \text{lateral triangle inscr.}\end{array}$$

9. *If the circumference of any circle*

$$\text{be}\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}\text{by}\left\{\begin{array}{l}.225079\text{, the product}\\ 4.442877\text{, the quotient}\end{array}\right\}\begin{array}{l}\text{is the side of the}\\ \text{square inscribed.}\end{array}$$

10. *If the square of the circumference of any circle*

$$\text{be}\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}\text{by}\left\{\begin{array}{l}.079577525\text{, the product}\\ 12.56636217\text{, the quotient}\end{array}\right\}\text{is the area.}$$

11. *If the area of any circle*

$$\text{be}\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}\text{by}\left\{\begin{array}{l}1.273241\text{, the product}\\ .785398\text{, the quotient}\end{array}\right\}\begin{array}{l}\text{is the square of}\\ \text{the diameter.}\end{array}$$

12. *If the area of any circle*

$$\text{be}\left\{\begin{array}{l}\text{multiplied}\\ \text{divided}\end{array}\right\}\text{by}\left\{\begin{array}{l}12.56636217\text{, the product}\\ .079577525\text{, the quotient}\end{array}\right\}\begin{array}{l}\text{is the square of the}\\ \text{circumference.}\end{array}$$

13. *When the diameter of one circle is* 1, *and the diameter of another is* 2, *the circumference of the first is equal to the area of the second,*— 3.141592.

14. *If the circumference be* 4, *the diameter and area are equal,*— 1.273241.

MENSURATION OF SOLIDS.

Art. 211. We have shown, Art. 191, that the unit of measure for plane surfaces is a square inch, foot, &c. We use similar units of measure in finding the area of curved surfaces.

The unit of measure for solids is a cube whose edge is a linear inch, foot, &c.,—and the content of any solid body is the number of times it contains the measuring unit.

Art. 212. To find the area or surface of a cube.

Rule. *Find the area of one of its sides, and multiply this area by 6; the product will be the area, or whole surface.*

1. Suppose each side of the cube A to be 20 inches, what is the area or whole surface?

20 × 20 = 400 × 6 = 2400 square inches, or 16⅔ square feet.

2. There is a cubic block of granite, each side of which is 6 feet; what is its area, or whole surface?

Art. 213. To find the solidity of a cube.

Rule. *Find the continued product of its length, breadth, and depth; this product will be its solid content.*

1. Suppose each side of the cube A to be 20 inches, what is its solid content?

20 × 20 × 20 = 8000 cubic inches, or 4$\frac{17}{27}$ cubic feet.

2. What number of cubic feet are there in a block of granite whose side is 6 feet?

Art. 214. To find the surface of a prism, or parallelopipedon.

Rule. *Multiply the perimeter of the base by the altitude or length, and add the area of the two ends to the product; the sum will be the required surface.*

1. Suppose the triangular prism B to be 60 inches in length, each side 12 inches, and the perpendicular 10.39 inches; what is the area or surface?

12 × 3 = 36 × 60 = 2160 square inches, the area of the sides; and 12 × 5.195 = 62.34 × 2 = 124.68 square inches, the area of the ends; and 2160 + 124.68 = 2284.68 square inches, the required area or surface.

2. What is the surface of a square prism, whose side is 18 inches, and length 15 feet?

Art. 215. TO FIND THE SOLIDITY OF A PRISM, OR PARALLELOPIPEDON.

RULE. *Find the area of the end or base, and multiply this area by the length or perpendicular height; the product will be the solid content.*

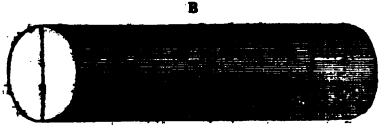
1. Suppose the triangular prism B to be 60 inches in length, each side 12 inches, and the perpendicular 10.39 inches, what is its solidity?

$12 \times 5.195 = 62.34 \times 60 = 3740.40$ cubic inches, the required solidity.

2. What is the number of cubic feet in a log of mahogany, 12 feet in length, and 2 feet square at each end?

Art. 216. TO FIND THE SURFACE OF A CYLINDER.

RULE. *Multiply the circumference of the end or base by the length or height, and add the area of the two ends to the product; their sum will be the area or surface.*



1. Suppose the cylinder B to be 12 feet in length, and its circumference 6.2832 feet, what is its area or surface?

$6.2832 \times 12 = 75.3984$ square feet, and $3.1416 \times 1 = 3.1416 \times 2 = 6.2832$ square feet, and $75.3984 + 6.2832 = 81.6816$ square feet, the required area or surface.

2. What is the surface of a cylinder whose length is 16 feet, and diameter 2.25 feet?

Art. 217. TO FIND THE SOLIDITY OF A CYLINDER.

RULE. *Find the area of the end or base, and multiply this area by the length or perpendicular height; the product will be the solidity.*

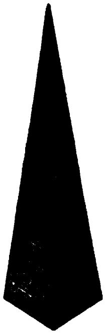
1. Suppose the cylinder B to be 12 feet in length, and its circumference 6.2832 feet, what is its solidity?

$3.1416 \times 1 = 3.1416 \times 12 = 37.6992$ cubic feet, the required solidity.

2. What is the solid content of a cylinder, the length of which is 16 feet, and the diameter of the base or end 2.25 feet?

Art. 218. TO FIND THE SURFACE OF A PYRAMID.

RULE. *Find, separately, the area of the base, and the areas of the triangles which constitute its sides; the sum of the area of the base and areas of the triangles will be the whole area or surface.*



C

1. Suppose the slant height of the triangular pyramid C to be 18 feet, each side of the base 6 feet, and the perpendicular of the base 5.196 feet; what is the whole area or surface of the pyramid?

$6 \times 2.598 = 15.588$ square feet, the area of the base; $6 \times 9 = 54 \times 3 = 162$ square feet, the area of the three sides of the pyramid; and $15.588 + 162 = 177.588$ square feet, the required surface of the pyramid.

2. What is the area or surface of a square pyramid, the slant height of which is 24 feet, and the side of its base 8 feet?

Art. 219. To find the solidity of a pyramid.

Rule. *Find the area of the base; multiply this area by one third of its altitude or perpendicular height; the product will be its solidity.*

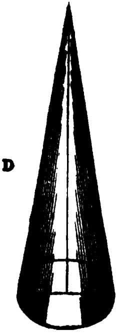
1. Suppose the altitude or perpendicular height of the triangular pyramid C to be 15 feet, and each side of the base 6 feet, and the perpendicular of the base 5.196 feet, what is its solidity?

$6 \times 2.598 = 15.588$ square feet, area of the base; and $15.588 \times 5 = 77.94$ cubic feet, the required solidity.

2. What is the number of cubic feet in a square pyramid, the altitude of which is 36 feet, and the side of its base 12 feet?

Art. 220. To find the surface of a cone.

Rule. *Multiply the circumference of the base by half the sum of the slant height and the radius of the base; the product will be the whole surface.*



D

1. Suppose the slant height of the cone D to be 18 feet, and the radius of the base 3 feet, what is its whole surface?

$18.8496 \times 10.5 = 197.9208$ square feet, the required surface.

2. What is the surface of a cone, the slant height of which is 36 feet, and its diameter 12 feet?

Art. 221. TO FIND THE SOLIDITY OF A CONE.

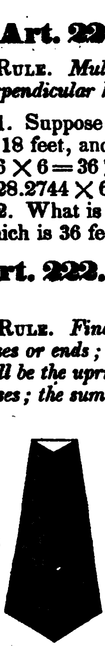
RULE. *Multiply the area of the base by one third of its altitude or perpendicular height; the product will be its solidity.*

1. Suppose the altitude or perpendicular height of the cone D to be 18 feet, and the diameter of the base 6 feet; what is its solidity?
$6 \times 6 = 36 \times .7854 = 28.2744$ square feet, the area of the base.
$28.2744 \times 6 = 169.6464$ cubic feet, the required solidity.

2. What is the number of cubic feet in a cone, the altitude of which is 36 feet, and the diameter of its base 12 feet?

Art. 222. TO FIND THE AREA OR SURFACE OF A FRUSTUM OF A PYRAMID OR CONE.

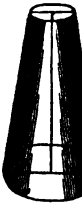
RULE. *Find the sum of the perimeters or circumferences of the two bases or ends; multiply the sum by half the slant height; the product will be the upright or curve surface; to which add the areas of the two bases; the sum will be the whole surface.*



E

1. Suppose the slant height of the triangular frustum E to be 18 feet, each side of the base 12 feet, the perpendicular of the base 10.39 feet, each side at the top 6 feet, and the perpendicular 5.2 feet; what is its area or surface?
$36 + 18 = 54 \times 9 = 486$ sq. feet, the upright surface.
$12 \times 5.195 = 62.34$ sq. feet, the area of the base.
$6 \times 2.6 = 15.6$ sq. feet, the area of the top.
$486 + 62.34 + 15.6 = 563.94$ sq. feet, the required surface.

2. What is the area or surface of the frustum of a square pyramid, the slant height of which is 24 feet, the side at the base 18 feet, and the side at the top 9 feet?



F

3. Suppose the slant height of the frustum of a cone F to be 20 feet, the diameter at the base 6 feet, and the diameter at the top 4 feet; what is its area or whole surface?
$6 \times 3.1416 = 18.8496$ feet, the circumference at the base; $4 \times 3.1416 = 12.5664$ feet, the circumference at the top; $18.8496 + 12.5664 = 31.416 \times 10 = 314.16$ sq. feet, the curve surface; $9.4248 \times 3 = 28.2744$ sq. feet, the area of the base; $6.2832 \times 2 = 12.5664$ sq. feet, the area of the top; $314.16 + 28.2744 + 12.5664 = 355.0008$ sq. feet, the required area or whole surface.

4. What is the surface of the frustum of a cone, the slant height of which is 60 feet, its diameter at the base 24 feet, and its diameter at the top 12 feet?

5. What is the area or surface of the frustum of a cone, the slant height of which is 75 inches, its diameter at the base 30 inches, and its diameter at the top 15 inches?

Art. 223. TO FIND THE SOLID CONTENT OF A FRUSTUM OF A PYRAMID.

RULE. *If it be the frustum of a square pyramid, multiply the side of the greater base by the side of the less; to this product add one third of the square of the difference of the sides; the sum will be the mean area between the bases: but if the base be any other regular figure, multiply this sum by the proper multiplier of its figure in the table, Art. 200, and the product will be the mean area between the bases; then multiply this mean area by the perpendicular height of the frustum, the product will be the solid content.*

1. Suppose the perpendicular height of the triangular frustum E to be 12 feet, the side of the base at the bottom 4 feet, the side at the top 2 feet; what is its solid content?

4 × 2 = 8 + 1.333 = 9.333 × .433 = 4.041189 square feet, the mean area. And 4.041189 × 12 = 48.494268 cubic feet, the solid content.

2. What is the number of cubic feet in the frustum of a square pyramid, the perpendicular height of which is 36 feet, the side at the base 12 feet, and the side at the top 6 feet?

Art. 224. TO FIND THE SOLID CONTENT OF THE FRUSTUM OF A CONE.

RULE. *Multiply the diameter of the base by the diameter of the top; to the product add one third of the square of the difference of the diameters; then multiply the sum by* .7854; *the product will be the mean area between the two ends. Multiply the mean area by the perpendicular height; the product will be the solid content.*

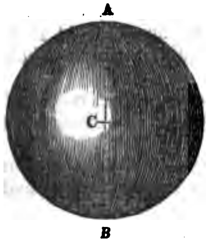
1. Suppose the frustum of a cone F to be 15 feet in height, 6 feet in diameter at the base, and 3 feet in diameter at the top, what is its solidity?

6 × 3 = 18 + 3 = 21 × .7854 = 16.4934 square feet, the mean area; and 16.4934 × 15 = 247.401 cubic feet, the required solidity.

2. What is the number of cubic feet in a mast, 60 feet in length, 3 feet in diameter at the base, and 1.5 feet in diameter at the top?

Art. 225. TO FIND THE AREA OR CONVEX SURFACE OF A SPHERE OR GLOBE.

RULE. *Multiply the circumference by the diameter.*



1. Suppose the diameter A B of the globe in the margin to be 12 feet, what is its convex area or surface?

12 × 3.1416 = 37.6992 × 12 = 452.3904 square feet, the surface required.

2. What is the surface of a sphere whose diameter is 7912 miles?

Art. 226. TO FIND THE SOLIDITY OF A SPHERE OR GLOBE.

RULE. *Multiply the surface by one sixth of the diameter; the product will be the solidity.*

Or, multiply the cube of the diameter by .5236; the product will be the solid content.

1. Suppose the diameter A C of the globe in the margin to be 9 feet, what is its solidity?

9 × 9 × 9 = 729 × .5236 = 381.7044 cubic feet, the required solidity.

2. What is the solid content of the globe on which we live, supposing the diameter to be 7912 miles?

Art. 227. TO FIND THE SOLIDITY OF A WEDGE.

RULE. *To twice the length of the base add the length of the edge; multiply the sum by the breadth of the base, and the product by one sixth of the perpendicular from the edge upon the base; this product will be the solidity.*

1. What is the number of cubic inches in a wedge, of which the perpendicular height is 8.7 inches, the edge 3.5 inches, and the base or head 3 inches in length and 2 inches in breadth?

Ans. 27.55 cubic inches.

Art. 228. THE FIVE REGULAR BODIES.

A REGULAR BODY is a solid bounded by similar and regular plane figures. There are five such regular solid bodies, viz. the Tetraedron, the Hexaedron, the Octaedron, the Dodecaedron, and the Icosaedron.

The Tetraedron, or equilateral pyramid, is a solid bounded by four equilateral triangles.

The Hexaedron, or cube, is a solid bounded by six equal squares.

The Octaedron is a solid bounded by eight equilateral triangles.

The Dodecaedron is a solid bounded by twelve regular and equal pentagons.

The Icosaedron is a solid bounded by twenty equilateral triangles.

The following table shows the surface and solidity of each of the five regular solid bodies, the linear edge of each being unity or 1.

No. of sides.	Names.	Surfaces.	Solidities.
4	Tetraedron.	1.7320508	0.1178511
6	Hexaedron.	6.0000000	1.0000000
8	Octaedron.	3.4641016	0.4714045
12	Dodecaedron.	20.6457788	7.6631189
20	Icosaedron.	8.6602540	2.1816950

Similar surfaces and solids are those which have their corresponding dimensions proportional. Since the areas of similar surfaces are to each other as the squares of their corresponding dimensions, we have the following rule for finding the surface of any regular solid, when the length of its linear edge is given.

Rule. *Multiply the tabular number in the column of surfaces by the square of its linear edge; the product will be the surface required.*

1. The linear edge of an octaedron is 6 feet; what is its surface?

The tabular area is 3.4641016. 6 × 6 = 36. And 3.4641016 × 36 = 124.7076576 square feet, the surface required.

Since the contents of similar solids are to each other as the cubes of their corresponding dimensions, we have the following rule for finding the content of any regular solid, when the length of its linear edge is given.

Rule. *Multiply the tabular number in the column of solidities by the cube of its linear edge; the product will be the solidity required.*

2. What is the solidity of a regular icosaedron, whose linear edge is 5 feet?

5 × 5 × 5 = 125. And 2.181695 × 125 = 272.711875 cubic feet, the required solidity.

3. What is the solidity of a regular octaedron, whose linear edge is 6 feet?

PRACTICAL QUESTIONS.

Art. 229. 1. What is the solidity of an octangular prism, whose side measures 6 inches, and whose length is 6 feet? What is the area of its surface?

2. There is a cylindrical granite pillar, 32 feet in length, 6 feet in diameter at the base, and 4 feet in diameter at the top. What is the area of its surface in square feet? What is its solidity in cubic feet?

3. What is the content of a cylindrical cistern in imperial gallons, whose depth is 6 feet, inside diameter at bottom 5 feet 9 inches, at top 5 feet 3 inches?

4. What is the content of a square pyramid, 750 feet in perpendicular height, and each side of its base measuring 620 feet?

5. What number of bushels of grain will a cubical bin contain, whose length inside is 8 feet, breadth 5 feet, and depth 4 feet?

6. There is a globe composed of India rubber cloth, 3 feet in diameter; what number of square yards of cloth were required to make the globe, making no allowance for waste; and what is its content in imperial gallons?

MENSURATION OF BOARDS AND TIMBER.

Art. 220. The unit of measure for boards, plank and joist, is the square foot.

TO FIND THE NUMBER OF SQUARE FEET IN A BOARD.

RULE. *Multiply the length in feet by the width in inches; divide the product by 12; the quotient will be the square feet. If the board is tapering, add the width of the two ends, and half the sum is the mean width.*

1. What is the number of square feet in a board, 16 feet in length, 15 inches wide at one end, and 13 inches at the other?

$15 + 13 = 28 \div 2 = 14$ in., the mean width; and $16 \times 14 = 224 \div 12 = 18\frac{2}{3}$ square feet.

TO FIND THE NUMBER OF FEET, BOARD MEASURE, IN A PLANK OR JOIST.

RULE. *Multiply the length in feet by the width in inches, and the product by the depth in inches; divide this product by 12; the quotient will be square feet.*

2. What number of square feet in a plank, 12 feet in length, 15 inches in width, and 2 inches in thickness? Ans. 30 square feet.

3. What number of square feet in a joist, 16 feet in length, 3 inches in width or thickness, and 9 inches in depth?

Art. 231. TO FIND THE SOLIDITY OF SQUARE OR RECTANGULAR TIMBER WHICH DOES NOT TAPER.

RULE. *Find the area of one end by multiplying the width by the depth; then multiply this area by the length; the product will be the solidity.*

1. What is the solidity of a stick of timber, 16 feet in length, 15 inches in width, and 12 inches in depth? Ans. 20 cubic feet.

2. What is the number of cubic feet in a log of mahogany, 12 feet in length, 30 inches in width, and 27 inches in depth?

Art. 232. TO FIND THE CONTENT OF ANY SOLID, OF WHICH THE BASES ARE PARALLEL, AND THE GREATEST AND LEAST THICKNESSES OR DIAMETERS ARE AT ITS ENDS.

RULE. *Find the area of each end, also the mean area between the two ends; multiply the mean area by 4; to the product add the area of each end, and multiply the sum by one sixth of the length; this product will be the solid content.*

NOTE 1. When the sides of the solid are straight between the ends, half the sum of two corresponding sides or diameters will be the mean corresponding side or diameter.

1. What number of cubic feet in a stick of timber 24 feet in length, the ends of which are 24 inches by 21, and 18 inches by 15?

Ans. 63.5 cubic feet.

2. What is the content of a pine log which is 18 feet in length, 21 inches in diameter at the larger end, and 18 inches in diameter at the smaller?

Art. 233. TO FIND THE SIDE OF THE LARGEST SQUARE STICK OF TIMBER THAT CAN BE HEWN OR SAWN FROM A ROUND LOG.

RULE. *Multiply the diameter of the smaller end of the log by .7071, the product will be the side of the inscribed square or the required side.*

1. What will be the side of the largest stick of square timber which can be hewn from a round log, 18 inches in diameter at the smaller end? $18 \times .7071 = 12.7278$ inches.

2. What will be the side of the largest stick of square timber that can be sawn from a round log 12 inches in diameter at the smaller end?

Art. 234. TO FIND THE NUMBER OF SQUARE FEET OF BOARDS WHICH CAN BE SAWN FROM A LOG OF ANY GIVEN DIMENSIONS.

RULE. *Find the side of the largest square stick of timber which can be sawn from the given log, Art.* 233. *Divide the side by the thickness of the board,* plus *the thickness of the saw; the quotient will be the number of boards. Multiply the number of square feet in one board by the number of boards, the product will be the required number of square feet.*

1. What is the number of feet of boards, 1.25 inches in thickness, including the thickness of the saw, which can be sawn from a log 12 feet in length, and 14.25 inches in diameter at the smaller end?

$14.25 \times .7071 = 10.076+$ inches, the side of the inscribed square; $10.076 \div 1.25 = 8+$, the number of boards 10 inches in width, and $12 \times 10 = 120 \div 12 = 10$, the number of square feet in one board, and $10 \times 8 = 80$, the number of feet required.

2. What is the number of feet of boards, 1.25 inches in thickness, which can be sawn from a log 16.75 inches in diameter at the smaller end, and 16 feet in length?

MENSURATION OF MASONRY.

Art. 235. Masonry includes all kinds of work of which stone and brick are the principal materials.

The measuring unit for walls of stone or brick, columns, blocks of stone or marble, is the cubic foot.

The measuring unit for pavements, slabs, chimney pieces, &c., is the square foot.

Art. 236. TO FIND THE SOLID CONTENT OF THE WALLS OF A RECTANGULAR BUILDING, OF ANY GIVEN DIMENSIONS.

RULE. *From the outside perimeter of the building, subtract four times the thickness of the walls; the remainder will be the length of the walls. Multiply the length by the height, and the product by the thickness; the result will be the solid content.*

1. What is the number of cubic feet in the walls of a house 40 feet by 24, the walls being 16 inches in thickness, and 32 feet in height, no deduction being made for doors and windows?

$40+40+24+24=128-5\frac{1}{3}=122\frac{2}{3}$, and $122\times 32\times 1\frac{1}{3}=5233\frac{7}{9}$ cubic feet.

2. What is the number of cubic feet in the walls of a school-house 75 feet by 54, and 40 feet in height, the walls being 20 inches in thickness to the height of 28 feet, and the remaining 12 feet 16 inches, making no deduction for doors and windows?

Art. 237. TO FIND THE NUMBER OF BRICKS WHICH WILL BE REQUIRED TO BUILD A WALL OF ANY GIVEN DIMENSIONS.

RULE. *Find the content of the wall in cubic feet, find also the number of bricks of the given dimensions that will be required to build a cubic foot, multiply the number of cubic feet in the wall by the number of bricks in a cubic foot, the product will be the required number of bricks. Bricks of the usual size are* 8 *inches in length,* 4 *inches in width, and* 2 *inches in thickness.*

$8\times 4\times 2=64$, the number of cubic inches in one brick, and $1728\div 64=27$, the number of bricks in a cubic foot.

A deduction of about one tenth of the required number of bricks is usually made for the thickness of the mortar between the courses.

1. What number of bricks of the usual size will be required to build a garden wall 75 feet in length, 6 feet in height, and the width of 4 bricks in thickness?

$75\times 6\times 1\frac{1}{3}=600\times 27=16200$, the required number of bricks.

2. What number of bricks will be required to build the walls of the school-house, of which the dimensions were given in question 2d, Art. 236?

Art. 238. TO FIND THE CONTENTS OF CIRCULAR WALLS, SUCH AS THE WALLS OF RESERVOIRS, CISTERNS, DRAINS, AND WELLS.

The length of a circular wall is the circumference of a circle whose diameter is the thickness of the wall longer than the inside diameter.

1. What number of bricks will be required to build a circular cistern, 8 feet in depth, 6 feet 4 inches in diameter inside, the circular walls being 8 inches in thickness, and the bottom 4 inches?

6 ft. 4 in. + 8 in. = 7 ft. $\times 3.1416=21.9912$ ft., the circumference or length of the wall. $21.9912\times 8=175.9296\times\frac{8}{12}=117.2864$ cubic ft. in the circular wall.

6 ft. 4 in. + 1 ft. 4 in. = 7 ft. 8 in., the diameter of the bottom And $7.666 \times 3.1416 = 24.0835$, and $12.04175 \times 3.833 \times \frac{1}{3} = 15.3853+$, the number of cubic feet in the bottom. And $117.2864 + 15.3853 = 132.6717 \times 27 = 3582.1359+$, the required number of bricks.

Art. 239. To find the number of perches of stone in a wall of any given dimensions.

Rule. *Find the number of cubic feet in the given wall, and divide by* 24.75, *the quotient will be the number of perches.*

1. What number of perches of stone will be required for the walls of a cellar whose length is 33 feet, and breadth 18 feet, within the walls, height of walls 8 feet, thickness 1.5 feet?

Art. 240. To find the solidity of any irregular body whose dimensions cannot be ascertained.

Rule. *Immerse the solid in some regular formed vessel, partly filled with water, and the contents of that part of the vessel filled by the rise of the water will be the solid content of the body immersed.*

A school-boy, wishing to find the solid content of an irregular piece of granite, immersed it in a vessel 15 inches square, partly filled with water, and found that the water rose 4 inches; what was the solid content of the piece of granite?

Ans. 900 cubic inches.

Art. 241. The unit of measure for excavations, such as canals, cellars, wells, &c., is a cube whose side is 6 feet, called a square of earth, and contains 216 cubic feet. Excavations are also estimated by the cubic yard.

TO FIND THE NUMBER OF SQUARES OF EARTH IN ANY GIVEN EXCAVATION.

Rule. *Find the content in cubic feet, and divide it by* 216; *the quotient will be the number of squares.*

1. What number of squares of earth must be removed in digging a rectangular cellar 36 feet in length, and 21 feet in width; depth at the corners 8 feet, 7 feet, 6 feet, and 5 feet?

Note. The mean depth may be found by dividing the sum of the depths at the corners by 4; the quotient will be the mean depth.

$8 + 7 + 6 + 5 = 26 \div 4 = 6.5$ ft., the mean depth. $36 \times 21 \times 6.5 = 4914 \div 216 = 22.75$ squares.

2. What number of cubic yards of earth must be removed in digging a well 24 feet in depth, 6 feet in diameter at the top, and 5 feet at the bottom?

Art. 242. Artificers compute the contents of their work by different units of measure. *Painting, plastering,* and *paving,* are estimated by the square yard. *Slating* and *tiling* are usually estimated by the square of 100 square feet. *Glazing* is estimated by the square foot. *Plumbers'* work is usually estimated by the pound or hundred weight.

Sheet-lead, used in roofing, &c., weighs from 7 to 12 lb. per square foot. Leaden pipes of $\frac{3}{4}$ inch bore weigh 10 lb.; of 1 inch bore, 12 lb.; of $1\frac{1}{4}$ inch bore, 16 lb.; of $1\frac{1}{2}$ inch bore, 18 lb.; of $1\frac{3}{4}$ inch bore, 21 lb.; and of 2 inches bore, 24 lb. per yard, in length.

1. What is the number of square yards of paving in a side-walk 66 feet in length, and 8 feet in width? Ans. $58\frac{2}{3}$ square yards.

2. How many squares of slating are there in the roof of a house 44 feet in length, and each side of the roof 24 feet in breadth?

Ans. $21\frac{3}{25}$ squares.

GAUGING.

Art. 243. GAUGING IS THE METHOD OF FINDING THE CAPACITY OF CASKS, AND OTHER VESSELS, IN GALLONS.

RULE. *Take the length of the cask in inches, also the diameter at the bung and head; subtract the head diameter from the bung diameter, and note the difference.*

If the staves of the cask are much curved, multiply the difference noted by .7; *if they are but little curved, by* .6; *if they are of a medium curve, by* .65; *add the product to the head diameter; the sum is the mean diameter, which changes the form of the cask to a cylinder.*

Find the square of the mean diameter and multiply it by .7854; *the product is the mean area; then multiply this mean area by the length of the cask; the product is the capacity of the cask in cubic inches.*

Divide this product by 231, *the quotient will be the number of wine gallons; or divide it by* 282 *the quotient will be the number of beer gallons, that the cask will contain.*

The length of the cask is usually taken with callipers, and from 1 to 2 inches is deducted for the thickness of the heads, according to the size of the cask.

The head diameter should be taken within the chimes and close to the head, and from .2 to .5 of an inch, according to the size of the cask, must be added, on account of the diameter being greater inside the head.

1. What number of wine gallons will a cask contain, 36 inches in length, 20 inches in diameter at the bung, and 17 inches in diameter at the head; the staves being much curved?

Ans. 44.65 + wine gallons.

2. What number of beer gallons will a cask contain, whose bung diameter is 32 inches, head diameter 27 inches, and length 45 inches; the staves being of a medium curve?

STRENGTH OF MATERIALS.

Art. 244. MATERIALS are exposed to four different kinds of strains.

1st. They may be torn asunder, as in the case of ropes, tie-beams, and stretchers. The strength of a body to resist this kind of strain is called its resistance to tension, or absolute strength.

2d. They may be crushed or compressed in the direction of their length, as in the case of columns and posts.

3d. They may be broken across, as in the case of beams and joists. The strength of a body to resist this kind of strain is called its lateral strength.

4th. They may be twisted or wrenched, as in the case of axles and screws.

Extensive and accurate experiments are necessary to determine the several measures of these strengths in the different materials.

TABLE I.

Showing the weight in pounds that will pull asunder a prism one inch square, of the following materials, according to the experiments of Mr. Rennie:

	Pounds.		Pounds.
Cast steel, . .	113077	Yellow Brass, . .	17958
Swedish iron, . .	72064	Cast tin, . . .	4736
English iron, . .	55872	Cast lead, . . .	1824
Cast iron, . .	19096	Good hemp rope, .	6400
Cast copper, . .	19072	Do. 1 inch in diameter,	5026

TABLE II.

Showing the cohesive strength of a bar of iron of different kinds, one inch square, obtained from different experimentists, expressed in pounds.

	Pounds.		Pounds.
Iron wire, . .	113077	English iron, . .	61600
" " . .	93964	" " . .	65772
Swedish iron, . .	78850	Welsh iron, . .	64960
" " . .	72064	" " . .	55776
" " . .	54960	French iron, . .	61001
" " . .	53244	Russian iron, . .	59472
German iron, . .	69133	Cast iron, . .	18295
English iron, . .	66900	" " . . .	19488
" " . .	55000	Welsh cast iron, .	16255

TABLE III.

Showing the lateral strength of the following materials, the bar being one foot long, and one inch square, expressed in pounds.

	Weight that will break them.		Weight which they can bear with safety.
Cast iron, . . .	3270	. . .	1092
Oak, . . .	627	. . .	209
Memel fir, . .	390	. . .	130
American white pine,	206	. . .	69

Art. 245. *The strength of materials to resist a strain lengthwise is in proportion to the squares of their diameters, or similar sides.*

1. If a good hemp rope, 1 inch in diameter, will resist a strain of 6400 pounds, what will be the resistance of a cable of the same materials, 3 inches in diameter?

$3 \times 3 = 9 \times 6400 = 57600$ lbs. Ans.

2. If the cohesive strength of a bar of English iron 1 inch square be 66900 pounds, what is the cohesive strength of a bar 4 inches square? Ans. 1070400 lbs.

Art. 246. *The lateral strength of bars and beams of the same material, and of the same length, is in proportion to their width multiplied by the square of their depth.*

In square bars and beams, the lateral strengths are in proportion to the cubes of the sides.

1. Suppose a bar of cast iron 1 inch square, and 10 feet in length, to bear a weight of 320 pounds, what number of pounds will a bar of the same length, and 3 inches square, support?

$3 \times 3 \times 3 = 27 \times 320 = 8640$ lbs. Ans.

2. If a joist 10 feet long, 2 inches in thickness, and 4 inches in depth, sustain a weight of 1280 pounds, what number of pounds will a joist of the same length sustain, which is 3 inches in thickness, and 9 inches in depth? Ans. 9720 lbs.

Art. 247. *The strength of bars, joists, and beams, of the same width and depth, is diminished in proportion to their length.*

1. There are two joists of the same depth and thickness; one is 15 feet in length, the other 12; the shorter joist will sustain a weight of 1500 pounds; what number of pounds will the longer joist sustain?

Ans. 1200 lbs.

2. There are two beams of the same depth and thickness; one is 24 feet in length, and the other 18 feet; the longer beam will sustain a weight of 9000 pounds; what number of pounds will the shorter beam sustain? Ans. 12000 lbs.

Art. 248. *The strength of a hollow cylinder is to that of a solid cylinder of the same length, and the same quantity of matter, as the greater diameter of the hollow cylinder is to the diameter of the solid cylinder; and the strength of hollow cylinders of the same length, weight, and materials, are as their greater diameters.*

1. What is the comparative strength of a solid cast iron cylinder 12 feet in length, and 3 inches in diameter, with a cast iron hollow cylinder of the same weight and length, whose greater diameter is 6 inches?

Ans. The solid cylinder is one half the strength of the hollow cylinder.

2. There are two hollow cylinders of the same length, weight, and material; the greater diameter of one of them is 3 inches, and the greater diameter of the other is 5 inches; what is their comparative strength?

Art. 249. *The strength of upright columns, or posts, to resist being crushed, is in proportion to the squares of their diameters or similar sides.*

1. There are two solid cast iron cylinders of the same length; one is 3 inches in diameter, and the other 4 inches; what is their comparative strength of resistance against being crushed?

2. There are two upright oak posts of the same length; one is 6 inches square, and the other is 8 inches; what is their comparative strength of resistance against being crushed?

Art. 250. *The strength of homogeneous cylinders to resist being twisted round their axes is as the cubes of their diameters; and this holds true of hollow cylinders, if their quantities of matter be the same.*

1. There are two similar screws of the same length and material; one is 5 inches in diameter, and the other 6 inches; what is their comparative strength against being twisted?

Art. 251. *The strength or stiffness of beams and joists to resist flexure, which are of the same length and thickness, is in proportion to the cubes of their depth. If the length and depth are the same, the strength or stiffness is in proportion to the width.*

1. There are two joists of equal length and thickness; one is 6 inches in depth, and the other 9 inches; what is their comparative strength or stiffness?

2. There are two beams of the same length and depth; one is 8 inches wide, and the other 10 inches; what is their comparative strength or stiffness?

PRACTICAL QUESTIONS.

1. If the cohesive strength of a rod of English iron 1 inch in diameter be 43880 pounds, what will be the cohesive strength of a rod of the same kind of iron 3 inches in diameter?

2. If a joist 12 feet in length, 3 inches in thickness, and 9 inches in depth, sustain a weight of 9650 pounds, what number of pounds will a joist of the same length sustain, which is 4 inches in thickness, and 12 inches in depth?

3. If a beam of oak 8 inches square, and 10 feet in length, sustain a weight of 32100 pounds, what number of pounds will a beam of oak of the same length and 12 inches square sustain?

4. There are two beams of pine, of the same depth and thickness; one is 15 feet in length, and the other 21 feet; if the shorter beam will support a weight of 3584 pounds with safety, what number of pounds will the longer beam support?

MECHANICAL POWERS.

Art. 252. THE MECHANICAL POWERS are simple machines which are used to apply or regulate moving power, or to produce motion so as to save time or force.

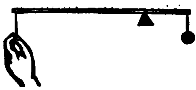
The mechanical powers are six in number, viz.: the *Lever*, the *Wheel and Axle*, the *Pulley*, the *Inclined Plane*, the *Wedge*, and the *Screw*.

The *weight* signifies the body to be moved, or the resistance to be overcome; and the *power* is the force employed to overcome that resistance, or to move that body. Weight and power are usually expressed by the letters W and P.

THE LEVER.

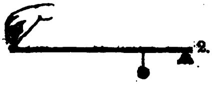
Art. 253. A LEVER is an inflexible bar of wood or metal, supported by a *fulcrum* or prop, on which it rests or turns.

There are three kinds of levers, which vary according to the relative positions of the weight, power, and fulcrum.



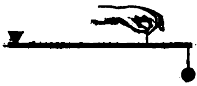
1.

In the first kind, the fulcrum is between the power and weight, as in figure 1.



2.

In the second, the weight is between the power and fulcrum, as is shown by figure 2.



3.

In the third, the power is between the weight and fulcrum, as is represented by figure 3.

In the first and second kinds, there is a gain of power, but a proportionate loss of velocity; and in the third kind, there is a gain in velocity, but a loss of power.

When the weight multiplied by its distance from the fulcrum equals the power multiplied by its distance from the fulcrum, the lever will be at rest, or in equilibrium; but if one of these products be greater than the other, the lever will turn on the fulcrum towards that side whose product is the greater.

1. *Hence, if the weight be multiplied by its distance from the fulcrum, and the product divided by the distance of the power from the fulcrum, the quotient will be the power required to balance the weight.*

2. *If the power be multiplied by its distance from the fulcrum, and the product divided by the distance of the weight from the fulcrum, the quotient will be the weight.*

3. *If the weight be multiplied by its distance from the fulcrum, and the product divided by the power, the quotient will be the distance of the power from the fulcrum.*

4. *If the power be multiplied by its distance from the fulcrum, and the product divided by the weight, the quotient will be the distance of the weight from the fulcrum.*

1. If a lever 12 feet in length rest on a fulcrum 2 feet from the weight, what power will be required to balance a weight of 2400 pounds?

$2400 \times 2 = 4800 \div 10 = 480$ lbs., the power required.

2. If a lever 10 feet in length rest on a fulcrum 1 foot from a weight of 1800 pounds, what power will be required to balance it?

3. In a lever of the second kind, supposing the weight to be 2 feet from the fulcrum, and a power of 480 pounds to be applied at the other end of the lever, 10 feet from the fulcrum, what weight would be raised? $480 \times 10 = 4800 \div 2 = 2400$ lbs., the weight.

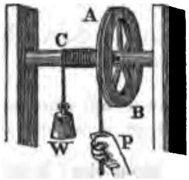
4. If a power of 200 pounds act 9 feet from a weight, on a lever of the second kind, and the weight 1 foot from the fulcrum, what weight will be raised?

5. If a weight of 2400 pounds is balanced by a power of 480 pounds, and the weight 2 feet from the fulcrum, what is the distance of the power from the fulcrum? Ans. 10 feet.

6. In a lever of the third kind, suppose the weight to be 300 pounds, and 6 feet from the fulcrum, and it requires a power of 400 pounds to balance the weight, what is its distance from the fulcrum?

THE WHEEL AND AXLE.

Art. 254. THE WHEEL AND AXLE is a kind of lever, so contrived as to have a continued motion about its fulcrum, or centre of motion, where the power acts at the circumference of the wheel, whose radius is one arm of the lever, and the other arm is the radius of the axle, on which the weight acts.



The wheel and axle consists of a wheel having a cylindrical axis passing through its centre. The power is applied to the circumference of the wheel, and the weight is attached to the circumference of the axle.

In the wheel and axle, an equilibrium takes place when the power multiplied by the radius of the wheel, is equal to the weight multiplied by the radius of the axle. Hence it follows,

1. *If the weigh be multiplied by the diameter of the axle, and the product divided by the diameter of the wheel, the quotient will be the power required to balance the weight.*

2. *If the power be multiplied by the diameter of the wheel, and the product divided by the diameter of the axle, the quotient will be the weight that will be balanced by the power.*

3. *If the weight be multiplied by the diameter of the axle, and the product be divided by the power, the quotient will be the diameter of the wheel.*

4. *If the power be multiplied by the diameter of the wheel, and the product divided by the weight, the quotient will be the diameter of the axle.*

1. In a store, the wheel used in raising merchandise is 6 feet in diameter, and its axle 1 foot; what power will be required to raise a bag of cotton weighing 360 pounds? Ans. 60 pounds.

2. If a power of 120 pounds be applied to the circumference of a wheel 5 feet in diameter, whose axis is 6 inches in diameter, what weight will be balanced? Ans. 1200 pounds.

3. If it be required to raise a weight of 360 pounds attached to an axle 1 foot in diameter, by applying a power of 60 pounds to the circumference of the wheel, what must be its diameter?

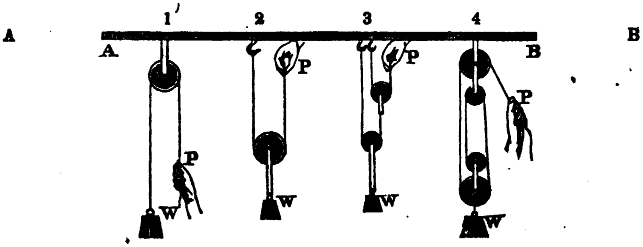
Ans. 6 feet.

4. If a power of 120 pounds be applied to the circumference of a wheel 5 feet in diameter, what must be the diameter of its axis to balance a weight of 1200 pounds? Ans. .5 of a foot.

THE PULLEY.

Art. 255. A Pulley is a grooved wheel revolving on an axle, around which a rope is passed, having a weight attached at one end, and a power applied at the other. The pulley may be either *fixed* or *movable*. If the pulley be fixed, no mechanical advantage is gained, since the power moves just as fast as the weight, and it only serves to change the direction of the power. But if the pulley be movable, the power acts with a mechanical advantage of two to one.

"The accompanying engraving exhibits various forms of the pulley. A B is a beam from which they are suspended.



"No. 1 is the fixed pulley in which there is no other advantage gained than that the power P and weight W move in a contrary direction. No. 2 is a movable pulley, in which the power P, by moving upwards, raises the pulley to the block to which the weight W is attached; but one end of the rope being attached to the beam A B, the power must move twice as fast as the weight, and there will be a proportional gain of power. No. 3 is a combination of two movable pulleys, in which the gain of power will be four; and No. 4 is a combination of two fixed and two movable pulleys, in which the gain of power will be the same as in No 3."

In a system of pulleys, where each pulley is embraced by a cord attached at one end to a fixed point, and at the other to the centre of the movable pulley next above it, and the weight attached to the lowest pulley, the effect produced will be equal to twice the number of movable pulleys in the system multiplied by the power. When there is a number of movable pulleys in one block, and the same number in a fixed block, and all connected by a rope passing around each, the effect will be the same. Therefore,

1. *If the power be multiplied by twice the number of movable pulleys, the product will be the weight.*

2. *If the weight be divided by twice the number of movable pulleys, the quotient will be the power.*

1. If a power of 160 pounds be applied to a rope connecting a system of of 4 movable pulleys, what weight can be raised?

Ans. 1280 pounds.

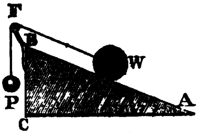
2. A weight of 2560 pounds is to be raised by using two blocks of pulleys, one fixed, the other movable, in which a rope passes over 8 movable pulleys; what power will be required?

Ans. 160 pounds.

THE INCLINED PLANE.

Art. 256. An Inclined Plane is a slope, or declivity, used in raising and lowering weights.

The inclined plane becomes a mechanical power in consequence of its supporting a part of the weight, and leaving only a part of it to be supported by the power.



When a power acts upon a body supported partly by an inclined plane, so as to keep that body at rest; then the weight, the power, and the pressure on the plane, will be as the length, the height, and the base of the plane, when the power acts in a direction parallel with the plane. The weight will be as A B, the power will be as C B, and the pressure on the plane will be as A C. Therefore,

1. *If the weight be multiplied by the height of the plane, and the product divided by the length of the plane, the quotient will be the power.*

2. *If the power be multiplied by the length of the plane, and the product divided by the height of the plane, the quotient will be the weight.*

3. *If the weight be multiplied by the base of the plane, and the product divided by the length of the plane, the quotient will be the pressure on the plane.*

1. An inclined plane is 80 feet in length, and 16 feet in perpendicular height; what power will be required to balance a weight of 4000 pounds? Ans. 800 pounds.

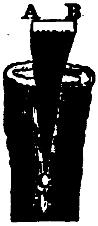
2. The height of an inclined plane is 16 feet, its length is 80 feet; what weight will a power of 800 pounds balance?

Ans. 4000 pounds.

3. The length of an inclined plane is 80 feet, its base is 78.38 feet. If a power balance a weight of 4000 pounds on this plane, what will be the pressure on the plane? Ans. 3919 pounds.

THE WEDGE.

Art. 257. The Wedge is a double inclined plane. It is used in splitting logs of wood and masses of stone.

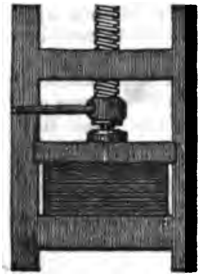


A B C in the diagram is the form of a wedge, and the diagram exhibits the action of a wedge in splitting a log. The resistance to be overcome is the cohesive attraction of the wood, or any other body which the wedge is used to separate, and the advantage gained by this power is in the proportion of half its thickness to its length. The wedge acts principally by being struck on the head with a hammer or mallet, and not by mere pressure. The proportion stated above is that which expresses its power when acting by pressure only.

Cutting instruments, such as axes, knives, and the like, act on the principle of the wedge. When long and proportionally thin, the wedge is a mechanical power of great force.

THE SCREW.

Art. 258. The Screw is a spiral inclined plane, cut round a cylinder, the height of the plane being the distance between the centres of two threads, and its length the circumference of the cylinder.



"The screw is represented in the adjoining diagram as acting upon a press, which is a very common use that is made of it. As the screw is turned, it advances lengthwise through a space just equal to the distance between the threads.

"Now if the power be applied directly to the head of the screw, then, in turning the screw once round, the power would move over as much more space than the screw advances, as the circumference of the head is greater than the distance between the threads. The mechanical advantage gained is in the same proportion; and we may increase the efficacy of the power either by diminishing the distance between the threads, or by increasing the space over which the power moves. If we attach a lever to the head of the screw, and apply the hand at the end, then we make the power move over a space vastly greater than that through which the screw advances, and the force becomes very powerful, and will urge down the press upon the books, or anything in the press, with great energy."

As the force or pressure is greater than the power in the same proportion that the circumference of the circle described by one revolution of the power is greater than the distance between two continuous threads of the screw, hence it follows,

1. *If the power be multiplied by the circumference of the circle described by one revolution of the power, and the product divided by the distance between two threads of the screw, the quotient will be the force or pressure.*

2. *If the force or pressure required be multiplied by the distance between two threads of the screw, and the product divided by the circumference of the circle described by one revolution of the power, the quotient will be the power.*

1. If the threads of a screw be .5 of an inch apart, and a power of 120 pounds be exerted at the end of a lever 72 inches in length, what will be the force or pressure produced?

Ans. 108573.696 pounds.

2. If the threads of a screw be .5 of an inch apart, what power must be exerted at the end of a lever 72 inches in length, to effect a pressure of 108573.696 pounds upon a quantity of books?

Ans. 120 pounds.

REMARKS ON THE MECHANICAL POWERS.

"The mechanical powers may be variously modified and applied, but still they form the elements of all machinery. In our calculations of their effects, we have made no allowance for friction, or the resistance arising from one body rubbing against another. There are no surfaces which are perfectly smooth. When, therefore, the surfaces of two bodies come in contact, the prominent parts of the one will often fall into the hollow parts of the other, and occasion more or less resistance to motion. In proportion as the surfaces of bodies are well polished the friction is diminished, but it is always considerable, and it is usually computed to destroy one third of the power of the machine. All the advantage that we can obtain from the mechanical powers, or their combinations, is to raise weights, or overcome great resistances, and this must be done at the expense of time; or to generate rapid velocities, as in turning-lathes, or cotton spinning machinery, and this is done at the expense of power."

SPECIFIC GRAVITY.

Art. 259. "The weight of a cubic foot of a body, in proportion to that of a cubic foot of water, is called its specific gravity.

"A cubic foot of water, at the temperature of 40° of Fahrenheit's thermometer, weighs 1000 ounces avoirdupois; and therefore the following table of specific gravities expresses in ounces the weight of a cubic foot of these bodies.

TABLE OF SPECIFIC GRAVITIES.

SOLIDS.

Platina, from 16000 to	23000	Spar, heavy,	4430
Pure Gold, hammered,	19326	Jargon of Ceylon,	4416
Guinea of George III.,	17629	Ruby, oriental,	4283
Tungsten,	17600	Garnet, precious,	4230
Mercury at 32° Fahr.,	13598	—— common,	3576
Lead,	11352	Topaz, from 3536 to	4061
Palladium,	11800	Sapphire, oriental,	3994
Rodium,	11000	Diamond, from 3523 to	3550
Pure Silver,	10744	Beryl, oriental,	3549
Shilling of George III.,	10534	English Flint Glass,	3329
Bismuth, molten,	9823	Tourmaline,	3155
Copper of Japan,	9000	Hornblende,	3000
—— wire-drawn,	8878	Asbestus,	2996
—— red, molten,	8788	Limestone,	2950
Cadmium,	8694	Basalt,	2860
Molybdena,	8611	Marble, Parian,	2837
Brass, wire-drawn,	8544	—— green Campanian,	2742
—— common,	7824	—— Egyptian,	2668
Arsenic,	8306	Chalk, British,	2784
Nickel, molten,	8279	Emerald of Peru,	2775
—— forged,	8666	Jasper,	2710
Uranium,	8109	Glass, white,	2892
Meteoric Iron, hammered,	7965	—— bottle,	2733
Steel,	7833	—— green,	2642
Cobalt, molten,	7812	Pearl, oriental,	2684
Bar Iron,	7788	Coral,	2680
Cast Iron, Carron,	7248	Slate,	2670
Wootz, hammered,	7787	Granite, Cornish,	2662
Pewter,	7471	—— Aberdeen,	2625
Tin, hardened,	7299	Rock Crystal,	2653
—— pure Cornish,	7291	Quartz,	2640
Zinc, molten,	7191	Pebble, English,	2619
Wolfram,	7119	Felspar,	2564
Manganese,	6900	Stone, common,	2500
Antimony,	6702	Porcelain, China,	2385
Tellurium,	6115	—— Limoges,	2341
Chromium,	5900	Obsidian,	2348
Mean of the Globe, about	5210	Gypsum,	2280
Loadstone,	4930	Clay,	2160

Opal,	2114	Butter,	942
Sulphur, native,	2033	Ice,	930
Brick,	2000	Gunpowder, shaken,	922
Ivory,	1917	Pumice Stone,	915
Nitre,	1900	Logwood,	913
Alabaster,	1874	Living Men,	891
Gunpowder, solid,	1745	Potassium,	865
Alum,	1714	Beech,	852
Phosphorus,	1714	Ash,	845
Bone, dry,	1660	Apple-tree,	793
Sand,	1500	Maple,	755
Gum Arabic,	1452	Citron,	726
Opium,	1337	Orange-tree,	705
Ebony, American,	1331	Walnut,	681
Lignumvitæ,	1327	Pear-tree,	661
Coal,	1250	Hazel,	600
Pitch,	1150	Linden-tree,	604
Rosin,	1100	Elm,	600
Amber,	1078	Cypress,	598
Mahogany,	1063	Cedar, American,	561
Brazil-wood, red,	1031	Fir, male,	550
Boxwood,	1030	—— female,	498
Sodium,	972	Poplar,	383
Oak, heart of,	950	Cork,	240

LIQUIDS.

Sulphuric acid,	1848	Wine of Bordeaux,	994
Baracic acid,	1830	Wine of Burgundy,	991
Nitric acid, or Aqua fortis,	1500	—— red port,	990
Nitrous acid,	1452	Castor-oil,	970
Honey,	1450	Linseed-oil,	940
Water of the Dead sea,	1240	Proof-spirit,	935
Aqua regia,	1234	Whale-oil,	923
Muriatic acid,	1170	Moselle wine,	916
Strong ale, from 1090 to	1050	Olive-oil,	915
Human blood,	1045	Muriatic ether,	874
Milk,	1030	Oil of turpentine,	870
Sea water,	1026	Brandy,	837
Tar,	1015	Alcohol, absolute,	792
Distilled water,	1000	Sulphuric ether,	739
White Champagne,	997	Air at earth's surface,	1.229

GASES.

Atmospheric air,	1.000	Muriatic acid,	1.260
Hydriodic acid,	4.300	Oxygen,	1.111
Fluosilicic acid,	3.611	Nitrous,	1.042
Chlorine,	2.500	Olefiant,	0.972
Sulphurous acid,	2.222	Nitrogen,	0.972
Cyanogen,	1.805	Ammonia,	0.590
Carbonic acid,	1.527	Hydrogen,	0.069

Prob. I. To find the magnitude of a body from its weight.

Rule. *Divide the weight of the body by its specific gravity, both being in ounces; the quotient is the content in cubic feet.*

1. How many cubic inches are in 1 lb. of gunpowder?
Ans. 1728 × 16 ÷ 922 = 30 inches, nearly.
2. What is the content of a block of Parian marble weighing 5 cwt.? Ans. 3.158 cubic feet.
3. What is the content of a ton weight of mahogany?
Ans. 33.716 cubic feet.
4. What is the content of a block of Cornish granite which weighs 4 tons? Ans. 53.85 cubic feet.
5. What is the content of a cast iron ball which weighs 100 lbs.?
Ans. 381.457 cubic inches.

Prob. II. To find the weight of a body from its magnitude.

Rule. *Multiply the content in feet by the specific gravity; the product is the weight in ounces.*

1. What is the weight of a stone of green Campanian marble, 63 feet long, and its breadth and thickness each 12 feet?
Ans. 63 × 12 × 12 × 2742 = 24875424 oz. = $694\frac{11}{160}$ tons.
2. What is the weight of a log of beech 10 feet long, 3 broad, and 2½ feet thick? Ans. 3993¾ lb.
3. What is the weight of a cast iron ball 2 inches in diameter?
Ans. 17.5697 ounces.
4. What is the weight of a log of mahogany 40 feet long, 3 broad, and 2½ thick? Ans. 8.898 tons.
5. What is the weight of a leaden ball 6 inches in diameter?
Ans. 46.4365 lbs.

Prob. III. To find the specific gravity of a body.

Case I. When the body is heavier than water.

Rule. *Weigh the body both in air and in water, and, annexing three ciphers to the weight in air, divide by the difference of the weights, to get the specific gravity.*

1. Suppose a piece of Aberdeenshire granite to weigh 10½lbs. in air, and 6½ lbs. in water. What is its specific gravity?
Ans. 10500 ÷ 4 = 2625 ounces the specific gravity.
2. A piece of copper weighs 36 oz. in air, and 32 in water. What is its specific gravity? Ans. 9000.
3. Suppose a piece of gold weighs 40 lbs. in air, and 37.93 lbs. in water. What is its specific gravity? Ans. 19323.67.
4. Suppose a piece of platina weighs 10 lbs. in air, and 9.5 lbs. in water. What is its specific gravity? Ans. 20000.

Case II. When the body is lighter than water.

Rule. *Having weighed the light body in air, and a body heavier than water both in air and in water, fasten them together with a slender tie, then weigh the compound in water, and subtract it from the* *weight of the heavy body in water; to the remainder add the weight of the light body in air, and by the sum divide the weight of the light body in air with three ciphers annexed; the quotient is the specific gravity of the light body.*

1. A piece of copper weighs 18 lbs. in air, and 16 lbs. in water; a piece of elm which weighs 15 lbs. in air is fixed to the copper, and the compound weighs 6 lbs. in water. What is the specific gravity of the elm?

Ans. $15000 \div (16 - 6 + 15) = 600$, the specific gravity of the elm.

2. A piece of copper which weighs 27 ounces in air, and 24 in water, is attached to a piece of cork which weighs 6 ounces in air, and the compound weighs 5 ounces in water. What is the specific gravity of the cork? Ans. 240.

3. A piece of lead weighs 60 lbs. in air, and 55 lbs. in water; a piece of poplar which weighs 30 lbs. in air is fixed to the lead, and the compound weighs 7 lbs. in water. What is the specific gravity of the poplar? Ans. $384\frac{8}{13}$.

4. A piece of steel weighs 140 lbs. in air, and 122 lbs. in water; a piece of fir which weighs 30 lbs. in air is fixed to the steel, and the compound weighs $97\frac{1}{2}$ lbs. in water. What is the specific gravity of the fir? Ans. $550\frac{50}{109}$.

Prob. IV. Given the specific gravity and the weight of a mass composed of two ingredients, and also the specific gravity of each ingredient, to find the quantity of each of them.

Rule. *As the specific gravity of the mass, multiplied by the difference between those of the ingredients, is to the specific gravity of the heaviest ingredient, multiplied by the difference between those of the mass and the other ingredient, so is the whole weight to the weight of the highest ingredient; and that of the other may be found in the same way.*

1. A composition of 112 lbs. is made of copper of Japan and tin. Required the quantity of each ingredient, the specific gravity of the mass being 8784.

Ans. $(9000 - 7291) \times 8784 : (8784 - 7291) \times 9000 :: 112 :$ 100.25 lbs. of copper.

2. A mixture of gold and silver weighed 170 lbs., and its specific gravity was 15630. Required the quantity of each metal in it.
Ans. 119.673 lbs. gold, 50.327 lbs. silver.

3. A composition of 100 lbs. is made of platina and steel, and its specific gravity is 15000. Required the quantity of each ingredient.
Ans. 78.54 lbs. platina, 21.46 lbs. steel.

4. A composition of silver and steel weighs 1000 lbs., and its specific gravity is 8000. Required the quantity of each ingredient.
Ans. 77.046 lbs. silver, 922.954 lbs. steel."

MISCELLANEOUS AND PRACTICAL QUESTIONS.

Art. 260. It *has often been remarked that those pupils who are frequently required to write questions from dictation, similar to the first four below, and to perform the operations required in solving them as rapidly as possible, soon acquire great facility of operation, which is very desirable.*

1. $\overline{20+25+30+35+40+45+50+55+60}-\overline{65+70+75}=$ what number?

2. $\overline{\$20.75+\$30.25+\$35.50}-\$40.45=$ what sum?

3. $\overline{475\times25+625\times45+750\times75}\div325=$ what qt.?

4. $\overline{\$3.25\times45+\$2.75\times50}-\overline{\$12\times15}=$ what rem.?

5. In a school there are four classes. In the first class, there are 120 pupils; in the second, 110; in the third, 100; and in the fourth, 90. Each class is composed of boys and girls. There are 80 boys in the first class, 60 in the second, 50 in the third, and 40 in the fourth. What number of girls are there in the school?

6. A farmer raised 120 bushels of wheat, 100 bushels of rye, 200 bushels of Indian corn, and 150 bushels of oats. He has sold 60 bushels of the wheat, 40 bushels of the rye, 120 bushels of the corn, and 75 bushels of the oats. Required the number of bushels of grain that the farmer raised; the number of bushels he has remaining unsold; also, the number of bushels of each kind that he has remaining unsold.

7. A mechanic purchased two lots of land. One of them contained 1575 square feet, for which he paid $1.25 a foot; the other contained 1625 square feet, for which he paid $1.45 a foot; he has since sold both lots, at $1.50 a foot. Required the number of dollars that he gained.

8. A gentleman has a fine orchard, in which there are 225 apple-trees. Suppose each tree to produce 5 barrels of apples each year, and that these apples will sell for $2.25 a barrel, what will be the annual income from this orchard?

9. A purchased of B 25 pounds of sugar, at 9 cents a pound; 12 pounds of tea, at 45 cents a pound; 14 pounds of coffee, at 12 cents a pound, and 35 pounds of butter, at 22 cents a pound. B agreed to take his pay in oats, at 37½ cents a bushel; what number of bushels did B receive in payment?

10. A man purchased a number of apple-trees at 25 cents each, for which he paid $144. He afterwards set them out in

24 equal rows. What number of trees did he purchase? What was the number of trees in each row?

11. A gentleman purchased a farm in the vicinity of Boston, containing 360 acres, at $125 an acre. He divided it into 8 equal sections, by streets, which included one ninth of the whole quantity of land purchased. He afterwards divided each section into 80 house-lots of equal size, and sold them at $100 each. The expense of grading the streets, surveying the land, and dividing it into house-lots, was $4000. What quantity of land did each house-lot contain? What did he gain by the transaction?

Art. 261. *Since diminishing the dividend and divisor proportionally* (Art. 134) *does not affect the quotient, hence, whenever it is required to divide the product of several numbers or factors by the product of several other numbers or factors, the operation may be abridged by cancelling and leaving out of the operation all the factors of the dividend and divisor which are common to both.*

If a remaining factor, or the product of two or more of the remaining factors of the dividend, is equal to a remaining factor, or to the product of two or more of the remaining factors of the divisor, all such factors may be cancelled.

If any two factors yet remaining, one of which is a factor of the dividend and the other a factor of the divisor, have a common measure, each factor may be divided by such common measure, and the quotient written in its place. Then, if the product of all the remaining factors of the dividend be divided by the product of all the remaining factors of the divisor, the true quotient will be obtained.

12. What will be the quotient of 36 × 30 × 25 × 11 × 9 × 8 × 7 × 4 × 3 divided by 63 × 24 × 18 × 15 × 12 × 11 × 4 × 2?

As every fraction indicates division, the numerator being the dividend and the denominator the divisor, we write all the factors of the dividend above a horizontal line, and all the factors of the divisor below it, in a fractional form.

We first cancel the common factors, 4 and 11, of the dividend and divisor, by crossing them.

$$\frac{\cancel{36}\times\overset{5}{\cancel{30}}\times\overset{5}{\cancel{25}}\times\cancel{11}\times\cancel{9}\times\cancel{8}\times\cancel{7}\times\cancel{4}\times\cancel{3}}{\cancel{63}\times\cancel{24}\times\cancel{18}\times\underset{3}{\cancel{15}}\times\underset{2}{\cancel{12}}\times\cancel{11}\times\cancel{4}\times\cancel{2}}=\frac{25}{6}=4\tfrac{1}{6}.$$

The product of the factors 8 × 3 of the dividend being equal *to the factor* 24 of the divisor, we cancel the factors 8 and 3 of the dividend, and the factor 24 of the divisor. The product of the factors 9 × 7 of the dividend cancels the factor 63 of the divisor. Also the product of the factors 18 × 2 of the divisor cancels the factor 36 of the dividend. The factor 30 of the dividend and 12 of the divisor have the common measure 6; therefore we divide each of them by 6, and write one quotient over 30 and the other under 12, crossing 30 and 12. Again, the factor 25 of the dividend and 15 of the divisor have the common measure 5; we divide each of them by 5, and write the quotients as before. Dividing the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, we obtain the required quotient, $4\frac{1}{6}$.

The method of cancelling equal or common factors, in the multiplication and division of fractions, will be subsequently explained.

13. Divide the product of 24 × 20 × 16 × 12 × 10 × 6 by 12 × 2 × 5 × 4 × 8 × 2 × 5 × 3.

14. What will be the quotient of 72 × 63 × 48 × 35 × 24 ÷ 9 × 8 × 9 × 7 × 8 × 6 × 7 × 5 × 4?

15. What number of blocks of granite, 24 inches long, 12 inches wide, and 8 inches thick, will be required to build a wall 120 feet long, 8 feet high and 2 feet thick?

$$\frac{120\times 12\times \cancel{8}\times \cancel{12}\times \cancel{2}\times \cancel{12}}{\underset{1\times 1\times 1}{\cancel{24}\times \cancel{12}\times \cancel{8}}}=\frac{1440}{1}=1440,$$ the number of blocks required.

16. What number of cubic yards of earth must be removed, in digging a cellar 36 feet long, 24 feet wide, and 9 feet deep?

17. What number of cords of wood are there in a pile 48 feet long, 4 feet wide, and 8 feet high?

18. What will be the expense of digging a cellar 36 feet long, 18 feet wide, and 6 feet deep, at $1.50 a square of 216 cubic feet?

19. There is a certain number which, being divided by 7, the quotient multiplied by 3, the product divided by 5, and 20 subtracted from the quotient, and 30 added to the remainder, half the sum shall be 35?

$35\times 2-30+20\times 5\div 3\times 7=700$, the number required.

20. A boy purchased a number of peaches, and gave away 10 of them to his companions; he afterwards purchased 30 more, and divided one half the number he then had among 5 of his class-mates, each of whom received 6 peaches. Required the number of peaches in the boy's first purchase.

21. A trader commenced business with a certain number of dollars; the first year he doubled his money, the second year he lost 300 dollars, the third year he gained 1500 dollars, and was then worth 3600 dollars. Required the number of dollars with which he commenced trade.

22. William asked his sister Mary how many problems she had solved during the preceding day. Mary replied, "If you multiply the number by 6, and divide the product by 7, and then multiply the quotient by 6, and divide the product by 36, the quotient will be 4." Required the number of problems solved by Mary.

23. A person went to 4 taverns in succession; upon entering each of which, he borrowed as much money as he carried to it; and upon leaving them, he paid the landlords one dollar each, which done, he finds himself without money. What sum of money did he carry to the first tavern?

24. A shepherd, in time of war, fell in with a party of soldiers, who plundered him of half his flock and half a sheep over; afterwards, a second party met him, who took half what he had left and half a sheep over; soon after this, a third party met him, and used him in the same manner; and then he had only 5 sheep left. It is required to find what number of sheep he had at first.

25. A merchant purchased 25 bales of sheetings, each bale containing 24 pieces, and each piece 32 yards, at 11½ cents a yard. What number of yards did he purchase? What was the amount paid?

26. What number of yards of carpeting, 3 quarters wide, will be required to cover a floor 21 feet long, and 18 feet wide? What will it cost, at $1.12½ per yard?

27. A farmer purchased a piece of land, 40 rods in length, and 36 rods in breadth, for which he paid at the rate of 30 dollars an acre. He has since enclosed it with a stone wall, which cost 87½ cents a rod. What was the whole amount paid for the land and wall?

28. A mechanic purchased a house-lot 24 feet in breadth, containing 1728 square feet, at $1.87½ a foot. He afterwards enclosed it with a picket fence, which cost 37½ cents a foot. What was the length of the lot? What was the whole amount paid for the land, including the fence?

29. A carpenter was directed to make 12 boxes of the same size, each of which was to contain 5 bushels of grain. The length of each box *inside* was to be 25 inches, its width 20 inches, and of sufficient depth to contain 5 bushels. The boxes were to be made of boards one inch in thickness. The depth

of each box is required; also the number of feet of boards necessary to make the 12 boxes, allowing one tenth of the whole quantity to be wasted in making.

30. A cabinet maker purchased a log of mahogany which contained 30 cubic feet. Its width was 24 inches, and its depth or thickness 21 inches. What was its length?

31. Suppose the distance from Boston to Albany by railroad to be 200 miles, and suppose the larger wheels of a locomotive engine to be 15 feet 9 inches in circumference, and the smaller wheels to be 12 feet 6 inches; how many more times will the smaller wheels turn round, in going from Boston to Albany, than the larger wheels?

32. Suppose that A leaves Boston for Albany at 7 o'clock in the morning, and travels by railroad at the rate of 18 miles an hour; and that B, on the same day, leaves Albany for Boston at 7½ o'clock in the morning, and travels by railroad at the rate of 20 miles an hour; at what time will they meet, and at what distance from each place, supposing the distance between the two cities to be 200 miles?

33. Suppose that a steamboat leaves New York for Albany at the same time that another steamboat leaves Albany for New York, and that each boat is propelled by steam-engines at the rate of 15 miles an hour. Suppose the current of the river to be 3 miles an hour, and the distance between the two cities to be 240 miles; in what number of hours will the two boats meet, and at what distance from each city?

34. The length of a degree of longitude, in the latitude of Boston, is about 52 statute miles. What number of miles are the inhabitants of Boston carried every hour by the revolution of the earth on its axis?

35. From 1 mile subtract 7 furlongs, 39 rods, 5 yards, 1 foot, 5 inches, and prove by addition.

	m.	fur.	rods.	yds.	ft.	in.
From	1	0	0	0	0	0
Subtract		7	39	5	1	5
		0	0	0	0	1
		7	39	5	1	6

In this problem, instead of adding 12 inches to the upper number, we add only 6 inches, which = ½ foot; from which we subtract 5 inches, and 1 inch remains; we then add ½ foot to the 1 foot, the sum is 1½ feet; then, instead of adding 3 feet to the upper number, we add 1½ feet, which are equal to ½ yard, and subtract 1½ from 1½, and 0 remains; we then add ½ yard to the 5 yards, and proceed as usual. In the proof, it is plain that the 1 foot and 6 inches are equal to ½ yard, which added to the 5 *yards* = 5½ yards, or 1 rod, which added to 39 rods = 40 rods, or 1 furlong, which added to 7 furlongs = 8 furlongs, or 1 mile.

36. From 55 miles subtract 13 miles, 7 furlongs, 39 rods, 5 yards, and 2 feet.

	m.	fur.	rods.	yds.	ft.
From	55	0	00	0	0
Subtract	13	7	39	5	2
	40	7	39	5	1

In this problem, we subtract the 2 feet as usual, and then add 1 yard to the 5 yards, and the sum is 6 yards, which cannot be taken from 5½ yards; we therefore add 11 yards to the upper number, which are equal to 2 rods, and subtract the 6 yards, and 5 yards remain; we then add 2 rods to the 39 rods; the sum is 41 rods, which cannot be taken from 40 rods; we therefore add 80 rods to the upper number, which are equal to 2 furlongs, and subtract the 41 rods, and 39 rods remain; we then add 2 furlongs to the 7 furlongs; the sum is 9 furlongs, which cannot be taken from 8 furlongs; we therefore add 16 furlongs to the upper number, and subtract the 9 furlongs, and 7 furlongs remain; we then add 2 miles to the 13 miles; the sum is 15 miles, which we subtract from the 55 miles, and 40 miles remain.

Problems 35 and 36 are designed to show that whenever the lower number of any denomination exceeds the upper, we do not always increase the upper number by so many of the same denomination as are equal to a unit of the next higher; but that it may be increased by a number greater or less, as circumstances may require.

37. Divide 97 miles, 7 furlongs, 39 rods, 4 yards, 2 feet, by 6, and prove by multiplication.

	m.	fur.	rods.	yds.	ft.
6)	97	7	39	4	2
	16	2	26	3	$1\frac{7}{12}$
					6
	97	7	39	4½	0½

It will be observed that the product, or proof line, does not correspond in numbers with the dividend; yet it does in value, for 4½ yards plus ½ foot = 4 yards 2 feet. Whenever the ratio of value between the denominations is not a whole number, this difficulty is liable to occur.

38. Divide 45 degrees, 10 miles, 7 furlongs, 37 rods, 5 yards, 1 foot, 3 inches, by 4, and prove by multiplication.

39. What is the greatest common measure of 2261 and 2737?

The greatest common measure of two or more numbers is the product of all the prime factors which they have in common, (Art. 73.)

As the factors of composite numbers always occur in pairs, if *a composite number has not a prime factor less than its square root, it cannot have one greater.*

40. What is the greatest common measure of 3773 and 3871?

41. A gentleman has a flower-garden which is 430 feet in length and 320 feet in width, and he wishes to enclose it with a board fence, all the boards of which shall be of equal length. What is the greatest length that he can use?

42. A speculator purchased a quarter section of new land, 320 rods in length, and 180 rods in breadth; he wishes to divide it into the smallest number of square fields. What will be their size and number?

43. What is the least common multiple of 2261 and 2737?

The least common multiple of two or more numbers is the product of all their prime factors which are not common to two or more of them; or, it is the quotient obtained by dividing their product by their greatest common measure, Art. 75.

44. What is the least common multiple of 765, and 1377?

45. What is the least common multiple of 3757, 3887, and 4823?

46. There is an island 60 miles in circumference, and 3 men start together and travel the same way around the island; A travels 10 miles an hour, B 8, and C 6. In what number of hours will they be together again? and what number of miles will each have travelled?

The usual mode of solving problems similar to the above is to find the time in which the first will overtake the second, the second the third, and so on; the least common multiple of the several periods of time will be the first time of their meeting.

Another method would be to find the greatest common measure of the differences of the several rates of travel, and divide the circuit by such common measure; the quotient will express the first time of their meeting. Then divide the number of miles each travels per hour by such common measure; the several quotients will express the number of revolutions performed by the several travellers.

47. A travels 30 miles a day, B 24, C 16, D 12, and E 10, in the same direction, around a lake 120 miles in circumference. In what number of days will they all meet? What number of miles will each have travelled? What number of times will each have travelled around the lake?

48. What is the least whole number that, being divided by 2, 3, 4, 5, 6, 7, 8, 9 and 10, will leave as remainders 1, 2, 3, 4, 5, 6, 7, 8 and 9, respectively?

The least common multiple of the above divisors is 2520; *and it is obvious that if we diminish this number one, we shall have the required result, viz.*, 2519, *which will answer the conditions of the question.*

49. What is the least number which, being divided by 5, 6, 7, 8, 9, 10 and 12, shall always leave a remainder of 4?

Sometimes one or more of the numbers have fractions annexed; in which case, the mixed numbers must be changed to improper fractions, and the numerators only are to be used in the calculation.

50. What is the least common multiple of $3\frac{1}{3}$, $6\frac{2}{3}$ and 30?

$3\frac{1}{3}$, $6\frac{2}{3}$ and $30 = \frac{10}{3}$, $\frac{20}{3}$ and 30; and using the numerators only, we find the prime factors of 10 to be 2 and 5; of 20, 2, 2 and 5; of 30, 2, 3 and 5. And $2 \times 2 \times 3 \times 5 = 60$, the least common multiple.

We may find the least common multiple of two or more fractions, by first reducing the fractions to their lowest terms, and then finding the least common multiple of the numerators.

51. What is the least common multiple of $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{8}{9}$?

The prime factors of 4 are 2 and 2; the prime factors of 8 are 2, 2 and 2. Then $2 \times 2 \times 2 \times 3 = 24$, the least common multiple.

52. What is the least common multiple of $5\frac{1}{2}$, $6\frac{2}{3}$ and $7\frac{3}{4}$?

53. What is the least common multiple of $\frac{5}{7}$, $\frac{7}{9}$ and $\frac{35}{43}$?

54. What is the least common multiple of $\frac{4\frac{1}{5}}{5}$, $\frac{\frac{3}{4}}{\frac{5}{6}}$, and $\frac{7\frac{1}{2}}{9\frac{4}{5}}$?

We will first change the complex fractions to simple ones.

A complex fraction may be changed to a simple fraction by multiplying both terms of the complex fraction by the least common multiple of their denominators.

$$\frac{4\frac{1}{5} \times 5}{5 \times 5} = \frac{21}{25}. \qquad \frac{\frac{3}{4} \times 12}{\frac{5}{6} \times 12} = \frac{9}{10}. \qquad \frac{7\frac{1}{2} \times 10}{9\frac{4}{5} \times 10} = \frac{75}{98}.$$

The prime factors of 21 are 3, 7. The prime factors of 9 are 3, 3. The prime factors of 75 are 3, 5, 5. And $3 \times 3 \times 5 \times 5 \times 7 = 1575$, the least common multiple required.

55. What is the least whole number that can be measured by $3\frac{1}{4}$, $4\frac{1}{3}$ and $5\frac{1}{2}$?

56. What is the least whole number that can be measured by $6\frac{3}{4}$, $7\frac{4}{5}$, $8\frac{5}{6}$ and $9\frac{6}{7}$?

PROBLEMS TO BE ANALYZED.

Art. 262. *In Art.* 69, *we have analyzed numbers and their factors; and we have also analyzed one or more problems in several of the other articles throughout the work.*

It is deemed important that the pupil should analyze every problem by naming each of its given conditions and terms; also each required result; and that he should give a reason for each successive step in the process of its solution.

In every problem, we have two or more numbers or terms given, coupled with one or more conditions; and we are required to find one or more numbers or terms answering the conditions of the problem.

57. If 5 yards of cloth cost 25 dollars, what will 9 yards of the same kind of cloth cost?

In this question, the given condition is that 5 *yards of cloth cost* 25 *dollars, and we are required to find the cost of* 9 *yards of the same kind of cloth.*

If 5 *yards of cloth cost* 25 *dollars,* 9 *yards will cost* 9 *fifths of* 25 *dollars.* 1 *fifth of* 25 *dollars is* 5 *dollars, and* 9 *fifths of* 25 *dollars are* 9 *times* 5 *dollars, which are* 45 *dollars; therefore,* 9 *yards will cost* 45 *dollars.*

58. If 9 tons of hay will keep 6 horses 5 months, what number of tons will be required to keep 12 horses 15 months?

59. If 25 men can perform a piece of work in 17 days, what number of days will it take 5 men to perform the same piece of work?

60. If 4 men can build a wall 48 rods in length in 8 days, what number of men will be required to build a wall 144 rods in length in 6 days?

61. If 200 men consume 80 barrels of flour in 24 weeks, what number of weeks will 120 barrels last 250 men?

62. If $3\frac{1}{2}$ pounds of sugar cost 42 cents, what will $10\frac{3}{4}$ pounds cost?

If $3\frac{1}{2}$ *pounds of sugar, or* 7 *half pounds, cost* 42 *cents,* 1 *half pound will cost* 1 *seventh of* 42 *cents, which is* 6 *cents; and* 2 *half pounds, or a whole pound, will cost* 2 *times* 6 *cents, or* 12 *cents; and* $10\frac{3}{4}$ *pounds will cost* $10\frac{3}{4}$ *times* 12 *cents, which are* 129 *cents, or* 1 *dollar and* 29 *cents.*

63. If $\frac{5}{8}$ of a yard of silk are worth 40 cents, what is the value of $12\frac{7}{8}$ yards?

If $\frac{5}{8}$ *of a yard of silk are worth* 40 *cents,* 1 *eighth is worth* 1 *fifth of* 40 *cents, which is* 8 *cents; and* 8 *eighths, or a whole yard, are worth* 8 *times* 8 *cents, or* 64 *cents. If* 1 *yard of silk is worth* 64 *cents,* $12\frac{7}{8}$ *yards will be worth* $12\frac{7}{8}$ *times* 64 *cents, which are* 824 *cents, or* 8 *dollars and* 24 *cents; therefore, the value of* $12\frac{7}{8}$ *yards is* 8 *dollars and* 24 *cents.*

64. What number of suits of clothes, each suit containing $3\frac{5}{8}$ yards of cloth, can be made from a piece of broadcloth measuring $43\frac{1}{2}$ yards?

65. A laborer, having undertaken to mow a quantity of grass, mowed $\frac{1}{4}$ of it the first day, $\frac{2}{7}$ of it the second day, $\frac{3}{14}$ of it the third day, and the remaining part of it the fourth day. What part of the whole quantity did he mow the first three days? What part of it did he mow the fourth day?

66. The united ages of a lady and her two daughters are 52 years. The elder daughter's age is three times the age of the younger, and the mother's age is 8 years less than twice the sum of the united ages of both daughters. What is the age of each?

67. A farmer, being asked how many sheep he owned, replied, that if he should purchase four fifths as many as he now has, and 10 sheep more, he should have 120 sheep. What number of sheep did he own?

68. A merchant gained a sum equal to three fifths of his original capital; he afterwards lost 1500 dollars, and then had 6500 dollars left. What was his original capital?

69. A mother, being asked the age of each of her two daughters, replied that the younger was 4 years old, and that the age of the elder was equal to one fourth of her own age; and that if her own age should be divided by the age of the younger, and one and a half the age of the younger subtracted from the quotient, the remainder would be equal to the age of the younger daughter. What was the age of the elder daughter? What was the age of the mother?

70. A number of bees left a hive. One fifth flew to a honeysuckle; one third, to a rose-bush; three times the difference of these two numbers flew to an arbor; and 5 bees continued flying about, attracted on each side by the fragrant syringa and lilac. What was the number of bees?

71. Francis gave one third of a collection of beautiful water-lilies to Mary, one fifth to Annie, one sixth to Emma, one fourth to Julia, and 6 which remained to his teacher. Required the whole number of water-lilies.

72. Two boys, A and B, were looking at the flag-staff on *Boston* Common; A asked B the height of it. B said that

his cane was 3 feet long, and that he would set it in a perpendicular position, and measure the length of its shadow, which he found to be 4 feet; he then measured the shadow of the flag-staff, and found it to be 120 feet. The height of the flag-staff is required.

73. The sum of the ages of B and C is 60 years; and if the age of A be divided by the age of B, the quotient will be 4. The age of each is required.

74. The sum of three numbers is 180, and they are proportionate to each other as 3, 4, and 5. What are the numbers?

Art. 263. *We have shown* (Art. 84) *that all equal numerators and denominators of a compound fraction may be cancelled; also, that when a numerator and denominator have a common measure, they may be divided by that common measure, and the quotients used as factors. A similar process may be pursued in multiplying and dividing one fraction by another.*

75. If $\frac{3}{4}$ of $\frac{4}{5}$ of a yard of silk cost $\frac{5}{6}$ of $\frac{6}{25}$ of a dollar, what will be the value of $\frac{2}{3}$ of $12\frac{1}{2}$ yards?

If $\frac{3}{4}$ of $\frac{4}{5}$ of a yard of silk cost $\frac{5}{6}$ of $\frac{6}{25}$ of a dollar, a whole yard will cost $\frac{4}{3}$ of $\frac{5}{4}$ of $\frac{5}{6}$ of $\frac{6}{25}$ of a dollar; and the value of $\frac{2}{3}$ of $12\frac{1}{2}$ yards will be $\frac{2}{3}$ of $12\frac{1}{2}$ times the value of one yard.

OPERATION.

$$\frac{\cancel{4}}{3} \times \frac{\cancel{5}}{\cancel{4}} \times \frac{\cancel{5}}{\cancel{6}} \times \frac{\cancel{6}}{\cancel{25}} \times \frac{\cancel{2}}{3} \times \frac{25}{\cancel{2}} = \frac{25}{9} = 2\tfrac{7}{9} \text{ dollars, the value of } \tfrac{2}{3} \text{ of } 12\tfrac{1}{2} \text{ yards.}$$

76. If $5\frac{3}{4}$ acres of land cost \$$143\frac{3}{4}$, what is the value of a farm containing $175\frac{7}{8}$ acres?

77. If a man can build $45\frac{1}{2}$ rods of wall in $9\frac{3}{4}$ days, how many days will it take him to build $227\frac{1}{2}$ rods?

78. If a man perform a journey in $7\frac{1}{2}$ days, travelling $12\frac{3}{4}$ hours each day, in what number of days will he perform the same journey, if he travel only $9\frac{1}{2}$ hours each day?

79. If a post, $5\frac{3}{4}$ feet in height, cast a shadow $7\frac{1}{2}$ feet in length, what is the height of a tree that casts a shadow $75\frac{1}{4}$ feet in length at the same time?

80. A regiment of 1200 men are to be clothed; each suit is to contain $3\frac{1}{2}$ yards of cloth $1\frac{1}{2}$ yard wide, and is to be lined with flannel $\frac{7}{8}$ of a yard wide. What number of yards of flannel will be required?

81. If 15.75 yards of cloth cost \$3.15, what is the value of 5 pieces, each measuring 35.875 yards, at the same rate?

82. If 12.25 acres of land are worth $375.5, what is the value of a farm containing 225.875 acres?

83. If .625 of a ship are worth $4525.375, what is the value of .875 of the same ship?

84. If it require 12¾ pounds of wool to make a piece of flannel 25½ yards in length, and ⅞ of a yard in width, what number of pounds of wool will be required to make 10 pieces of flannel, each piece measuring 27¾ yards in length, and ¾ of yard in width?

85. If a man travel 375 miles in 12½ days, when the days are 12½ hours long, in what number of days will he travel 750 miles, when the days are only 10¾ hours long?

86. If 12 men consume ½ of ⅔ of ¾ of ⅘ of 30 bushels of wheat, in ½ of $\frac{14}{39}$ of $\frac{6}{7}$ of $\frac{8}{15}$ of 20 months, what number of bushels will 4 men consume in ⅜ of $\frac{6}{7}$ of $\frac{16}{19}$ of $\frac{38}{39}$ of $\frac{14}{15}$ of 40 months?

87. If 36 men can dig a ditch 375¾ feet in length, 3½ feet in width, and 2¾ feet in depth, in 7½ days, of 9½ hours each, in what number of days can 24 men dig a ditch 751½ feet in length, 4½ feet in width, and 3¼ feet in depth, working 10¾ hours each day?

88. A merchant from the country purchased goods in Boston to the amount of $1500. He paid 24½ per cent. of the $1500 for broadcloths; 25¼ per cent. of the sum remaining for prints; 75¾ per cent. of the sum remaining after purchasing the prints, for shirtings and sheetings; and the balance still remaining for linens. What sum did he pay for each kind of goods purchased?

89. A gentleman deposited $1200 with a broker, and directed the broker to invest the same in railroad stock. After the broker had invested 33⅓ per cent. of the $1200, he purchased 5 shares in the Western Railroad, at 2½ per cent. advance; the original or par value of each share being $100. What per cent. of the $1200 did he pay for the 5 shares? What per cent. of the $1200 has he remaining to be invested? What will the broker's commission amount to, at 2⅛ per cent. on the $1200?

90. A merchant in Boston remits $5000 to his agent at Rochester, directing him to invest the same in flour, after deducting his commission of 3½ per cent. What will be the amount of his commission? What sum will he have to invest in flour?

91. A coal-dealer in Boston sends a draft for $2050 to his agent in Philadelphia, directing him first to deduct his com-

mission of 2½ per cent., and invest the remainder in coal at $3.25 a ton. What number of tons did he purchase?

92. The par value of a share in a certain railroad is $100. If I should purchase 25 shares at 7½ per cent. below their par value, and sell them at 5½ per cent. above, what should I gain by the transaction; and what per cent.?

93. What number of shares of railroad stock can be purchased with $2368.75, the original or par value of each share being $100, and the real or market value 5¼ per cent. below par?

94. A bookseller purchased 120 gross of steel pens, for three fourths of which he paid 62½ cents per gross, and for the remainder he paid 56¼ cents per gross. If he should sell them all at 66⅔ cents per gross, what per cent. profit would he make by the transactions?

95. A merchant deposited $6750 in the Merchants' Bank. He has since drawn two checks upon the bank; the first for 37½ per cent. of the whole sum deposited, and the second for 62½ per cent. of what remained in the bank after the first check had been paid. What sum has he remaining in the bank?

96. Purchased 75 reams of paper, for which I paid $290.62½; at what price per ream must I sell it, to make a profit of 15 per cent.?

97. A merchant purchased 50 hogsheads of molasses, each hogshead holding 118 gallons, at 17 cents a gallon; paid for freight $1.12½ per hogshead, for insurance $27.50, and the duties amounted to $115.64, the leakage being 2 per cent.; at what price per gallon must he sell it, to make a profit of 25 per cent.?

98. What amount of premium must I pay annually for the insurance of ⅗ of the value of my house against loss by fire, at the rate of ⅜ of 1 per cent., the house being valued at $7650?

99. A merchant purchased 350 barrels of flour for me, on commission, for which he paid $4.62½ a barrel, cash. The next day, he sold the same flour, for cash, at $4.87½ a barrel. The merchant charged me 1½ per cent. commission on the purchases, and 2 per cent. on the sales. How much did I gain by the transaction, and what per cent.?

100. Purchased 575 barrels of flour, at $4⅞ a barrel, on a credit of 6 months. Sold the flour, the same day, at \$4$\frac{15}{16}$ a barrel, on a credit of 4 months. What per cent. did I gain, estimating the present worth of the debts at the date of the transaction?

101. Boston, April 1, 1850.

For value received, I promise to pay C. D., or order, six hundred and seventy-five dollars, on demand, with interest at 6 per cent. A. B.

ENDORSEMENTS.

July 1, 1850,	received	$75.00.	Nov. 15, 1850,	rec'd,	$125.00.
Oct. 1, 1850,	"	$5.25.	Feb. 15, 1851,	"	$175.75.

What will be due on the above note April 1, 1851?

The interest on the above question is to be computed by the rule in Art. 148.

The following rule has been established by the Supreme Court of the State of Connecticut:

"*Compute the interest to the time of the first payment, if that be one year or more from the time the interest commenced; add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner from one payment to another, until all the payments are absorbed; provided the time between one payment and another be one year or more.*

"*But if any payments be made before one year's interest hath accrued, then compute the interest on the principal sum due on the obligation for one year, add it to the principal, and compute the interest on the sum paid from the time it was paid up to the end of the year, add it to the sum paid, and deduct that sum from the principal and interest added together. If any payments be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period.*"

NOTE. *If the year extends beyond the time of payment, find the amount of the remaining principal to the time of payment; find also the amount of endorsement, or endorsements, and subtract their sum from the amount of the principal.*

The following rules, for computing *annual* interest, are sanctioned by the Courts in Vermont; also, by some of the Courts in New Hampshire.

When the contract is for the payment of the interest annually, and no payments have been made —

RULE. "*Find the interest of the principal for each year, separately, up to the time of payment; then find the simple interest of these interests, severally, from the time they become due up to the time of payment; the sum of all the interests added to the principal will be the amount.*"

"*If payments have been made, find the amount of the principal, and from this amount subtract the payment or payments up to the end of the first year. Proceed in the same manner to the time of payment.*"

"When the contract is for a sum payable at a specified time, with annual interest, and payments are made before the debt becomes due, —

RULE. "*Find the interest of the principal up to the time of the first payment, and reserve this interest; then subtract the payment from the principal, and find the interest of the remainder up to the time of the next payment; add this interest to the reserved interest, and so continue up to the time the debt becomes due, and the sum of the interests, added to the last principal, will be the amount due at that time. After the debt becomes due, the interest is to be extinguished annually, if the payments are sufficient for that purpose.*"

102. Hartford, January 1, 1848.

For value received, I promise A. B. to pay him, or order, seven hundred and sixty dollars, on demand, with interest.

C. D.

On this note are the following endorsements: March 1, 1849, received one hundred dollars. May 1, 1849, received ninety dollars. June 1, 1850, received two hundred dollars. What will be due on this note July 1, 1850, computing interest by the Connecticut rule.

103. W. Foster has T. Smith's note; dated July 1, 1845, for $450, with interest at 6 per cent., to be paid annually. What was due on this note July 1, 1849, no payments having been made up to that time, computing interest according to the Vermont rule?

104. Montpelier, Vt., January 1, 1845.

For value received, I promise to pay C. D., or order, five hundred dollars, on demand, with interest annually.

On this note are the following endorsements: September 1, 1845, received one hundred dollars. July 1, 1846, received two hundred dollars. April 1, 1847, received ninety dollars. What was due on this note January 1, 1848, computing interest by the Vermont rule?

105. Rutland, Vermont, June 1, 1846.

For value received, I promise to pay J. Smith, or order, five hundred dollars, in three years, with interest annually.

W. B. Fessenden.

On this note are the following endorsements: Sept. 1, 1847, received two hundred dollars. Nov. 1, 1848, received one hundred and seventy-five dollars. What was due on this note June 1, 1849, computing interest by the Vermont rule?

A more equitable mode than the above, in computing interest on notes when partial payments have been made, would be to find the balance due on the note at the end of each successive year, up to the *time of paying the whole balance due on the note*, which may be done by the following

Rule. *When the interest is payable annually, and the payments have been made at the expiration of each successive year, find the amount of the principal or balance remaining due at the end of each successive year, and deduct the amount of each payment, whether it be more or less than the interest then due.*

If the payment is not made until after the expiration of a year, find the amount of the payment up to the end of the next succeeding year, and deduct the amount of the payment from the amount of the balance due on the note. If a payment is made before the expiration of a year, find the amount of the payment up to the end of the year, and deduct the amount of the payment from the amount of the balance due on the note. Proceed in a similar manner until the payment of the whole balance due on the note.

Remark. According to the above rule, if the interest is not paid at the end of each successive year, the debtor will have to pay compound interest, until he makes a payment equal to the interest then due; and if he makes a payment before the expiration of a year, he is allowed the interest on the payment for the time it was paid before the expiration of the year.

106. Boston, Jan. 1, 1845.

For value received, I promise to pay C. Dana, or order, five hundred dollars, on demand, with interest annually, at 6 per cent. E. Bancroft.

On the above note are the following endorsements: Jan. 1, 1846, received thirty dollars. March 1, 1847, received seventy-five dollars. July 1, 1847, received fifty dollars. Jan. 1, 1849, received twenty-five dollars. What was the amount due on this note, Jan. 1, 1850, computing interest by the above rule?

107. What will be the compound interest on $575, for 5 years and 9 months, the interest payable semi-annually, at the rate of 6 per cent. a year?

108. A merchant hired a store on a lease for 5 years, and agreed to pay a rent of $600 a year, in quarter-yearly payments. Suppose the merchant paid the rent punctually at the time it became due, and suppose the owner of the store to have put each quarterly payment at compound interest at 6 per cent. at the time he received it, what would have been the amount of rent at the expiration of the lease?

Art. 264. In computing interest, we have three *numbers or terms* given, viz., the *principal*, *time*, and *rate per cent.*, to find two other *numbers* or *terms*, viz., the *interest* and *amount;* and if any three of the above five terms are given, the other two may be found. The following questions will indicate the method:

109. What is the interest and amount of $500, for 5 years, at 6 per cent.?

The interest of 1 dollar for 5 years, at 6 per cent., is 30 cents. or 30 hundredths of a dollar, and $500 × .30 = $150.00, the interest of $500 for 5 years, and $500 + $150 = $650, the amount required. Art. 145.

110. A note, which has been on interest 5 years, at 6 per cent., amounts to $650. The principal and interest are required.

The amount of 1 dollar for 5 years, at 6 per cent., is $1.30, and $650 ÷ $1.30 = $500, the principal required; and $650 — $500 = $150, the interest required.

111. A note for $500 has been on interest at 6 per cent., until it amounts to $650. The interest and time are required.

$650 — $500 = $150, the interest required; $500 × .06 = $30, the interest of $500 for 1 year; and $150 ÷ $30 = 5, the number of years, or time required.

112. A note for $500 has been on interest 5 years, and amounts to $560. The interest and rate per cent. are required.

$650 — $500 = $150, the interest required; $150 ÷ 5 = $30, the interest of $500 for 1 year; and $30 ÷ $500 = $.06, the rate per cent. required. From the above illustrations, rules may be deduced.

113. What is the difference between the interest of $875, for 5 years, at 6 per cent., and the discount of the same sum, for the same time, at the same rate?

114. A merchant has a note for $1200, payable at the expiration of 90 days, which he gets discounted at a bank, and, to oblige his friend, lends him the amount received from the bank for the same time; the rate of discount and interest being the same, viz., 6 per cent., what does the merchant lose by obliging his friend?

115. A merchant has to make a payment of $994.50, and obtains this amount by having his endorsed note discounted at a bank, for 30 days. For what amount does he give his note?

116. A merchant purchased a quantity of merchandise, for which he paid $775; on the same day, he sold the same for $825. What per cent. profit did he make?

117. A merchant imported 850 yards of broadcloth, which cost £430 sterling, on which he paid a duty of 30 per cent.,

the freight and other charges amounted to $18.75. He sold the same at $4.75 cents a yard, on a credit of 6 months. What per cent. profit did he make, supposing he paid for the cloth 2 months before selling it, allowing money to be worth 6 per cent. a year in trade, estimating his profit at the time of selling it?

118. Sold a quantity of merchandise for $125, and lost 12½ per cent.; for what number of dollars should the merchandise have been sold. to have made a profit of 12½ per cent.?

119. A merchant sold flour at $5.75 a barrel, and made a profit of 10 per cent.; he has since sold the same kind of flour at $6.25 a barrel. What per cent. profit did he make, at the latter price?

120. A stationer sold quills at $3.75 a thousand, and gained ⅕ of the first cost; but quills being scarce, he raised the price to $4.25 a thousand. What per cent. did he gain by selling them at the latter price?

121. A merchant purchased goods to the amount of $1280, which he sold at a loss of 12½ per cent.; and invested the proceeds of the sale in other goods, which he sold at a profit of 15 per cent. Did he gain or lose by these transactions, and how much?

122. A merchant sold a quantity of coffee at 9 cents a pound, and lost 10 per cent. He afterwards sold another quantity of the same kind of coffee to the amount of $262.50, and gained 20 per cent. What number of pounds was there in the quantity last sold, and at what price per lb. did he sell it?

123. A merchant imported 125 tons of iron, which cost £24 10s. sterling, per ton; the freight and duties amounted to £35. 15s., and other charges to £7. 5s. At what price per ton must he sell the iron, to make a profit of 20 per cent.?

124. A merchant imported 1250 yards of silk, which cost 62½ cents a yard; one fifth of it was so much damaged that he was obliged to sell it at a loss of 12½ per cent.; at what price per yard must he sell the remainder, so as to make a profit of 15 per cent. on the whole amount purchased?

125. A. B. purchased of C. D., on a credit of 6 months, the following bills of merchandise:

Jan. 10, a bill amounting to $325.50. Feb. 15, a bill amounting to $450.75. April 10, a bill amounting to $225.50. June 15, a bill amounting to $175.25. On the 1st of July, A. B. wishes to give C. D. his note for the whole amount; at what time must it be made payable, so that no loss of interest shall be sustained by either party?

126. A trader purchased of an importing house the following bills of merchandise:

May 1, a bill amounting to 150 dollars, on a credit of 180 days.
June 15, a bill amounting to 175 dollars, on a credit of 120 days.
July 10, a bill amounting to 125 dollars, on a credit of 90 days.
Aug. 1, a bill amounting to 75 dollars, on a credit of 60 days.

On the 15th of September, the trader proposed to make a settlement, and give his note for the whole amount; at what time must the note be made payable?

127. Three persons formed a partnership for the transaction of business. A advanced $\frac{2}{5}$ of the capital stock, B $\frac{2}{7}$, and C the remainder. It is required to divide the gain, which was $3500, justly among them.

128. Two men purchased a farm in company. A paid $3250, B paid $2750. What yearly income, and what per cent., will each derive from the investment, the farm renting for $625, and the taxes and expenses for repairs being $125 a year?

129. A mechanic hired a large house for one year, with the liberty of letting any part of it. At the end of 3 months he admitted A, and at the end of 6 months he admitted B, as joint tenants, each occupying equal portions of the house. How should the rent, which was $560, be divided among them?

130. Three men, A, B, and C, made a contract with a railroad corporation to construct and grade 12 miles of their road for $17500. A agreed to superintend the work, and was to receive $475 for his services. A employed 25 workmen and 6 yoke of oxen 120 days; B employed 30 workmen and 8 horses 75 days; and C employed 35 workmen and 12 horses 60 days. It was agreed that 3 yoke of oxen should be reckoned equal to 2 men, and 4 horses equal to 3 men. What number of dollars did each of the three men who contracted to build the road receive?

131. What is the duty on 4 hogsheads of brown sugar, weighing gross as follows: No. 1, 1224 pounds; No. 2, 1236 pounds; No. 3, 1272 pounds; No. 4, 1296 pounds; draft 7 pounds per hogshead, tare 12 per cent., duty 2½ cents a pound?

132. What is the duty on an invoice of woollen goods which cost in London £750. 15s. 6d. sterling, at 35 per cent. ad valorem?

133. A tax of $\frac{7}{10}$ of 1 per cent. has been assessed upon the whole amount of taxable property in a town. What must A

pay, whose real estate is valued at $7650, and personal property at $1760?

134. A tax of $5600 is to be assessed on the town of D. The real estate of the town is valued at $700000, and the personal property at $270000; and there are 600 polls, each of which is taxed $1.25. What is the per cent., or tax, on 1 dollar? and what is A's tax, whose real estate is valued at $12000, and his personal property at $2500, he paying for 3 polls?

135. What is B's tax, who, living in the same town, pays for 2 polls, and whose real estate is valued at $4750, and his personal property at $1275?

136. A farmer has 20 cows; 3 of which are worth $35 each, 4 are worth $30 each, 6 are worth $24 each, 4 are worth $20 each, 2 are worth $18 each, and 1 is worth $13. What is the average value of the 20 cows?

137. On a certain day, the mercury in the thermometer was observed to average, from 6 o'clock in the morning to 9, 64 degrees; from 9 to 12, 74 degrees; from 12 to 3, 84 degrees; and from 3 to 6, 70 degrees. What was the mean temperature of the day?

138. Suppose 10 pounds of pure gold, when immersed in water, to displace 4 pounds of water, 10 pounds of pure silver to displace 7 pounds of water, and 10 pounds of a composition of gold and silver to displace 6 pounds of water; what are the proportions of gold and silver in the compound?

139. "Hiero, King of Sicily, ordered a crown to be made containing 63 ounces of pure gold; but suspecting that the workmen had debased it by using part silver, he recommended the detection of the fraud to the famous Archimedes, who, putting it into water, found that it displaced 8.2245 cubic inches of water. He next found that a cubic inch of gold weighed 10.36 ounces, and a cubic inch of silver weighed 5.85 ounces; then, from these data, he calculated the proportions of gold and silver of which the crown was composed. What must have been the result?"

140. A provision dealer purchased a lot of hogs, turkeys, and geese, paying 10 dollars for each hog, 1 dollar for each turkey, and 1 shilling for each goose. The whole number purchased was 100, for which he paid 100 dollars. What number of each did he purchase?

NOTE. *Questions similar to the four preceding ones may be performed by Alligation.*

141. What is the sum of the natural series of numbers 1, 2, 3, 4, 5, &c., up to 10000?

142. A man rents his store for $500 a year, the rent to be paid at the end of each year. What will the rent amount to at the expiration of 20 years, allowing 6 per cent., simple interest, for the use of the money?

143. A body falling by its own weight, and not resisted by the air, would descend, the first second, 16$\frac{1}{12}$ feet; in the next second, through three times that space; in the third, through five times, &c. Through what space, at the same rate of increase, would it fall in a minute?

144. What debt can be discharged in a year, by paying 1 cent the first month, 10 cents the second, 100 cents the third, and so on, increasing in a ten-fold proportion?

145. A young fellow, well skilled in numbers, agreed with a rich farmer to work for him 10 years, without any other reward for his services, but the produce of one wheat corn for the first year, and that produce to be sowed the second year, and so on till the end of the 10 years. Allowing the increase to be in a ten-fold proportion, and 7680 wheat corns to make a pint, and the whole produce to be worth $1.25 a bushel, what number of dollars did he receive for his services?

146. What sum of money must a man save annually, commencing at 21 years of age, to be worth $20000 when he is 45 years old, the investments being made at 6 per cent. compound interest?

147. What is the present worth of an estate valued at $12000, which is to come into possession on the death of a person who is now 50 years old, the average duration of life at that age being 21 years?

148. A man has a perpetual annual annuity of $360 dollars, which he wishes to exchange for a life annuity. Supposing money to be worth 6 per cent. a year, what annuity must he receive, it being probable that he will live 21 years?

149. Which is of the greater value, the income of an estate of $650 a year for 12 years, or the reversion of the same estate forever, at the expiration of the 12 years, estimating compound interest at 6 per cent.?

150. A man wishes to give his son, who is now 9 years old, $10000 when he shall be 21 years old. What sum must he invest annually for that purpose, estimating compound interest at 6 per cent.?

151. "A railroad has been constructed through a farm, in consequence of which, the owner of the estate is obliged to expend $400 in fencing. Suppose the fence must be renewed every 12 years, what sum should he now receive, to com-

pensate him for the required expenditure, money being worth 6 per cent. compound interest?"

152. There are two adjoining stores, each renting for $600 a year; the rent of one of them is payable semi-annually, and the rent of the other is payable quarterly. What will be the difference in the amount of income from the two stores, at the expiration of 10 years, provided all the rent is invested as fast as it becomes due, at 6 per cent. compound interest?

153. A, B, and C, purchased a vessel, for which they paid $3040. A paid 3½ times as much as B, and B paid 2¼ times as much as C. What number of dollars did each pay?

154. A merchant hired a sum of money, for which he agreed to pay the interest semi-annually, at the rate of 5½ per cent. a year. The amount of interest that he paid semi-annually was $99; what was the principal?

155. A cistern has three pipes, of different diameters; the largest pipe will fill it in 1⅓ hours, the next smaller pipe will fill it in 3⅓ hours, and the smallest will fill it in 5 hours. In what time will the cistern be filled, when all three of the pipes are discharging water into the cistern at the same time?

156. If A can reap an acre of wheat in 9¼ hours, B in 10½ hours, and C in 10¾ hours, in what time can they reap an acre, if they work together?

157. A farmer, having two equal flocks of sheep, sells 18 sheep from one flock, and 30 from the other; after which he finds twice as many left in one flock as in the other. How many were there originally in each flock?

158. A merchant retired from business when he had passed $\frac{9}{10}$ of his life, during ⅔ of which period he had been engaged in trade, having commenced at the age of 21. At what age did he die?

159. At the commencement of business, A had ⅔ as much money as B; but at the expiration of a year, A having gained 20 per cent., and B having lost $1200, they had equal sums of money. What was the original stock of each?

160. A man died, leaving an estate of $20000 to be divided between his two sons, one being 11 and the other 16 years old, in such proportion that the share of each, being on interest at 6 per cent., should amount to the same sum, when they should arrive at 21 years of age. What sum must each receive?

161. A man bought some lemons at 2 cents each, and three fourths as many at 3 cents each, and then sold them all at the rate of 5 cents for 2 lemons, and by so doing gained 25 cents. What number of lemons did he buy?

162. A gentleman distributed some money among four

poor children, giving the first as much as the second and fourth, the second as much as the third and fourth, the third half as much as the first, and the fourth 5 cents. How much money did he give to each ?

163. Two persons talking of their ages, one says, ⅔ of my age is equal to ¾ of yours, and the difference of our ages is 10 years. What were their ages ?

164. A market-woman purchased 12 apples and 6 pears, for which she paid 17 cents; she purchased, at another time, 3 apples and 12 pears, for which she paid 20 cents. What was the price of one apple and one pear, at the rate she purchased them ?

165. A grain-dealer purchased 16 bushels of corn and 20 bushels of rye, for which he paid $30. At another time, he purchased 24 bushels of corn and 10 bushels of rye, for which he paid $27. What did he pay per bushel for each kind of grain ?

166. A and B talking of their ages, A says to B, ⅙ of my age and ¼ of yours are equal to 13 years; B says to A, ½ of my age and ⅛ of yours are equal to 16 years. The age of each is required.

167. A gentleman left an estate, amounting to $26000, to his four sons, in such a manner that the third was to have once and one half as much as the fourth, the second was to have as much as the third and fourth, and the first was to have as much as the other three. What was the share of each ?

168. A man left an estate to his wife, son, and daughter, as follows: To his wife, ½ of the whole, and ⅓ as much as the share of the daughter; to his son, ⅓ of the whole; and to the daughter the remainder, which was $1200 less than the share of the son. What was the share of each ?

169. Two persons purchased a house for $8400. The first could pay for the whole, if the second would give him ⅓ of his money; and the second could pay for the whole, if the first would give him ⅕ of his money. How much money had each ?

170. A and B invested equal sums of money in trade; A gained a sum equal to one fifth of the sum which he invested, and B lost $300; then A had $\frac{12}{7}$ as much money as B. What number of dollars did each invest ?

171. There are three boys, A, B, and C, whose ages are as follows: A is 6 years old, B is as old as A and ½ the age of C, and C is as old as A and B both. What are the ages of B and C ?

172. A gentleman divided his fortune among his three sons, giving A 8 dollars as often as B 5, and B 7 dollars as often as C 4; the difference between the shares of A and C was $4500. What was the whole fortune, and the share of each?

173. A hare is 75 leaps before a greyhound, and takes 4 leaps while the greyhound takes 3; but 2 of the greyhound's leaps are equal to 3 of the hare's leaps. What number of leaps must the greyhound take to catch the hare?

174. A and B have the same income; A saves $\frac{1}{5}$ of his income, but B, by spending $300 a year more than A, at the end of 8 years, finds himself $400 in debt. What is their income, and what does each spend per year?

175. A merchant laid out $50 for linen and cotton cloth, buying 3 yards of linen for a dollar, and 5 yards of cotton for a dollar. He afterwards sold $\frac{1}{4}$ of his linen and $\frac{1}{5}$ of his cotton for $12, which was 60 cents more than it cost him. What number of yards of each kind of cloth did he purchase?

176. What is the hour of the day, when the time past from noon is equal to $\frac{11}{15}$ of the time to midnight?

177. The hour and minute hands of a clock are together at 12 o'clock. When will they be together the first time after 12 o'clock? When the fifth time?

178. At what time will the hour and minute hands of a clock point in opposite directions the first time after 12 o'clock? When will they point in opposite directions the fifth time?

179. A man bought apples at 5 cents per dozen, half of which he exchanged for pears, at the rate of 8 apples for 5 pears; he then sold all his apples and pears at 1 cent each, and by so doing gained 19 cents. How many apples did he buy, and how much did they cost?

180. A mechanic purchased a lot of land, containing 2400 square feet, at $3.75 a foot, on which he has built a store which cost $5750. The taxes and repairs amount to $250 a year. What yearly rent must he obtain for the store, to make a profit of 10 per cent. on the investment?

181. A manufacturer in Lowell remits to his agent in New Orleans a Bill of Exchange on London, which he is directed to sell, and invest the proceeds in cotton. On selling the bill at 10 per cent. advance, the agent received $4015. What number of pounds sterling was the bill drawn for, and what sum will the agent have remaining to invest, after deducting

¼ of 1 per cent. on the sum for which the bill was sold, and 2½ per cent. on the sum to be invested?

182. A merchant received on consignment 3 bales of sheeting, marked A, B, and C. A contained 420 yards, of a quality 15 per cent. better than B; B contained 380 yards, of a quality 10 per cent. poorer than C; and C contained 450 yards. The whole were sold together, at 12½ cents per yard. How much should be credited to each, after deducting 2½ per cent. commission?

183. A son having asked his father's age, the father replied, "Your age is 12 years; to which if five eighths of both our ages be added, the sum will be equal to mine." What was the father's age?

184. A and B set out from the same place, and travel in the same direction. A travels uniformly 18 miles a day, and after 9 days, turns and goes back as far as B has travelled in those 9 days; he then turns again, and, pursuing his journey, overtakes B in 22½ days from the time they first set out. It is required to find the rate at which B uniformly travelled.

185. A, B, and C bought a piece of land, the profits of which amount to $120 per annum. Now, the sum of money that each paid was in such proportion, that as often as A paid $5, B paid $7, and as often as B paid $4, C paid $6. How much must each have of the profits per annum?

186. A and B can dig a trench in 4 days, B and C in 6 days, and A and C in 5 days. In what time would A, B, and C dig the trench, and in what time would each of them dig it?

187. A mechanic engaged a lad to work for him 48 days, on the following conditions: — For each day that he worked he should receive 24 cents; and for each day that he was idle he should pay 12 cents for his board. At the expiration of 48 days, the account was settled, and the boy received $5.04. Required the number of days he worked, and the number of days he was idle.

188. A market-woman purchased a certain number of eggs at 2 for a penny, and as many more at 3 for a penny; and having sold them all at the rate of 5 for 2 pence, found that she had lost 4 pence. What number of eggs did she purchase?

189. A and B can perform a piece of work in 8 days, A and C can perform it in 9 days, and B and C can perform it in 10 days. In what number of days can each perform the work alone?

190. A and B have each the same income. A contracts an annual debt, amounting to ⅐ of it; B lives on ⅘ of it; and

at the end of 10 years, B lends A a sum sufficient to pay his debts, and has $160 left. What is the income of each?

191. A young man, at the age of 21 years, had a sum of money given him by his father. The young man supported himself during three successive years for $50 a year; and at the expiration of each year he increased that part of the sum his father gave him which he had not expended for his support, by a sum equal to one third of the same part. At the end of the third year, he had doubled the sum his father gave him. What number of dollars did his father give him?

192. A merchant died, leaving $60000; which he directed his excutor to divide among his 4 sons, whose ages were 8, 10, 12, and 15 years, in such proportion that the share of each, when invested at 5 per cent. compound interest, should amount to the same sum when each should attain the age of 21 years. The share of each son is required.

193. A young man, being asked what salary he had, replied, "At present I have $550; the first year of my apprenticeship I received only $100, but, on account of my industry, I have received, each year, an addition of $50 to my salary." Required the number of years since he commenced his apprenticeship.

194. A mechanic owns a wagon, having a mechanical apparatus attached to it, by which the difference of the number of revolutions of the wheels may be determined. Suppose each of the fore wheels to be 9 feet 9 inches in circumference, and each of the hind wheels to be 12 feet 3 inches in circumference. Now, on a journey, the mechanic has ascertained, by means of the apparatus, that the fore wheels have made 3600 more revolutions than the hind wheels. What distance has he travelled?

195. A colonel, wishing to form his regiment into a square, made two trials. At the first trial, he had 29 men left, after completing the square; at the second trial, having extended the side of the first square by one man, he wanted 32 men to complete the square. What was the number of men in the regiment?

196. A traveller, setting out on a journey, took a sum of money with him, to defray his expenses. The first week, he expended $50 more than the half of it. The second week, he expended $30 more than the fifth of what he had remaining. The third week, he expended $20 more than the fourth of what he had remaining at the end of the second week; when he had only $10 dollars left. The sum that he had at the commencement of his journey is required.

197. A and B can do $\frac{1}{10}$ of a piece of work in a day, B and C can do $\frac{1}{5}$ of it in $2\frac{1}{4}$ days, and A and C can do $\frac{1}{2}$ of it in $4\frac{1}{11}$ days. In what time would each do it alone, and in what time would it be done if they all worked together?

198. On a certain day, 20 farmers, 30 merchants, 24 lawyers, and 24 tailors, spent at a dinner $64; which was divided among them in such a manner that 4 farmers paid as much as 5 merchants, 10 merchants paid as much as 16 lawyers, and 8 lawyers as much as *12 tailors.* How much money did each class pay?

199. Two partners, Peter and John, bought goods to the amount

of $1000 ; in the purchase of which, Peter paid more than John, and John paid — I know not how much. They then sold their goods for ready money, and thereby gained at the rate of 200 per cent. on the prime cost ; they divided the gain between them in proportion to the purchase money that each paid in buying the goods ; and Peter says to John, My part of the gain is really a handsome sum of money ; I wish I had as many such sums as your part contains dollars ; I should then have $960000. I demand each man's particular stock in purchasing the goods.

200. A certain gentleman, at the time of marriage, agreed to give his wife $\frac{2}{3}$ of his estate, if at the time of his death he left only a daughter ; and if he left only a son, she should have $\frac{1}{3}$ of his property : but, as it happened, he left a son and a daughter, by which the widow received in equity $2400 less than if there had been only a daughter. What would have been his wife's dowry, if he had left only a son?

201. A merchant increases his capital yearly by 20 per cent., but takes from it annually $1000 for the support of himself and family. After he had carried on his business in this manner for three years, he finds, after deducting the usual sum of $1000, that his capital has increased $200 more than $\frac{3}{5}$ of the original sum. What was his original capital?

202. A, B, C, can build a house in 20 days, B, C, D, in 24 days, C, D, A, in 30 days, and A, B, D, in 36 days. Suppose they all commence the job together ; but that, after 10 days, A cut his foot, and ceased working ; two days afterwards, B was compelled to leave, and at the end of 18 days, C was compelled to leave ; in what time would D finish the job?

203. If 15 oxen eat $4\frac{1}{2}$ acres of grass in 2 weeks, and 24 oxen eat $14\frac{2}{5}$ acres in 5 weeks, how many oxen will eat 48 acres in 8 weeks, the grass being at first equal on every acre, and growing uniformly?

Art. 264. *We have shown, by a diagram,* (Art. 189,) *that the square described upon the hypothenuse of a right-angled triangle is equal to the sum of the squares described upon the base and perpendicular ; hence, when any two sides of a right-angled triangle are given, we can easily find the third side.*

When the base and perpendicular are given, we can find the hypothenuse by extracting the square root of the sum of the squares of the base and perpendicular.

When the hypothenuse and either of the other sides are given, we can find the third side by extracting the square root of the difference of the squares of the hypothenuse and given side.

204. There is a right-angled triangle, the hypothenuse of which is 90 feet, and the perpendicular 54 feet ; what is the length of the base?

205. What must be the length of a ladder that will reach a window 24 feet from the ground, the foot of the ladder being placed 7 feet from the wall of the building?

206. A line 244 feet in length will reach from the top of the walls

of a fort, situated on the bank of a river, to its opposite side; the river is known to be 240 feet in width; what is the height of the walls of the fort?

207. There is a rectangular school-room, 72 feet in length, and 44 feet in width; what is the number of feet from the centre of the room to either corner?

208. A and B start from the same place; A travels 121 miles directly north, and B, in the same time, travels 144 miles directly west. How many miles are they from each other?

209. There is a room 18 feet in length, 16 feet in breadth, and 12 feet in height; what must be the length of a line that will reach from a corner at the bottom to its opposite diagonal corner at the top?

210. Suppose a ladder 30 feet in length to be so placed in a street as to reach a window 24 feet from the ground, on one side of the street, and, without moving the foot of it, will reach a window 18 feet from the ground, on the other side; what must be the width of the street?

211. Suppose a pole to stand on a horizontal plane 96 feet in height above the surface: at what height from the ground must it be cut off so that the top of it may fall on a point 48 feet from the bottom of the pole, the end where it must be cut off resting on the upright part?

NOTE. *If the square of the base or perpendicular of any right-angled triangle be subtracted from the square of the sum of the other two sides, and the remainder divided by twice their sum, the quotient will be the length of the required perpendicular or base.*

212. There are two columns in the ruins of Persepolis left standing upright; one is 70 feet above the plane, and the other 50 feet; in a straight line between these, stands a statue, 5 feet in height, the head of which is 100 feet from the summit of the higher, and 80 feet from the top of the lower column. Required the distance between the tops of the two columns.

213. Three poles, each 50 feet long, were erected on a plain so that the upper ends met, and the lower ends were 60 feet apart. What length of rope was required to reach from their point of meeting to the ground?

214. What is a mean proportional number between the numbers 36 and 144?

NOTE. *A mean proportional between two given numbers is a number whose second power is equal to the product of those numbers. Hence, the square root of their product must be the mean proportional required.*

215. What is a mean proportional between 25 and 289?

216. A circular pavement contains 15625 square yards; what must be the length of the side of a square which shall contain the same number of square yards?

NOTE. *The area of a square surface being equal to the square of its side, hence the square root of any given area must be the length of the side of a square which shall contain that area.*

217. It is required to lay out 100 acres of land in the form of a rectangle, which shall be twice as many rods in length as it is in breadth; what must be its length and breadth?

NOTE. *If we extract the square root of one half the number of square rods in the given quantity of land, this root must be the required breadth.*

218. Suppose I wish to set out an orchard of 600 trees so that the length shall be to the breadth as 3 to 2; how many trees must it be in breadth, and how many in length?

219. A farmer owns 4 pieces of land; the first contains 4 A. 2 r. 10 rods; the second, 5 A. 1 r. 15 rods; and the third, 6 A. 3 r. 20 rods. The fourth is in the form of a right-angled parallelogram, its length being three times its breadth, and contains just as much land as the other three pieces. The length and breadth of the fourth piece are required. Suppose there is a barn in the centre of the fourth piece of land, what must be its distance from each side and end, and from each corner?

220. A surveyor wishes to lay out a township, containing 640 acres of prairie land, in the form of a right-angled parallelogram, the length of which is to be in proportion to its breadth as 4 to 3. What must be its length and breadth? Supposing there is a tree standing in the centre of the township, 165 feet in height, what must be the distance from the top of this tree to the centre of each side, and to each corner, supposing the land to be perfectly level?

QUESTIONS REQUIRING THE EXTRACTION OF THE CUBE ROOT.

Art. 265. 221. The bushel measure is 18.5 inches in diameter, and 8 inches in depth; and contains 2150.4 cubic inches. What must be the length of the side of a cubic box which will contain the same quantity?

NOTE. *If we extract the cube root of any given number, the root must be the length of the side of a cube which will contain the same quantity*

222. A carpenter was directed to make a cubical bin that should contain 25 bushels; what was its length, breadth and depth?

223. There is a granite pillar containing 187.5 cubic feet; its ends are equal and parallel squares, and its length is 12 times its breadth or depth. Its length, breadth and depth, are required.

PROPORTIONS AMONGST LINES, SURFACES, AND SOLIDS.

Art. 266. Lines are proportional to each other simply as their lengths, since they have but one dimension.

The areas of similar surfaces are proportional to each other as the squares of their like linear dimensions.

The contents of similar solids are proportional to each other as *the cubes of their* like linear dimensions.

COMPARISON OF THE CORRESPONDING LINES OF SIMILAR FIGURES.

Art. 267. Similar figures are those which have all their corresponding lines proportional.

If the diameter of one circle is twice the diameter of another, the circumference of the one is twice the circumference of the other.

If the side of a rectangle is three times the corresponding side of a similar rectangle, all the other sides of the one are three times the corresponding sides of the other.

If the side of a triangle is four times the corresponding side of a similar triangle, all the other sides of the one are four times the corresponding sides of the other; and the same principle is applicable to the corresponding dimensions of all similar surfaces and solids.

COMPARISON OF PARTS OF THE SAME FIGURE.

Art. 268. "Finding the ratio of the diameter of a circle to its circumference, called *squaring the circle*, is a problem that has long puzzled mathematicians; but its further investigation is now generally abandoned, they being satisfied that the exact ratio cannot be found, while, for all practical purposes, it was long since discovered with sufficient accuracy.

ARCHIMEDES discovered that if the diameter of a circle be 1, the circumference will be between $3\frac{10}{70}$ and $3\frac{10}{71}$, and hence he gave the practical ratio as 7 to 22, or 1 to $3\frac{1}{7}$. METIUS, a German mathematician, established the ratio, for practical purposes, at 113 to 355.

LUDOLF VAN CEULEN, a Dutch mathematician, investigated the subject with great care, and carried the calculation much further than any of his predecessors. He proved that if the diameter of a circle be 1, the circumference will be greater than

3.14159265358979323846264338327950288. but less than

3.14159265358979323846264338327950289."

Enclosing the circle between an inscribed and circumscribing polygon, was the mode adopted by Van Ceulen. As we increase the number of sides, the sum of all the sides, or the perimeter of the polygon, will approximate more and more nearly to the circumference of the circle; and if we enclose the circle between two similar and corresponding polygons, the length of the circumference will be somewhere between the perimeters of the two polygons.

For practical purposes, it is usual to consider the circumference 3.1416 when the diameter is 1, this being within less than .00001 of a unit of the *exact* ratio. Assuming this ratio as accurate, if the diameter of a circle be 1, $\frac{1}{6}$ of its circumference will be .5236, which is equal to the curve arc upon one side of a regular hexagon; and the side of a hexagon being equal to the radius of its circumscribing circle, hence *the side is .5*; so that the curve exceeds the straight line .0236. The *radius of* a circle is a straight line extending from the centre of the circle to the circumference, or half its diameter; hence its ratio to the circumference will be as 1 to 6.2832.

From this assumed ratio, or from the more accurate ratio obtained by Van Ceulen, a number of proportional multipliers and divisors have been found. See Art. 210.

The ratio of the circumference of a circle to its diameter being as 3.1416 to 1, as assumed, it follows that the circumference of any circle will be 3.1416 times its diameter; also, that its diameter will be $\frac{10000}{31416}$ or .3183 of its circumference: hence, if the diameter of any circle be multiplied by 3.1416, the product will be its circumference; also, if the diameter of any circle be divided by .3183, the quotient will be its circumference. It also follows, that if the circumference of any circle be divided by 3.1416, the quotient will be its diameter; also, if the circumference of any circle be multiplied by .3183, the product will be its diameter.

It can be demonstrated that the circumference of a circle $\times$ by one fourth of its diameter $=$ the area; also, that the square of the diameter $\times$ by one fourth of 3.1416 $=$ the area. And $3.1416 \times \frac{1}{4}$ or $.25 = .7854$; hence, if the square of the diameter of any circle be multiplied by .7854, the product will be the area. And $1 \div .7854 = 1.2732$; hence, if the square of the diameter of any circle be divided by 1.2732, the quotient will be the area.

It has been shown that the area of a circle whose diameter is 1 is .7854; therefore, its square root must be the side of a square whose area is equal to the area of the circle. The $\sqrt{.7854} = .8862$; hence, if the diameter of any circle be multiplied by .8862, the product will be the side of a square whose area is equal to the area of the circle.

The diameter of a circle being 1, its square is $1 \times 1 = 1$, and the diameter 1 is equal to the diagonal of the inscribed square. Since the diagonal divides the square into two equal right-angled triangles, therefore the square of the diagonal must be equal to the sum of the squares of the base and perpendicular, which are two of the sides of its inscribed square; hence the square of the base or perpendicular must be .5, and $\sqrt{.5} = .7071$, the length of the side of the inscribed square. Then it follows that if the diameter of any circle be multiplied by .7071, the product will be the side of the inscribed square.

We have shown that the ratio of the diameter of a circle to a side of its inscribed square is as 1 to .7071, and we have also shown that the circumference of a circle to its diameter is as 3.1416 to 1; hence, the ratio of the circumference of a circle to a side of its inscribed square is as 3.1416 to .7071, and $.7071 \div 3.1416 = .225$, a decimal ratio sufficiently exact for practical use; therefore, if the circumference of any circle be multiplied by .225, the product will be the side of an inscribed square. If we divide the side of the inscribed square by .225, the quotient will be the circumference of its circumscribing circle.

Since the side of the inscribed square is .7071 of the diameter of its circumscribing circle, the diameter will be $1 \div .7071 = 1.4142$ times the side of the inscribed square.

The area of a sphere whose diameter is 1 is 3.1416; and $3.1416 \times \frac{1}{6} = .5236$, the solidity of a sphere whose diameter is 1. Hence, if the surface of any sphere be multiplied by $\frac{1}{6}$ of its diameter, the product will be its solidity. Or if the cube of the diameter of any sphere be multiplied by .5236, the product will be its solidity.

We might illustrate the method of finding many other ratios which are sometimes used in calculation. Whenever we find a decimal ratio having reference to a circle, we may rest assured that it is derived in some way from the ratio of the diameter of a circle to its circumference.

COMPARISON OF SIMILAR SURFACES.

Art. 269. The areas of surfaces, and the contents of solids, when expressed in numbers of the same denomination, may be compared, and their proportional magnitudes expressed in a fractional form.

Similar surfaces and solids are those which have all their corresponding dimensions proportional.

The areas of similar surfaces are to each other as the squares of their corresponding dimensions.

The ratio of the *square* of the diameter of one circle to the *square* of the diameter of another, is equal to the ratio of the *area* of the one to the *area* of the other.

In two *similar* rectangles, the ratio of the *area* of the one to the *area* of the other, is equal to the ratio of the *square* of the side of the one to the *square* of the corresponding side of the other; and this principle is applicable to all similar surfaces.

224. There are two similar rectangles; their corresponding dimensions are as 3 to 4; the area of the smaller is 54 square yards. What is the area of the greater?

225. There is a square township of new land, each side of which measures 1920 rods; what must be the side of a square piece of land that shall contain $\frac{1}{36}$ as many square rods?

226. There is a circular pavement 15 feet in diameter; what must be the diameter of a circle which shall contain 8 times the number of square feet?

COMPARISON OF SIMILAR SOLIDS.

Art. 270. *The contents of similar solids are to each other as the cubes of their corresponding dimensions.*

227. There are 2 cubes; the side of one of them is 2 inches, the side of the other is 4 inches. How many times greater is the cube whose side is 4 inches than the cube whose side is 2 inches?

$2 \times 2 \times 2 = 8$, the number of cubic inches in the smaller cube.

$4 \times 4 \times 4 = 64$, the number of cubic inches in the greater cube; and 64 inches are 8 times 8 inches. Hence, we perceive that when the corresponding dimensions of two *similar* solids are to each other as 2 to 1, the contents are as 8 to 1.

When the corresponding dimensions are as 3 to 1, the contents are as 27 to 1.

When the corresponding dimensions are as 5 to 6, the contents are as 125 to 216, &c.

228. There is a cubic box whose side is 18 inches; what must be the side of a cubic box which shall contain 8 times as many cubic inches?

229. There is a cubic block of granite whose side is 24 inches; what must be the side of a block that shall contain $\frac{1}{64}$ as many cubic inches?

230. If a cubic block of granite, whose side is 12 inches, weigh 166 pounds, what will a cubic block of the same kind weigh, the side of which measures 36 inches?

231. If a cube of Swedish iron, whose side is 12 inches, weigh 486.876 pounds, what will be the weight of a cube of the same kind of iron, the side of which is 18 inches?

COMPARISON OF DISSIMILAR SURFACES.

Art. 271. The area of a circle is equal to the area of a triangle of which the radius of the circle is the perpendicular, and the circumference of the circle is the base.

From this analogy we derive the rule for finding the area of a circle, viz., Multiply half the circumference by half the diameter; or, multiply the circumference by one fourth of the diameter.

The area of a circle is .7854 of its circumscribing square, and the area of an ellipsis is .7854 of its circumscribing parallelogram. The square root of the product of the transverse and conjugate diameters of an ellipsis, is the diameter of a circle equal in area to the ellipsis. The area of a triangle is equal to half the area of a parallelogram of equal base and altitudes; hence the rule for finding the area of a triangle is to multiply the base by half its perpendicular height.

COMPARISON OF DISSIMILAR SOLIDS.

Art. 272. "As the *circle*, in surfaces, contains the greatest surface within the same bounds, so the *sphere*, in solids, contains the greatest solidity within the same surface; there being no corners in the one or the other. Both in solidity and surface, the sphere is $\frac{2}{3}$ its circumscribing cylinder, if we take into consideration the surface of the ends of the cylinder; the surface of the sphere being equal to the curve surface of the cylinder, without its ends."

"If the diameter of the sphere be 1 inch, its solidity will be .5236 of an inch; and the solidity of the circumscribing cylinder will be .7854 of a cubic inch; of which .5236 is just $\frac{2}{3}$."

"The surface of an inch globe is 3.1416 square inches; and the curve surface of an inch cylinder is 3.1416 square inches, to which .7854 being added for each end, we have the whole surface 4.7124 square inches, of which 3.1416 the surface of the globe is just $\frac{2}{3}$; so that both in solidity and surface the globe is $\frac{2}{3}$ of its circumscribing cylinder. The surface of a sphere is equal to four times the area of a great circle of it."

"Surface increases as the square of its linear dimensions only, while solidity increases as the cube of the linear dimensions of the solid body; a globe 2 inches in diameter has four times the surface of one only an inch in diameter, and it has 8 times the solidity. If the diameter of the base of a cone be equal to the perpendicular height of the cone, then the solidity will be half that of a sphere whose diameter is equal to the base of the cone, and $\frac{1}{3}$ of a cylinder of the same base and altitude as the cone."

From which it appears, that, the height and diameter being equal, the solidity of a *cone*, *sphere* and *cylinder*, are as 1, 2, 3. If a cylinder of equal diameter and height be made to contain 3 pints of water, and a cone of equal diameter and height as the cylinder be introduced, one pint will run out; and if the cone be taken out, and a sphere of the same diameter be introduced, another pint will run over."

PRACTICAL QUESTIONS IN MENSURATION.

Art. 273. 232. There is a circle 60 feet in diameter; what is its area? What is the difference between its inscribed and circumscribing squares?

233. There is a hollow cylinder 9 inches in diameter, and 12 inches in length; what part of this cylinder would be filled by the largest square prism that could be placed within it?

234. There is a hollow globe whose inside diameter is 9 inches; what is the side of the largest cube that can be placed within it?

235. There are 4 cubic boxes; the inside dimensions of each measure 12 inches. Suppose a globe, 12 inches in diameter, to be placed in one of them; a cylinder, 12 inches in diameter and 12 inches in length, in a second; a square pyramid, whose side and perpendicular height are each 12 inches, in a third; and a cone, whose diameter and perpendicular height are each 12 inches, in the fourth. What number of quarts of water would be required to fill each of the boxes, the solids remaining in them?

236. What will be the expense of painting the walls of a room which measure 75.5 feet in compass, and 12.5 feet in height, at 10 cents a square yard?

237. The length of a room is 20 feet, its width 18 feet, its height 12 feet; what will the coloring of the walls come to, at 12½ cents a square yard; deducting a fireplace, 6 feet by 4 feet, and 4 windows, each 6 feet 6 inches by 3 feet 9 inches?

238. What will be the cost of paving a rectangular yard, the length being 21 yards, and the breadth 15 yards, in which there is a foot-path, 5 feet 3 inches wide, running the whole length of the yard: the path to be paved with broad stones, at 36 cents a square yard: and the remainder of the yard with bricks, at 24 cents a square yard?

239. What will be the expense of painting a conical spire, of which the height is 60 feet, and the diameter at the base is 10 feet, at 15 cents a square yard?

240. A gentleman has a bowling-green 300 feet long and 200 feet broad, which he wishes to raise a foot higher, by means of the earth dug out of a ditch which surrounds it. To what depth must the ditch be dug, supposing its breadth to be 8 feet?

241. What would be the cost of excavating the New York and Erie Canal, supposing it to be 360 miles in length, 40 feet in width at the surface, 32 feet in width at the bottom, and 6 feet in depth, at 25 cents a cubic yard?

242. Suppose the wheels of a railroad car to be 3 feet 9 inches in diameter, what number of revolutions do they make in a minute, when the cars are running at the rate of 25 miles an hour?

243. The court-house in Boston has 8 granite pillars, each 25 feet 6 inches in length, 4 feet 6 inches in diameter at the base, and 3 feet 6 inches in diameter at the top. What number of cubic feet do they contain; and what is their weight, allowing a cubic foot to weigh 2760 ounces?

244. There is an elliptical pavement; its longer diameter is 90 feet, and its shorter diameter 60 feet. What is its area? What would be the area of its circumscribing parallelogram?

245. A gentleman has a flower-garden, 180 feet in length, and 120 feet in breadth, which is enclosed by a brick wall, (except the entrances,) 4.5 feet in height, and 12 inches in thickness, the wall to be included within the limits of the garden. Opposite the centre of the garden, on each side and end, there is a gate 6 feet in width, and in the centre of the garden there is an elliptical pond, whose longer diameter is 60 feet, and shorter diameter 40 feet, around which there is a gravel-walk 6 feet in width; there is also a walk 6 feet in width, next the wall around the garden, and from each gate to the walk around the pond. What number of square feet are there in the garden, exclusive of the pond and walks, and what number of bricks will be required to build the wall, allowing each brick to be 8 inches long, 4 inches wide, and 2 inches thick?

246. A tract of land is to be laid out in the form of an equal square, and to be enclosed with a post and rail fence, 5 rails high, so that each rod of fence shall contain 10 rails. How large must said square be, to contain just as many acres as there are rails that enclose it?

247. A carpenter was directed to make an ice-chest 6 feet in length, 3 feet 6 inches in breadth, and 2 feet 9 inches in depth, measuring inside, and to make it of plank 2 inches in thickness. What number of feet of plank, board measure, will be required to make it, allowing 10 per cent. for waste? What will be the cost of the plank, at $24 per thousand feet, board measure? What is the capacity of the chest in beer gallons?

NOTE. *Merchantable boards are usually 1 inch in thickness; hence, to reduce the superficial contents of plank to board measure, multiply the superficial contents by the thickness of the plank.*

248. There is a garden 100 feet in length, and 80 feet in breadth. On each side and end of the garden there is a border of uniform width, which contains just one fourth of the whole area of the garden; what is the width of the border? Within the border, on each side and end, there is a graveled walk which contains just one eighth of the whole area of the garden; what is the width of the walk?

249. Four men, A, B, C and D, purchased a grindstone 60 inches in diameter, and of uniform thickness; how much of its diameter must each grind off, to have an equal share of the stone, if A first grinds his share, then B, then C, then D, making no allowance for the eye?

250. There are 2 apple-trees in an orchard, standing 60 feet apart, and each tree 40 feet from the wall which encloses the orchard; what is the shortest distance that a boy must pass over, in running from one tree to the wall, and then to the other tree?

251. There is a square pyramid; each side of its base is 30 inches, its perpendicular height is 120 inches; which is to be divided, by sections parallel to its base, into three equal parts; the perpendicular height of each part is required.

252. The sides of two square pieces of land are as 3 to 5, and the sum of their superficial contents is 30600 square feet. What is the length of a side of each piece?

253. A lady purchased three pieces of cloth, paying as many cents a yard for each piece as there are yards in the piece. The whole number of yards purchased was 36, and the prices of the three pieces were proportionate to each other as 4, 9, 16. The whole cost of the three pieces was $4.64. The whole number of yards in each piece is required.

254. There is a box in the form of an exact cube; each of its sides is 16 inches, measuring inside. What is its capacity in cubic inches? What is the solid contents of the largest globe that can be placed in this box? What is the solid contents of the largest cube that could be placed within this globe, if it were hollow, supposing its inside diameter to be 14 inches?

NOTE. *The side of a cubic box, and the diameter of the largest globe that can be placed within it, are equal.*

The inside diameter of a hollow globe, and the diagonal of the largest cube that can be placed within it, are also equal; and one third of the square of the diagonal of a cube is equal to the square of the side of the cube; hence, its square root must be the side of its inscribed cube.

255. Suppose a ball, 36 inches in diameter, to be dressed down to a cube; what would be its size?

256. How large a globe may be turned out of a cube, whose side is 36 inches in length?

257. Required the side of an equilateral triangle, that may be inscribed in a circle 90 feet in diameter.

258. There is a certain piece of round timber, 24 inches in diameter; required the side of an equilateral triangular beam, that may be hewn from it.

259. In turning a one horse chaise in a ring, it was observed that the outer wheel made two turns, while the inner wheel made but one; the wheels were both 4 feet high, and supposing them fixed at 5 feet asunder on the axle, what was the circumference of the track described by the outer wheel?

260. A farmer, having a quantity of wheat to measure, finds himself without a vessel to measure it in. He accordingly made a box 15 inches square, but could not tell precisely the depth necessary, that it might contain just a bushel. What must be its depth?

The capacity of a standard bushel is 2150.4 cubic inches; and this, divided by 15 × 15 = 225, will give 9.55 inches, as the depth.

261. What must be the depth of a circular measure, whose inside diameter is 13¾ inches, to contain a half bushel?

262. What must be the depth of a circular measure, whose inside diameter is 10¾ inches, to contain a peck?

263. What must be the depth of a circular measure, whose inside diameter is 8¾ inches, to contain a half peck?

264. What must be the depth of a circular measure, whose inside diameter is 6¾ inches, to contain two quarts?

265. What must be the depth of a circular measure, whose inside diameter is 5¾ inches, to contain a quart?

266. A carpenter was directed to make one dozen cubic boxes, each to contain 5 bushels; what must be the inside dimensions of each box? What number of feet of boards would be required to make them, allowing 12½ per cent. for waste?

267. How many acres are contained in a square field, the diagonal of which is 20 rods longer than either of its sides?

We may assume a square of any size, and find the excess of the diagonal; then institute the proportion, as the excess thus found is to the given excess, so is the side of the assumed square to the side of the required square.

268. What will be the side of the largest stick of square timber that can be hewn from a log 12 inches in diameter at the smaller end?

269. A pine log is 30 feet in length, and 3 feet 9 inches in circumference, at the smaller end; what number of cubic feet of timber will it make when hewn square?

270. What will be the expense of painting a conical spire, of which the perpendicular height is 75 feet, and the diameter at the base 15 feet, at 20 cents a square yard?

271. A stone-house is 50 feet long, 34 feet wide, 20 feet high, to the eaves, and the gables rise 9 feet above the eaves: 9 feet of the walls are 2 feet thick, the remainder to the eaves 1½ feet thick, and the gables 1 foot thick. How many cubic feet in the walls? What did the building of them come to, at $1.25 per perch, no allowance being made for windows, doors, &c.?

272. A church spire is to be built in the form of a hexagonal pyramid, each side of the base being 10 feet, and its perpendicular height 80 feet. Within the spire is to be a hollow cone, 15 feet in diameter at the base, and so running up as to leave the walls of the

spire as thick at the top of the cone as at the bottom. How many cubic feet will the spire contain? What will be the expense of painting the surface of this spire, at 25 cents per square yard?

273. A New England farmer left the place of his nativity, and emigrated to the new state of Iowa, where he purchased a square mile of new land, and built a house in its centre. He gave equal portions of this square mile to each of his four sons; and the share of each son was enclosed within one of the four largest circles that could be described within the square mile, and not intersect each other. Each son built a house in the centre of his circular piece of land. What part of the square mile has the father left, and what number of acres does it contain? What is the number of acres in each son's share? What is the distance from each son's house to his father's house? What is the distance from each son's house to the house of each of the other three?

274. A mason was directed to build a brick cistern, of sufficient capacity to contain 2000 imperial gallons. Supposing its inside diameter to be 8 feet, and the top to be arched in the form of a hemisphere, what must be its depth inside, measuring from the centre of the arch? Supposing the walls to be 12 inches in thickness, what number of bricks 8 inches long, 4 inches wide, and 2 inches thick, will be required to build it?

275. A had a circular meadow, and agreed that B should have the privilege of grazing his three horses, at $2 per acre, as follows: He was to drive 3 stakes in the ground, at such distances asunder that when the horses were attached severally to them by ropes, they might graze over the greatest possible quantity of ground in the circle, without encroaching on each other's premises. After the grazing was over, it was found that just one acre of grass remained untouched, at the centre of the meadow. It is required to give the distance asunder of the three stations; to find the length of the ropes used to confine the horses; the quantity of land in the meadow; and the amount paid by B for what his horses grazed over.

276. A gentleman has a garden 180 feet in length, and 120 feet in width. In the centre of the garden there is an elliptical pond, whose longer diameter is 36 feet, and shorter diameter 24 feet; the pond is enclosed by a wall of granite, 3 feet in depth, and 18 inches in thickness. There is a graveled walk around the pond, 4 feet in width, including the wall. What number of square feet of land does the garden contain, exclusive of the pond and graveled walk? What is the number of square feet in the pond? What number of square feet in the walk? What number of cubic feet in the wall? What number of square feet in the face of the wall and top?

NOTE. *The circumference of an ellipsis may be found by multiplying the square root of half the sum of the squares of the two diameters by* 3.1416.

MISCELLANEOUS PROBLEMS AND RULES.

Art. 274. "The sum and quotient of two numbers given, to find the numbers.

Rule. *Divide the sum by the quotient* + 1; *the result will be the less number sought; from which, by subtraction, the greater number is readily found.*

Example. A and B played at marbles. After A had won several, they found that each had 70. A wishes to know how many he has won; but only knows that B had, at the commencement of the play, four times as many as he. How many did each begin with?

Art. 275. The sum and product of two numbers given, to find the numbers.

Rule. *From the square of their sum, take four times their product, and the square root of the remainder will be the difference of the numbers. Then proceed as in Art.* 123.

Example. Five hundred rods of fencing are necessary to enclose a certain rectangular farm, which, at $22 per acre, will cost $1980. Required the length and breadth of the farm.

Art. 276. The sum of two numbers and the sum of their squares given, to find the numbers.

Rule. *From the square of the sum take the sum of the squares, and half the remainder will be the product. Then proceed as in Arts.* 275 *and* 123.

Example. A and B have 50 guineas between them, which are to be so divided that the sum of the squares shall be 1300. How many had each, supposing A to have the greater number?

Art. 277. Given the sum of two numbers, and the difference of their squares, to find the numbers.

Rule. *Divide the difference of their squares by their sum, and you will have the difference of the numbers.*

Example. A man has two square farms, one containing 1000 acres more than the other; and to enclose both with a fence 10 rails high, and 2 panels to the rod, requires 64000 rails. How many acres in each?

Art. 278. The sum of the squares and the difference of the squares of two numbers given, to find the numbers.

Rule. *From the sum take the difference, and half the remainder is the square of the less; which, taken from the sum of the squares, will, of course, give the square of the greater. Then extract the square root to find the numbers.*

Example. Two companies of boys went in search of nuts, and each boy got as many nuts as there were boys in his company. Moreover, all the boys in the larger company got 295 nuts more than all the boys in the smaller company, and the whole number collected was 1025. How many boys were there in each company?

Art. 279. Given the difference of two numbers, and their product, to find the numbers.

Rule. *To the square of the difference add four times the product, and the square root of the sum will be the sum of the numbers. Then proceed as in Art.* 123.

Example. Two travellers, A and B, were asked how far they had travelled. A said he had travelled fifty miles further than B, and B said that if the number of miles that he had travelled were repeated once for every mile A had travelled, the distance would be no less than 75000 miles, which would reach three times round the earth. How far had each travelled?

Art. 280. The difference of two numbers and their quotient given, to find the numbers.

Rule. *Divide the difference by the quotient, less* 1, *for the smaller number, and add it to the difference, for the larger.*

Example. Says a father to his son, "I am 10 times as old as you, and I was 45 years old when you were born." What is the age of each?

Art. 281. Given the difference of two numbers, and the sum of their squares, to find the numbers.

Rule. *From twice the sum of the squares, take the square of the difference; the square root of the remainder will be the sum of the numbers. Then proceed as in Art.* 123.

Example. The perpendicular of a right-angled triangle measures 10 rods more than the base, and the hypothenuse is 50 rods; required the area.

Art. 282. The difference of two numbers, and the difference of their squares given, to find the numbers.

Rule. *Divide the difference of the squares by the difference of the numbers; the quotient will be the sum of the numbers. Then proceed as in Art.* 123.

Example. A field of corn is 10 rods longer than it is wide, and the square of the width is 700 less than the square of the length. What is the area of the field?

Art. 283. The product of two numbers, and the sum of the squares given, to find the numbers.

Rule. *Add twice the product to the sum of the squares; the square root of the sum will be the sum of the numbers. Then proceed as in Art. 275.*

Example. The diagonal of a rectangular 30 acre lot is 100 rods, and it is ten rods longer than wide. What are the length and breadth of the lot?

Art. 284. Given the product of two numbers, and the difference of their squares, to find the numbers.

Rule. *Add one fourth of the square of the difference of squares to the square of the product; find the square root of the sum; from this root take half the difference of squares, and the square root of the remainder will be the less number.*

Example. A bought a piece of cloth, giving as many dollars per yard as there were yards; while B bought a finer piece, giving likewise as many dollars per yard as there were yards: and it was found that B's cost $28 more than A's; and had B paid the same rate per yard that A paid, his cloth would have cost him $48. What did each pay, and what quantity of cloth did each purchase?

Art. 285. The product of two numbers and their quotient given, to find the numbers.

Rule. *Divide the product by the quotient, and the square root of the result will be the less number.*

Example. A bought a quantity of calico for $3.60, and the number of yards was 2½ times the number of cents he gave per yard. What did he give per yard, and how many yards did he buy?

Art. 286. Given the product of two numbers, and the square of the difference, to find the numbers.

Rule. *Add four times the product to the square of the difference, and the square root of the sum will be the sum of the numbers. Then, having the sum and product, proceed as in Art. 275.*

Example. Being engaged in laying out an oblong garden, which contained just an acre and a half, I found that having cut off from one end a square area, the greatest square that I could form in the remainder contained just 64 square rods. What were the length and breadth of my garden?

Art. 287. The quotient of two numbers and the sum of their squares given, to find the numbers.

Rule. *Divide the sum of the squares by the square of the quotient, plus 1; and the square root of the result will be the less number.*

Example. A and B paid $100 for cloth, each paying as many dollars per yard as he bought yards, and A bought ¾ as many yards as B. How many yards did each buy?

Art. 288. Given the quotient of two numbers, and the difference of their squares, to find the numbers.

Rule. *Divide the difference of their squares by the square of the quotient less 1; the square root of the resulting quotient will be the less number.*

Example. A and B bought cloth, each paying as many dollars a yard as he bought yards; and B bought 1½ times as many yards as A, and paid for it $28 more than A paid. How many yards did each buy?

Art. 289. The quotient of two numbers and the square of their sum given, to find the numbers.

Rule. *Divide the square of the sum by the square of the quotient plus* 1; *the square root of the resulting quotient will be the less number.*

Example. Says John to Henry, 'I have three times as many cents as you have, and if both were added together, and expended in apples, at as many apples for a cent as there are cents, they would purchase 400.' How many cents had each boy?

Art. 290. The quotient of two numbers, and the square of their difference given, to find the numbers.

Rule. *Divide the square of their difference by the square of the quotient less one. The square root of the resulting quotient will be the less number.*

Example. The ratio of two numbers is 7, and the square of their difference 324. What are the numbers?

Art. 291. Given the square of the sum, and the sum of the squares, of two numbers, to find the numbers.

Rule. *From the square of the sum take the sum of the squares, and twice the product of the numbers will remain. Then subtract four times the product from the square of the sum, and the remainder will be the square of the difference. Then, having the square of the sum and difference, extract the roots, and proceed as in Art.* 123.

Example. The square of the sum of two numbers is 144; the sum of their squares is 80. What are the numbers?"

Art. 292. "Knowing the price for which a thing is sold, in which there is as much gained per cent. as the article cost, to find the original cost.

"Rule. *Multiply the price or sum sold for by* 100, *add the square of* 50 *to the product; from the square root of this sum subtract* 50, *and the remainder will be the first cost.*"

1. "Sold a piece of cloth for £56, and gained as much per cent. as the cloth cost me. What was the original cost?"

2. "A, B and C, each contributed $200 to a common stock, and went to the west to buy land. They bought 300 acres, and in dividing it equitably it was determined that A allow 75 cents per acre more for his land than B and C gave. How many acres did each receive, and at what price?"

Note. *This, and like questions, can be solved by the following;*

Rule. "*From the whole amount paid for the land, take the value at the given difference per acre; and reserve the remainder. Multiply the aforesaid value, at the difference in price, by four times the whole amount paid by him who paid least per acre, and to the product add the* *square of the reserved remainder; of the sum extract the square root, and to it add the reserved remainder Divide the sum by twice the whole number of acres, and the quotient will be the price per acre paid by him who paid the least per acre. The quantity is then easily found.*"

EXCHANGE.

Art. 298. Exchange, in commerce, is a term which designates that class of mercantile transactions by which the debts of individuals residing at a distance from their creditors are paid by means of Bills of Exchange.

A Bill of Exchange is a written order, addressed to some person at a distance, directing him to pay a specified sum of money to a third person, or to his order, either at sight, or after the expiration of some specified time. For forms of Bills of Exchange, see Art. 300.

Bills of Exchange are bought and sold, and pass from one individual to another, like any other circulating medium.

"The intrinsic *Par of Exchange* is the value of the currency of one country estimated in the currency of another, by comparing the quantity of gold and silver in their respective coins."

The intrinsic value of the English sovereign, which represents the pound sterling, is $4.861 in our gold, taken as a standard.

The *exchange* value of the English pound is $\$4.44\frac{4}{9}$, and all premiums are computed upon this standard.

The *commercial* value of a Bill of Exchange on England is its exchange value plus the premium it will bring in the market.

Suppose bills on England sell at 9 per cent. premium. If we add to the exchange value of the pound sterling 9 per cent., the sum is its commercial value.

Exchange value of 1 £ = $4.444 +.
Premium 9 per cent. = .399 +.
Commercial value = $4.843 +.

1. A merchant in Boston wishes to remit to Liverpool £475. 15s. 6d., and purchases a Bill of Exchange for that amount, at a premium of 8¼ per cent. What does he pay for the bill, in Federal money?

2. A merchant in New York wishes to remit to London $5675.25, exchange being at a premium of 9 per cent. What will be the amount of his Bill of Exchange, in pounds sterling?

In France, accounts are kept in *francs* and *centimes*, 1 franc = 100 centimes. The value of a franc is 18.6 cents, on which value the rate of exchange is computed. The rate of exchange is sometimes computed on the exchange value of a dollar in francs and centimes.

3. What must a merchant pay for a bill on Paris for 5000 francs, at a premium of 1½ per cent.?

4. What must a merchant pay for a bill on Havre for 7875 francs, *exchange* being at the rate of 5 francs 40 centimes to the dollar?

BOARD MEASURE.

BOARD RULES.

Art. 294. The rule used by surveyors of lumber, in Boston and vicinity, is 3 feet in length, of a square form, having four equal and parallel sides.

Each side is divided into four equal spaces by lines running the whole length of the rule, and each space is divided into equal parts, which are all numbered; each number indicating the contents of a board, in square feet, corresponding with its length in feet and width in inches.

At one end of the rule, the spaces on the sides are all numbered in succession, as follows: 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27; these numbers correspond with the lengths of boards to be measured, in feet. The upper space on the first side, numbered 12, is used in measuring the width of boards, in inches; this rule is also used with a ten foot pole, in measuring the length of boards, in feet.

To find the content of a board, first ascertain its length in feet; then apply the rule across the board, and ascertain its mean width in inches. Suppose the board is 16 feet in length, and 15 inches in width; on the space numbered 16, on the second side of the rule, and under 15 inches, we find 20, the number of square feet in the board.

Suppose a board is 22 feet long, and 18 inches wide; on the space numbered 22 on the third side, and under 18 inches, we find 33, the number of square feet in the board.

NOTE. *Planks, joists and timber, less than six inches square, are to be reduced to board measure.*

There is another board rule 2 feet in length, of an octagonal form, having eight equal sides or faces.

On a line running round the rule at the centre, the faces are numbered in succession, as follows: 8, 9, 10, 11, 12, 13, 14, 15; and each face is designated by the number on it, which numbers correspond to the lengths of the boards to be measured.

If the board is 13 feet in length, place the thumb on face 13, and apply the measure across the board, and the number on face 13 corresponding with the width of the board will express the number of square feet in the board. Thus, if the width of the board extends from 1 to 15, the board contains 15 square feet.

If the board is 14 feet long, it must be measured on face 14; if the board is 18 feet long, measure its width on face 10, and also on face 8, and the sum of the two results will be the content of the board in square feet.

There is also a *folding* rule, on the face of which is a table to show the content of a board from 1 to 20 feet in length, and from 6 to 20 inches in width.

The upper line of the table shows the length of the board in feet, and the column at the left shows the width of the board in inches. If a board is 16 feet long, and 10 inches wide, find the length, 16 feet, on the upper line, then under 16 on line 10 is 13-4, which shows that the board contains 13 and 4 twelfths square feet.

TO MEASURE ROUND TIMBER.

Art. 295. "*The practical rule, used by surveyors, is to multiply the length in feet by the square of one fourth of the mean girth in inches, and to divide the product by* 144; *the quotient is the number of cubic feet.*

The girths of a piece of round timber are 62.832 and 31.416 inches, and its length 50 feet. What number of cubic feet does it contain?

$$\frac{62.832+31.416}{2} \div 4 = 11.781^2 \times 50 \div 144 = 48.1916$$

cubic feet.

Pine and spruce spars, from 10 to 4.5 inches in diameter inclusive, are to be measured by taking the diameter, within the bark, at one third of their length from the larger end. Spars are usully purchased by the inch diameter, and all under 4 inches in diameter are called poles.

Spars not more than 7 inches in diameter should be 5 times as many feet in length as they are inches in diameter. Those which are more than 7 inches in diameter should be 4 times as many feet in length as they are inches in diameter."

TONNAGE OF VESSELS.

Art. 296. By a law of Congress, the tonnage of vessels is found by the following

RULE. *If the vessel is double-decked, take the length from the fore part of the stem to the after side of the stern post above the upper deck, the breadth at the broadest part above the main wales; half of this breadth must be taken as the depth of the vessel; then deduct from the length three fifths of the breadth; multiply the remainder by the breadth, and the product by the depth; divide this last product by* 95, *and the quotient is the tonnage.*

What is the tonnage of a ship of the line, measuring as above, 210 feet on deck, and 59 feet in breadth?

$59 \div 2 = 29.5$, the depth.

$210 - \frac{3}{5}$ of $59 = 174.6 \times 59 \times 29.5 \div 95 = 3198.8 +$ tons.

If the vessel is single-decked, take the length and breadth as above directed for a double-decked vessel, and deduct from the length three fifths of the breadth, and take the depth from the under side of the deck plank to the ceiling of the hold; then proceed as before.

The length of a single-decked vessel is 223 feet, the breadth 39.5 feet, and the depth of hold 23.5 feet; what is its tonnage?

$223 - \frac{3}{5}$ of $39.5 = 199.3 \times 39.5 \times 23.5 \div 95 = 1947.3 +$ tons.

CARPENTERS' RULE. *Measure the length of the keel, breadth of the beam, and depth of the hold, all in feet.* Divide the continued product *of the three dimensions by* 95; *the* quotient will be the tonnage. If the *vessel is double-decked, half the breadth of the beam is taken as the depth of the hold.*

What is the tonnage of a single-decked vessel, whose length is 65 feet, breadth 20 feet, and depth 10 feet?

$$65 \times 20 \times 10 \div 95 = 136\tfrac{16}{19} \text{ tons.}$$

HORSE POWER.

Art. 297. "A *horse* travels 400 yards, at a walk, in 45 minutes; at a trot, in 2 minutes; at a gallop, in one minute."

" A *draft horse* can draw 1600 lbs. 23 miles a day, weight of carriage included."

" The ordinary work of a horse may be stated at 22500 pounds, raised 1 foot in a minute, for 8 hours a day."

" A *horse power* in machinery is estimated at 33000 lbs. raised 1 foot in a minute; but as a horse can exert that force but 6 hours a day, 1 machinery horse power is equal to that of 4.4 horses."

The strength of a horse is equivalent to that of 5 men.

USEFUL TABLES.

GRAVITATION.

Art. 298. In bodies descending freely by their own weight, the velocities are as the times, and the spaces as the square of the times.

The times, then, will be 1, 2, 3, 4, &c.;
The velocities, then, will be 1, 2, 3, 4, &c.;
The spaces passed through as 1, 4, 9, 16, &c.;
And the spaces for each time as 1, 3, 5, 7, 9, &c.

A body falling freely will descend through 16.0833 feet in the first second of time, and will then have acquired a velocity which will carry it through 32.166 feet in the next second.

TABLE *exhibiting the Relation of Time, Space, and Velocities.*

Seconds from the beginning of the descent.	Velocity acquired at the end of that time.	Squares.	Space fallen through in that time.	Spaces.	Space fallen through in the last second of the fall.
1	32.166	1	16.08	1	16.08
2	64.333	4	64.33	3	48.25
3	96.5	9	144.75	5	80.41
4	128.665	16	257.33	7	112.58
5	160.832	25	402.08	9	144.75
6	193.	36	579.	11	176.91
7	225.166	49	788.08	13	209.08
8	257.333	64	1029.33	15	241.25
9	289.5	81	1302.75	17	273.42
10	321.666	100	1608.33	19	305.58
11	353.832	121	1946.08	21	337.75
12	386.	144	2316.	23	369.92

To find the Area of a Regular Polygon, when the side only is given.

Rule.—Multiply the square of the side by the multiplier opposite to the name of the polygon in the following table, and the product will be the area.

No. of sides.	Names of Polygon.	Angle.	Angle of Polygon.	Area.	A	B	C
3	Trigon	120°	60°	0.433012	2.	1.732	.5773
4	Tetragon	90	90	1.000000	1.41	1.414	.7071
5	Pentagon	72	108	1.720477	1.238	1.175	.8506
6	Hexagon }	60	120	2.598076	1.156	= Radius	{ = l'gth of side
7	Heptagon	$51\frac{3}{7}$	$128\frac{4}{7}$	3.633912	1.11	.8677	1.152
8	Octagon	45	135	4.828427	1.08	.7653	1.3065
9	Nonagon	40	140	6.181824	1.06	.6840	1.4619
10	Decagon	36	144	7.694208	1.05	.6180	1.6180
11	Undecagon	$32\frac{8}{11}$	$147\frac{3}{11}$	9.365640	1.04	.5634	1.7747
12	Dodecagon	30	150	11.196152	1.037	.5176	1.9318

Additional uses of the foregoing Table.

The third and fourth columns of the table will greatly facilitate the construction of these figures, with the aid of the sector. Thus, if it is required to describe an *octagon*, opposite to it, in column third, is 45; then with the chord of 60 on the sector as radius, describe a circle, taking the length 45 on the same line of the sector; mark this distance off on the circumference, which, being repeated around the circle, will give the points of the sides.

The fourth column gives the angle which any two adjoining sides of the respective figures make with each other.

Take the length of a perpendicular drawn from the centre to one of the sides of a polygon, and multiply this by the numbers in column A, the product will be the radius of the circle that contains the figure.

The radius of a circle multiplied by the number in column B, will give the length of the side of the corresponding figure which that circle will contain.

The length of the side of a polygon multiplied by the corresponding number in the column C, will give the radius of the circumscribing circle.

Note.—1. diameter × .8862 = side of an equal square.
2. circumference × .2821 = " " "
3. diameter × .7071 = " of the inscribed square.
4. circumference × .2251 = " " "
5. area × .9003 = " " "
6. side of a square × 1.4142 = diam. of its circums. circle.
7. " " × 4.443 = circum. " "
8. " " × 1.128 = diam. of an equal circle.
9. " " × 3.545 = circum. " "
10. square inches × 1.273 = round inches.

TABLE *showing the Weights, Evaporative Powers per Weight, Bulk and Character of Fuels, from Report of Professor Walter R. Johnson*, 1844.

Designation of Fuel.	Specific gravity.	Weight per cubic foot.	Lbs. of steam from water at 212° by 1 *lb.* of fuel.	Lbs. of steam from water at 212° by 1 *cub. ft.* of fuel.	Weight of clinker from 100 lbs. of coal.	Number of cubic feet required to stow a ton.
BITUMINOUS.						
Cumberland, *maximum* .	1.313	52.92	10.7	573.3	2.13	42.3
" *minimum* .	1.337	54.29	9.44	532.3	4.53	41.2
Blossburgh	1.324	53.05	9.72	522.6	3.40	42.2
Midlothian, *screened* . .	1.283	45.72	8.94	438.4	3.33	49.
" *average* . .	1.294	54.04	8.29	461.6	8.82	41.4
Newcastle	1.257	50.82	8.66	453.9	3.14	44.
Pictou	1.318	49.25	8.41	478.7	6.13	45.
Pittsburgh	1.252	46.81	8.20	384.1	.94	47.8
Sydney	1.338	47.44	7.99	386.1	2.25	47.2
Liverpool	1.262	47.88	7.84	411.2	1.86	46.7
Clover Hill	1.285	45.49	7.67	359.3	3.86	49.2
Cannelton, Ia.	1.273	47.65	7.34	360.	1.64	47.
Scotch	1.519	51.09	6.95	369.1	5.63	43.8
ANTHRACITE.						
Peach Mountain . . .	1.464	53.79	10.11	581.3	3.03	41.6
Forest Improvement . .	1.477	53.66	10.06	577.3	.81	41.7
Beaver Meadow, No. 5 .	1.554	56.19	9.88	572.9	.60	39.8
Lackawanna	1.421	48.89	9.79	493.	1.24	45.8
Beaver Meadow, No. 3 .	1.610	54.93	9.21	526.5	1.01	40.7
Lehigh	1.590	55.32	8.93	515.4	1.08	40.5
COKE.						
Natural Virginia . . .	1.323	46.64	8.47	407.9	5.31	48.3
Midlothian	. . .	32.70	8.63	282.5	10.51	68.5
Cumberland	. . .	31.57	8.99	284.	3.55	70.9
WOOD.						
Dry Pine Wood	. . .	21.01	4.69	98.6	. . .	106.6

The above table exhibits the ultimate effects. As a safe estimate for practical values, a deduction (for the coals) of $\frac{14}{100}$ should be made.

BRICKS, LATHS, ETC.

Dimensions.

Common brick . . . . 8 to 7¾ × 4¼ × 2¼ inches.
Front brick . . . . 8¼ × 4⅛ × 2⅜ "

20 common bricks to a cubic foot, when laid;
15 " " " a foot of 8 inch wall, when laid.

Laths are 1¼ to 1½ inches by four feet in length, are usually set one *quarter of an* inch apart, and a bundle contains 100.

Stourbridge fire-brick, 9⅝ × 4⅝ × 2⅝ inches.

WEIGHTS OF VARIOUS SUBSTANCES.

	Cubic foot in pounds.	Cubic inch in pounds.
Cast Iron	450.55	.2607
Wrought Iron	486.65	.2816
Steel	489.8	.2834
Copper	555.	.32118
Lead	708.75	.41015
Brass	537.75	.3112
Tin	456.	.263
White Pine	29.56	.0171
Yellow Pine	33.81	.019
White Oak	45.2.	.026
Live Oak	70.	.040
Salt Water (sea)	64.3	.03721
Fresh Water	62.5	.03616
Air	.07529	—
Steam	.03689	—

Weights of a Cubic Foot of various Substances in ordinary use.

	Pounds.		Pounds.
Loose earth or sand	95	Clay and Stones	160
Common soil	124	Cork	15
Strong soil	127	Tallow	59
Clay	135	Brick	125

HAY.

10 cubic yards of meadow hay weigh a ton. When the hay is taken out of large or old stacks, 8 to 9 yards will make a ton.

11 to 12 cubic yards of clover, when dry, weigh a ton.

HILLS IN AN ACRE OF GROUND.

40 feet apart	27 hills.	8 feet apart	680 hills
35 " "	35 "	6 " "	1210 "
30 " "	48 "	5 " "	1742 "
25 " "	69 "	3½ " "	3556 "
20 " "	108 "	3 " "	4840 "
15 " "	193 "	2½ " "	6969 "
12 " "	302 "	2 " "	10890 "
10 " "	435 "	1 " "	43560 "

TABLE OF GOLD AND SILVER COINS.

Art. 299. Coins of all nations, met with in circulation, that have been issued within a century; showing their real worth in dollars and cents, also their commercial value in trade.

Denomination.		Dates.	Reign.	$ c.
		AUSTRIA.		
Gold	Ducat	1762 to 1838	Various	2.27
"	Do. Quadruple	1830	Do.	9.13
"	Sovereign	1778 to 1838	Do.	6.75
Silver	Rix Dollar	1753 to 1840	Do.	.97
"	Florin	1788 to 1840	Do.	.49
"	Brabant Coin	1753 to 1790	Francis II.	1.07
		BRAZIL.		
Gold	Moidore	1779 to 1819	Various	4.92
"	Half Joe	1822 to 1838	Peter I. and II.	8.72
Silver	964 Reis	1810 to 1826	John VI. and Peter I.	1.08
"	1200	1837	Peter II.	.994
		GREAT BRITAIN.		
Gold	Guinea (halves &)	1719 to 1760	George I. to III.	5
"	Do. } quarters }	1760 to 1785	Do. III.	5.026
"	Do. (in pro.)	1785 to 1813	Do. "	5.055
"	Seven Shillings	1806 to 1813	Do. "	1.65
"	Sovereigns (halves	1817 to 1829	Do. III. and IV.	4.83
"	Do. } in	1831 to 1836	William IV.	4.85
"	Do. (pro.	1836 to 1839	Victoria	4.861
"	Double do.	1826	George IV.	9.719
Silver	Shilling	1721 to 1840	Various	.22
"	Crown	1822	George IV.	1.09
"	Token 5 shilling	1804	Bank of England	.997
"	Do. 6 " Irish	1804	" Ireland	.992
		BRUNSWICK.		
Gold	X Thaler	1745	Charles	7.812
"	" do.	1813 to 1838	Various	7.90
		DENMARK.		
Gold	Christian d'or.	1775	Christian VII.	4.014
"	Double Frederick d'or	1811 to 1839	Frederick VI.	7.882
"	Frederick d'or	1813 to 1839	Do.	3.932
		FRANCE.		
Gold	Louis d'or	1776 to 1773	Louis XV.	4.79
"	Double do.	1744	Do.	9.717
"	Louisd'or	1786 to 1792	Louis XVI.	4.516
"	Double do.	" "	Do.	9.119
"	Napoleon of 20 f.	1803 to 1814	Napoleon	3.841
"	Double 40 f.	" "	Do.	7.685
"	Forty Franc	1814 to 1840	Various	7.70
"	Twenty do.	" "	Do.	3.85
Silver	Crown	1726 to 1773	Louis XV.	1.081
"	Half do.	" "	Do.	.521
"	Crown (half in pro.)	1774 to 1792	Louis XVI.	1.091
"	Six Livres	1793	Louis XVI. and Rep.	1.093
"	Five Franc	1803 to 1838	Various	.932
		HESSE.		
Gold	Ten Thaler	1773 to 1785	Frederick II.	7.742
"	Five do.	1771 to 1817	Various	3.88

	Denomination.	Dates.	Reign.	$ c.
		MEXICO.		
Gold	Doubloon	Promiscuous dates, coined at eight different mints, varying in value from $15.444 to $15.679, average		15.534
Silver	Dollars or 8 Reals	Promiscuous dates, coined at four different mints, varying in value from $00.94 to $1.0105, average		1.003
	Parts in proportion	Will average short weight		
		NETHERLANDS.		
Gold	Ducat	1770 to 1840	Various	2.258
"	Ten Guilders	1816 to 1839	William I.	4.007
"	Five do.	do.	Do.	1.994
Silver	Ducatoon and halves	1766 to 1795	William, Stadth	1.263
"	Rix Dollar	1806	Louis Napoleon	1.033
"	Guilder	Various	William I. and V.	.40
		PORTUGAL.		
Gold	Joannese	1730	John V.	17.24
"	Half Joe	in propor'n	Various	8.65
"	Moidore	1714 to 1726	John X.	6.488
"	Crown halves	1838	Maria	5.812
		PRUSSIA.		
Gold	Frederickd'or	1752 to 1812	Fred. William	3.95
"	Double do.	1800 to 1811	Do. III.	7.928
"	Do. do.	1831	Do.	7.972
		RUSSIA.		
Gold	Imperial	1756 to 1762	Elizabeth&Catharine	9.97
"	Do.	1766 to 1781	Catherine II.	7.842
Silver	Double, half in Pro.	1722 to 1819	Various average	.80
		SAXONY.		
Gold	Double Augustd'or	1784 to 1826	Fred. Augustus	7.90
"	Do. do.	1837	Do. VI.	7.946
		SPAIN.		
Gold	Doubloon and halves	1789 to 1808	Charles VI. America	15.59
"	Pistoles or quarters	1813 to 1822	Ferdinand	3.90
"	Doubloons	1751	Ferdinand VI.	16.265
"	Escudo or eighths	1809 to 1820	Do. do.	1.906
Silver	Cob Dollar	1736 to 1770	Philip and Chas.	1.01
"	Globe do.	1736 to 1771	Do.	1.008
"	Pillar'd Dollar	1772 to 1825	Chas. Ferdinand	1.
"	Pistareen	1707 to 1833	Various average	.185

Doubloons, to command the nominal par $15.60, should, and generally do, weigh the same as a dollar, 17 dwt. 8½ gr.

UNITED STATES OF N. A.

Gold Eagles and Pts. old Liberty cap, previous to 1834	$10.66⅔
Carolina and Georgia Gold.	
" 5 $ do. before 1834	$5.28 to 5.39
" 5 " " since 1834, N. C. gold	4.89 to 4.93
" 5 " " " " Carolina gold	4.82 to 4.93
" 5 " " " " Georgia "	4.70 to 4.94
" One dollar, N. C. "	.94 to .97

FORMS OF BILLS OF EXCHANGE, DRAFTS, NOTES, &c.

BILLS OF EXCHANGE.

Art. 306. A Foreign Bill of Exchange is a written order from a party in one country addressed to another party in a different country, to pay a certain sum of money, at a specified time.

A Set of Exchange is three copies of the same bill, all of the same tenor and date, excepting the words, "first," "second," "third," by which they are distinguished from each other.

Exchange for £5000. *New York, October* 1, 1847.

Three months after date, pay this our first of exchange (second and third of the same tenor and date not paid) to the order of Brown, Brothers & Co., Five Thousand Pounds sterling, value received, as advised.

Edward Collins & Co.

Messrs. Samuel Johns & Co., }
Merchants, Liverpool. }

Exchange for £5000. *New York, October* 1, 1847.

Three months after date, pay this our second of exchange (first and third of the same tenor and date not paid) to the order of Brown, Brothers & Co., Five Thousand Pounds sterling, value received, as advised.

Edward Collins & Co.

Messrs. Samuel Johns & Co., }
Merchants, Liverpool. }

Exchange for £5000. *New York, October* 1, 1847.

Three months after date, pay this our third of exchange (first and second of the same tenor and date not paid) to the order of Brown, Brothers & Co., Five Thousand Pounds sterling, value received, as advised.

Edward Collins & Co.

Messrs. Samuel Johns & Co., }
Merchants, Liverpool. }

FORMS OF DRAFTS.

A Draft is an order drawn on some person, directing him to pay to a third person, named in the draft, or to the order of said third person, a specified sum of money, either at sight, or at the expiration of some specified time.

$175. *Boston, September* 1, 1847.

At sight, pay to the order of James French, One Hundred and Seventy-five Dollars, value received, and oblige

Your Obedient Servant,
William Edwards.

Messrs. James B. Eldridge & Co., }
Merchants, New York. }

$350. *Salem, September* 1, 1847.

Thirty days after date, pay to the order of William Briggs, Three Hundred and Fifty Dollars, value received, and charge the same to the account of Your Obedient Servant,

Benjamin Billings.

Messrs. J. K. Mills & Co.,
Merchants, Boston.

FORMS OF PROMISSORY NOTES.

A **Promissory Note** is an engagement, in writing, to pay a given sum of money at a specified time, to a person named in the note, or to his order.

Negotiable Note. No. 1.

$575. *Boston, September* 1, 1847.

For value received, I promise to pay William Andrews, or order, Five Hundred and Seventy-five Dollars, on demand, with interest.

Francis Paywell.

Negotiable Note. No. 2.

$320. *Boston, September* 4, 1847.

Six months after date, I promise to pay James French, or order, Three Hundred and Twenty Dollars, for value received.

Francis Paywell.

Negotiable Note. No. 3.

$500. *Boston, September* 10, 1847.

For value received, we, jointly and severally, promise to pay Henry Johnson, or order, Five Hundred Dollars, in ninety days from date, with interest.

Francis Paywell,
Richard Paywell.

Note Payable at a Bank.

$650. *Boston, October* 1, 1847.

For value received, I promise to pay to the order of Benjamin Dutton, Six Hundred and Fifty Dollars, at the Merchants' Bank, in sixty days from date.

Henry Paywell.

Note for Borrowed Money.

$50. *Boston, October* 5, 1847.

Borrowed and received of Thomas B. Wales, Fifty Dollars, which I promise to pay on demand, with interest.

Richard Paywell.

FORMS OF ORDERS AND RECEIPTS.

An Order is a written request to pay a specified sum of money, or amount of merchandise, to the person in whose favor the order is drawn, on the presentation of the order.

Order for Money.

Salem, *September* 1, 1847.

Mr. Samuel Carter,

Please to pay John R. Poor, or order, Thirty Dollars, value received, and charge the same to the account of

$30. Your Obedient Servant,

John W. Carleton.

Order for Goods.

Boston, *September* 1, 1847. — Mr. E. Dana, please to deliver to Mr. B. Bennett, Goods to the amount of Seventy-five Dollars, and charge the same to the account of

$75. Your Obt. Servt.

Thomas Shaw.

Receipt for Money on Account.

Boston, *September* 1, 1847. — Received of J. Colby, One Hundred Dollars on account. Asa Richardson.

$100.

Receipt for Money on a Note.

Boston, *October* 1, 1847. — Received of J. Robinson, Seventy-five Dollars, which is endorsed on his note of January 1, 1847.

$75. John Stevens.

Receipt in full of all Accounts.

Boston, *October* 1, 1847. — Received of B. Bennett & Co., One Hundred and Twenty Dollars, in full of all accounts.

$120. Charles Norris.

Receipt for Money received for Another Person.

Boston, *October* 1, 1847. — Received of J. Williams, Fifty Dollars, for account of T. Newman.

For T. Newman,

$50. Charles Johnson.

Receipt for Rent.

Boston, *October* 1, 1847. — Received of R. Parker, Fifty Dollars, in full for one quarter's Rent of House No. 158 Washington Street, ending this day.

$50. William Lovering.

Leavitt's Fourth Reader.
12mo, [illegible] pages, embossed back, cloth sides, each, ... 50

Robinson's American Arithmetic.
Sheep, ... 50

Robinson's Primary School Arithmetic.
Sheep, ... 33

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By Rev. G. B. Perry, D.D. Cloth, ... 20

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Sheep, ... 50

Bliss's Geography of New England.
Sheep, ... 25

Bliss's Outline Map of New England.
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By Rev. A. [illegible] [illegible], ... [illegible]

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☞ The best work of the kind ever issued from the American press. [illegible] copies have been published.

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By S. W. Cole, author of the "American Veterinarian." Full sheep, ... [illegible]
THE SAME, fancy cloth, gilt back and sides, ... [illegible]
☞ This is undoubtedly one of the most valuable works on the subject ever published in this country. [illegible] copies were published the first three months.
☞ We have been induced, in order to facilitate the sale of Mr. Cole's valuable works in the Middle, Southern, and Western States, to place them upon the exchange list of our list.)

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The Art of Painting, ... 1 [illegible]
Stuart's Hints on Prophecy, ... [illegible]
Winer's Chaldee Grammar, ... [illegible]
Noehden's German Grammar, 1 [illegible]
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[illegible] vols. 8vo, ... [illegible] 00
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Selections from Joseph Hall,